Impact of magnetic fields on stellar structure and evolution

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Impact on stellar structure
Are magnetic fields strong enough to play a role in the structure of stars?

Main forces that rule hydrostatic equilibrium in stars:

- Pressure and gravity
  \[ \frac{P}{H_p} = \rho \frac{GM}{r^2} \]

- Sound speed in solar interior
  \[ c_s^2 \approx \frac{P}{\rho} \approx \frac{GM}{R} \Rightarrow c_s \approx 5 \times 10^7 \text{ cm/s} \]

Magnetic force

\[ \vec{j} \times \vec{B} \]

Lorentz, or rather Laplace?

Magnetic pressure is of same order as gas pressure when Alfvén velocity equals sound speed:

\[ c_A = \frac{B}{\sqrt{4\pi \rho}} \approx c_s \Rightarrow B \approx 2 \times 10^8 G = 2 \times 10^4 T \]
Laplace or Lorentz?

Pierre-Simon Laplace
1749 - 1827

Hendrik Lorentz
1853 - 1928

\[ d \vec{F} = I \, dl \times \vec{B} \]

force exerted by a magnetic field on an element of electric courant

\[ \vec{F} = q \vec{V} \times \vec{B} \]

force exerted by an magnetic field on a moving charged particle
Can a field of MegaGauss strength exist in the Sun?

Models built without magnetic field compared with helioseismic data

new composition (Asplund et al.)
old composition (Grevesse et al.)

observed - model sound speed

due to a field of $\approx 3$ MG?

Bahcall & coll. 2005
Can a field of MegaGauss strength exist in the Sun?

A closer look at helioseismology: frequencies are split by mag. field

→ upper limit ≈ 300 kG

Splitting coefficients expected from a 4 MG toroidal field located in the tachocline ($\Delta=0.04 R_\odot$)

same, averaged over 30 neighbouring modes

Anita, Chitre & Thompson 2000
Magnetic pressure dominates gas pressure in the surface layers

magnetic pressure: \[ P_m = \frac{B^2}{8\pi} \]

gas pressure: \[ P_g = \frac{R\rho T}{\mu} \]
Magnetic fields interfere with thermal convection
Strong magnetic fields block convective heat transport in sunspots
Strong magnetic fields suppress the convective instability

Linear instability in a unstably stratified magnetized medium

perturb by \( \xi \propto \exp[st + ik \cdot \vec{x}] \)

dispersion relation, neglecting thermal and Ohmic diffusion:

\[
s^2 = \left(\frac{k_h}{k}\right)^2 \frac{g}{H_p} \left[ \nabla - \nabla_{ad} \right] - \left(\vec{k} \cdot \vec{V}_A\right)^2
\]

Chandrasekhar; Weiss 1960’s

→ most unstable for horizontal wave-vector,
    may be stabilized by sufficiently strong horizontal field
    \( \sim 10^7 \text{G} \) below the solar CZ
    \( \sim 10^3 \text{G} \) at surface

• explains why inhomogeneities in surface composition of Ap stars
  are not smoothened out by convection

• displaces somewhat the boundary of CZ;
  effect on Li burning during PMS?
Magnetic fields couple stars to their environment
Magnetized star coupled to accretion disk

→ explains the relatively slow rotation of TT stars

Bouvier et al. 1997

Fendt 1994
Magnetized winds → strong angular momentum loss

If Sun loses matter at equator:

\[
\frac{d}{dt} I \Omega = R^2 \Omega \frac{d}{dt} M
\]

\[
\frac{d}{dt} k^2 M R^2 \Omega = R^2 \Omega \frac{d}{dt} M
\]

\[
\frac{(R^2 \Omega)_f}{(R^2 \Omega)_i} = \left[ \frac{M_f}{M_i} \right]^p
\]

\[
p = k^{-2} - 1 = 16
\]

\[
\left[ \frac{M_f}{M_i} \right] = 0.99 \quad \frac{(R^2 \Omega)_f}{(R^2 \Omega)_i} = 0.85
\]

but Sun loses matter at distance D (Alfvén radius):

\[
\frac{d}{dt} I \Omega = D^2 \Omega \frac{d}{dt} M
\]

\[
\frac{d}{dt} k^2 M R^2 \Omega = D^2 \Omega \frac{d}{dt} M
\]

\[
\frac{(R^2 \Omega)_f}{(R^2 \Omega)_i} = \left[ \frac{M_f}{M_i} \right]^p
\]

\[
D/R = 5
\]

\[
p = (D/R)^2 k^{-2} - 1 = 425
\]

\[
\left[ \frac{M_f}{M_i} \right] = 0.99 \quad \frac{(R^2 \Omega)_f}{(R^2 \Omega)_i} = 0.014
\]

Fessenkov 1949    Schatzman 1954    Schatzman 1962
Disc-coupling and mass loss by magnetized wind determine the rotation of stars

Monitor Project (Irwin et al. 2006, 2007) → the young Sun was a fast rotator
Rotational mixing in radiation zones

Meridional circulation

Classical picture: circulation is due to thermal imbalance caused by perturbing force (centrifugal, magn. field, etc.)
Eddington (1925), Vogt (1925), Sweet (1950), etc

Eddington-Sweet time

\[ t_{ES} = t_{KH} \frac{GM}{\Omega^2 R^3} \quad \text{with} \quad t_{KH} = \frac{GM^2}{RL} \]

Revised picture: after a transient phase of about \( t_{ES} \), circulation is driven by the loss (or gain) of angular momentum and structural changes due to evolution

• AM loss by wind: need to transport AM to surface → strong circulation
• no AM loss: no need to transport AM → weak circulation

shear-induced turbulence and internal gravity waves contributes to AM transport
Rotational mixing in magnetized radiation zones

Transport of angular momentum

\[ \rho \frac{d}{dt} (r^2 \sin^2 \theta \Omega) = -\nabla \cdot \left( \rho r^2 \sin^2 \theta \Omega \vec{U} \right) + \frac{\sin^2 \theta}{r^2} \partial_r \left( \rho v_r r^4 \partial_r \Omega \right) - \nabla \cdot \left( \rho r^2 F_{IGW} \right) + r \sin \theta \vec{e}_\phi \cdot \vec{L} \]

advection thru MC  turbulent diffusion  internal gravity waves  Laplace torque

Even a weak field can inhibit the transport of AM

\[ B^2 > 4\pi \bar{\rho} \frac{R^2 \Omega}{t_{AML}} \quad t_{AML}: \text{characteristic time for AM loss} \]

For  \( \bar{\rho} = 1 \text{ g/cm}^3 \quad R = 7 \times 10^{10} \text{ cm} \quad R\Omega = 10^7 \text{ cm/s} \quad t_{AML} = 10^9 \text{ yr} \)

\[ \rightarrow \quad B_{\text{crit}} \approx 20 \text{ G} \]

But the exact figure depends sensitively on the topology of magnetic field
Rotational mixing in magnetized radiation zones

Evolution of an axisymmetric field

poloidal (meridian) field

\[ \vec{B}_p = \nabla \times \vec{A}, \quad \vec{A} = A \vec{e}_\phi \]

toroidal (azimuthal) field

\[ \vec{B}_T = B_T \vec{e}_\phi \]

\[ \partial_t A + \frac{1}{s} \vec{U} \cdot \nabla(sA) = \eta \left( \nabla^2 A - \frac{A}{s^2} \right) \quad s = r \sin \theta \]

induction equations

\[ \partial_t B_T + s \vec{U} \cdot \nabla \left( \frac{B_T}{s} \right) = -B_T \nabla \cdot \vec{U} + s \vec{B}_p \cdot \nabla \Omega + \eta \left( \nabla^2 B_T - \frac{B_T}{s^2} \right) \]

\[ \Omega \text{-effect} \]

suppressed when \( \Omega \) cst on field lines of \( B_p \) (Ferraro law)

2D equations are projected on spherical harmonics
to be implemented in stellar evolution codes (thesis S. Mathis)
Rotational mixing in radiation zones

- Microscopic diffusion
- Distribution of chemical elements
- Meridional circulation
- Turbulent transport
- Magnetic field
- Convection
- Internal gravity waves
- Rotation
- Penetration, overshoot

Mathis & Zahn 2005
The solar tachocline problem

Hydrostatic and geostrophic equilibrium
conservation of angular momentum
conservation of thermal energy
Boussinesq approximation

solutions are separable: \( \Omega(r, \theta) = \Omega(r) + \sum_i \tilde{\Omega}_i(r) f_i(\theta) \)

In thin layer approximation, for \( t \gg r_0^2/K \)
\[
\frac{\partial \tilde{\Omega}}{\partial t} = -K \left( \frac{2\Omega}{N} \right)^2 \left( \frac{r_0}{\lambda} \right)^2 \frac{\partial^4 \tilde{\Omega}}{\partial r^4} + \nu \frac{\partial^2 \tilde{\Omega}}{\partial r^2}
\]

In present Sun, differential rotation would have spread down to \( r = 0.3 R_\odot \)

\( \rightarrow \) not observed - why is the tachocline so thin?

Another physical process must confine the tachocline

Anisotropic turbulence? Spiegel & Z 1992
Fossil magnetic field? Gough & McIntyre 1998
Can the tachocline be confined by a fossil field?

Numerical simulations by Sacha Brun

diff. rotation imposed at top of RZ
initial dipolar penetrates in CZ

⇒ Ferraro
Ω ~ cst on field lines of $B_{\text{pol}}$

ASH code
tuned for RZ
optimized for massively parallel machines

193x128x256
Can the tachocline be confined by a fossil field?

No: such a field eventually connects with the CZ and imprints its differential rotation on the RZ

Brun & Z 2006

initial dipolar field buried in RZ

⇒ Ferraro
Magnetic fields generate instabilities
MHD instabilities

Theoretical results, mostly by Tayler & collaborators

• A purely poloidal field is unstable to non-axisymmetric perturbations (Markey & Tayler 1973)

• A purely toroidal field is unstable to non-axisymmetric perturbations (Tayler 1973; Wright 1973; Goossens et al. 1981)

• Stable fields are probably a mix of poloidal and toroidal fields of comparable strength

• Rotation stabilizes somewhat a purely toroidal field, but it cannot suppress entirely the instability (Pitts & Tayler 1973)

Results obtained in the ideal case (no thermal and Ohmic diffusions)
MHD instabilities

Linear analysis, adding diffusion  (Acheson 1978; Spruit 1999, 2002)

Radiation zone, stable stratification
buoyancy frequency :

\[ N^2 = N_t^2 + N_\mu^2 = \frac{g}{H_P} (\nabla_{ad} - \nabla) + \frac{g}{H_P} \left( \frac{d \ln \mu}{d \ln P} \right) \]

Purely toroidal field
Alfvén frequency :

\[ \omega_A^2 = \frac{B_v^2}{4 \pi \rho s^2} \quad s = r \sin \theta \]

Diffusivities - thermal: \( \kappa \approx 10^7 \, cm^2 / s \)  Ohmic: \( \eta \approx 10^3 \, cm^2 / s \)

Perturbation, near axis: \( \xi \propto \exp(i (l s + m \varphi + n z - \sigma t)) \)

Instability for

\[ \omega_A^4 > C \Omega \eta l^2 \left[ \frac{\eta}{\kappa} N_t^2 + N_\mu^2 \right] \quad \text{Im}(\sigma) = 0 \]
\[ C = O(1) \]

Spruit’s conjectures :

• instability saturates when turbulent \( \eta \) ensures marginal stability
• turbulence operates a dynamo in radiation zone
Tayler instabilities in the solar radiation zone
(magnetic tachocline simulation)
Tayler instabilities in the solar radiation zone
(magnetic tachocline simulation, cont.)
Tayler instabilities in the solar radiation zone
(magnetic tachocline simulation)

Brun & JPZ 2006
JPZ, Brun & Mathis 2007

Poloidal field is not regenerated
→ no dynamo

Decay of poloidal field not enhanced by instability
→ no eddy diff.
→ no mixing
The dynamo loop

→ It cannot work as explained by Spruit and Braithwaite

Z, Brun & Mathis 2007
Why does Braithwaite find a dynamo?

How Braithwaite’s 2006 simulation (in black) differs from ours (in red)

boundary conditions:
- field normal to surface
- potential field

geometry

resolution
- 64x64x63
- 128x256x192

Euler, numerical dissipation: Rm=?
DNS, enhanced diffusion: Rm=10^5

was the simulation pursued long enough?
Magnetic fields may inhibit instabilities

Another example: thermohaline instability in RG
Thermohaline mixing in red giant stars

- first dredge-up
- extra mixing due to inversion of $\mu$-gradient produced by $^3\text{He}(^3\text{He},2p)^4\text{He}$

Eggleton, Dearborn & Lattanzio et al. 2006
In fact, Eggleton et al. observed **convective instability**, which occurs when
\[ \nabla > \nabla_{ad} + \frac{d\ln \mu}{d\ln P} \]
Ledoux criterion

In reality, as the \( \mu \)-gradient builds up, the first instability to arise as soon as
\[ \frac{d\ln \mu}{d\ln P} < 0 \]

is the **thermohaline instability**.

Charbonnel & Z 2007

who use Ulrich’s 1972 prescription
Such extra-mixing destroys $^3$He; which explains its Galactic abundance

However, observations show that a small fraction of stars ($\sim 4\%$) avoid this extra-mixing (Charbonnel & do Nascimento 1998)

Moreover, 2 PNe have been observed with high $^3$He abundance $\sim 10^{-3}$ (NGC 3242, J320) (Balser et al. 2006)

Our explanation: the thermohaline instability is suppressed by magn. field $\sim 10^5$G in those RGB stars that are the descendants of Ap stars (Charbonnel & Z, submitted to A&A)
Conclusions

Magnetic fields play little rôle in the structure of stars but they have an impact on their evolution

• by determining their rotation state
• by suppressing instabilities
• by interfering with mixing processes operating in RZ: rotational mixing, thermohaline mixing
• possibly by triggering MHD instabilities

Obviously, the effect depends on field strength
→ observational constraints are highly needed