SURPRISES OF THE HAPKE PHOTOMETRIC MODEL

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and

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Planetary photometry: models

Very complicated structure of planetary surfaces

Structure is multiscale;
each scale requires several parameters for description

Refractive index depend on the composition
(widely varied both regionally and locally in a microscopic scale)

Regolith particles:
different irregular shapes and different sizes (additional parameters are needed to characterize particles)

Theory has to operate with many physical parameters

Problem of uniqueness of fitting, when many parameters are used

Brightness phase curves of planets are very simple

Hapke model (H-model): main points

- Most developed and popular in planetary photometry
- Works within a framework of geometrical optics approximation
- Accounts for the shadow-hiding effect
- ***8 parameters***:
  (1) the single-scattering albedo of a medium $\boldsymbol{\omega}$,
  (2) the packing density of a regolith medium $\boldsymbol{\rho}$ ($\boldsymbol{\rho} < 1$)
  (3) the typical angle of relief inclination $\bar{\theta}$,
  (4) the amplitude of the opposition peak $\boldsymbol{B_{0s}}$,
  (5),(6) the parameters of single-particle scattering indicatrix, ***b*** and ***c***,
  (7),(8) the parameters of the effect of coherent backscattering enhancement, $\boldsymbol{B_{0c}}$ and $\boldsymbol{h_C}$
- $\boldsymbol{B_{0c}}$ and $\boldsymbol{B_{0s}}$ are actually empirical and, hence, the H-model suggests a semi-empirical description of the lunar surface

H-model: main formulas

Apparent albedo of a particulate surface with a macroscopic random topography:

$$A\left(\alpha, i, e, \omega, B_0, \rho, b, c, \bar{\theta}\right) = \frac{\omega}{4} \frac{B_{CB}\left(\alpha, B_{0c}, h_C\right)\cos e}{\cos i + \cos e} \times$$
$$\times\left(B_{SH}\left(\alpha, B_{0s}, \rho\right) p\left(\alpha, b, c\right) + M\left(i, e, \omega, b, c\right)\right) S\left(\alpha, i, e, \bar{\theta}\right)$$

Single scattering indicatrix (double Henyey-Greenstein function):

$$p\left(\alpha, b, c\right) = \frac{1+c}{2}\left[\frac{1-b^2}{\left(1+2b\cos\alpha+b^2\right)^{3/2}}\right] + \frac{1-c}{2}\left[\frac{1-b^2}{\left(1-2b\cos\alpha+b^2\right)^{3/2}}\right]$$

H-model: main formulas

Apparent albedo of a particulate surface with a macroscopic random topography:

$$A\left(\alpha, i, e, \omega, B_0, \rho, b, c, \overline{\theta}\right) = \frac{\omega}{4} \frac{B_{CB}\left(\alpha, B_{0c}, h_C\right)\cos e}{\cos i + \cos e} \times$$

$$\times \left(B_{SH}\left(\alpha, B_{0s}, \rho\right) p\left(\alpha, b, c\right) + M\left(i, e, \omega, b, c\right)\right) S\left(\alpha, i, e, \overline{\theta}\right)$$

Term describing the interparticle shadow-hiding effect:

$$B_{SH}\left(\alpha, B_{0s}, \rho\right) = 1 + \frac{B_{0s}}{1 - \frac{8}{3\ln\left(1-\rho\right)} \tan\frac{\alpha}{2}}$$

H-model: main formulas

Apparent albedo of a particulate surface with a macroscopic random topography:

$$A\left(\alpha, i, e, \omega, B_0, \rho, b, c, \bar{\theta}\right) = \frac{\omega}{4} \frac{B_{CB}\left(\alpha, B_{0c}, h_C\right)\cos e}{\cos i + \cos e} \times$$
$$\times \left(B_{SH}\left(\alpha, B_{0s}, \rho\right) p\left(\alpha, b, c\right) + M\left(i, e, \omega, b, c\right)\right) S\left(\alpha, i, e, \bar{\theta}\right)$$

Term describing the coherent backscattering enhancement:

$$B_{CB}\left(\alpha, B_{0c}, h_C\right) = 1 + B_{0c} \frac{1 + \left(1 - \exp\left(-\frac{1}{h_C}\tan\frac{\alpha}{2}\right)\right) \Big/ \frac{1}{h_C}\tan\frac{\alpha}{2}}{2\left(1 + \frac{1}{h_C}\tan\frac{\alpha}{2}\right)^2}$$

$$h_C = \frac{\lambda \sigma n Q_S}{4\pi}$$

λ - wavelength, n - number of particles per unit volume, σ - mean particle cross-sectional area, Q_S - mean particle scattering efficiency

H-model: main formulas

Incoherent multiple scattering term :

$$M(i,e,\omega,b,c) = \Re(i,b,c)\big(H(e,\omega)-1\big) + \Re(e,b,c)\big(H(i,\omega)-1\big) +$$
$$+ \Re_0(b,c)\big(H(e,\omega)-1\big)\big(H(i,\omega)-1\big)$$

where:

$$H(x,\omega) = \left(1 - \omega x\left(\frac{1-\sqrt{1-\omega}}{1+\sqrt{1-\omega}}\right) + \frac{1}{2}\left(1 - 2x\frac{1-\sqrt{1-\omega}}{1+\sqrt{1-\omega}}\right)\ln\frac{1+x}{x}\right)^{-1}$$

$$\Re(t,b,c) = 1 + \sum_{n=1}^{15} A_n \rho(n,b,c) P_n(t) \qquad \Re_0(b,c) = 1 + \sum_{n=1}^{15} A_n^2 \rho(n,b,c)$$

$$A_n = \begin{cases} 0, & \text{if } n \text{ is even} \\ \dfrac{(-1)^{(n+1)/2}}{n}\,\dfrac{1\cdot 3\cdot 5 \ldots n}{2\cdot 4\cdot 6 \ldots (n+1)}, & \text{if } n \text{ is odd} \end{cases} \qquad \rho(n,b,c) = c(2n+1)b^n$$

Chandrasekar's function $H(x,\omega)$ represent multiple scattering for isotropic scatterers, $P_n(t)$ - Legendre functions

H-model: main formulas

The term describing shadow-hiding on the planetary surface random relief (topography) :

$$\left.\begin{aligned}
S(\alpha,i,e,\bar{\theta}) &= \frac{\mu_e(i,e,\varphi)}{\mu_e(0,e,0)}\frac{\cos i}{\mu_{0e}(i,0,\pi)}C(\bar{\theta})\left\{1-f(\varphi)\left[1-C(\bar{\theta})\frac{\cos i}{\mu_{0e}(i,0,\pi)}\right]\right\}^{-1} \\
\mu_{0e}(i,e,\varphi) &= C(\bar{\theta})\left[\cos i+\sin i\tan\bar{\theta}\,\frac{E_2(e)\cos\varphi+\sin^2(\varphi/2)E_2(i)}{2-E_1(e)-(\varphi/\pi)E_1(i)}\right] \\
\mu_e(i,e,\varphi) &= C(\bar{\theta})\left[\cos e+\sin e\tan\bar{\theta}\,\frac{E_2(e)-\sin^2(\varphi/2)E_2(i)}{2-E_1(e)-(\varphi/\pi)E_1(i)}\right]
\end{aligned}\right\} \quad \text{if } e \le i$$

$$f(\varphi)=\exp\left(-2\tan\left(\frac{\varphi}{2}\right)\right) \qquad C(\bar{\theta})=\left(1+\pi\tan\bar{\theta}\right)^{-1/2}$$

$$E_1(t)=\exp\left(-\frac{2}{\pi}\cot\bar{\theta}\cot t\right) \qquad E_2(t)=\exp\left(-\frac{1}{\pi}\cot^2\bar{\theta}\cot^2 t\right)$$

H-model: main formulas

The term describing shadow-hiding on the planetary surface random relief (topography) :

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S\left(\alpha,i,e,\bar{\theta}\right) &= \frac{\mu_e(i,e,\varphi)}{\mu_e(0,e,\pi)}\frac{\cos i}{\mu_{0e}(i,0,0)}C\left(\bar{\theta}\right)\left\{1-f(\varphi)\left[1-C\left(\bar{\theta}\right)\frac{\cos e}{\mu_e(0,e,\pi)}\right]\right\}^{-1} \\
\mu_{0e}(i,e,\varphi) &= C\left(\bar{\theta}\right)\left[\cos i+\sin i\tan\bar{\theta}\,\frac{E_2(i)-\sin^2(\varphi/2)E_2(e)}{2-E_1(i)-(\varphi/\pi)E_1(e)}\right] \\
\mu_{0e}(i,e,\varphi) &= C\left(\bar{\theta}\right)\left[\cos e+\sin e\tan\bar{\theta}\,\frac{E_2(i)\cos\varphi+\sin^2(\varphi/2)E_2(e)}{2-E_1(i)-(\varphi/\pi)E_1(e)}\right]
\end{aligned}\right\} \quad \text{if } i \le e$$

$$f(\varphi)=\exp\left(-2\tan\left(\frac{\varphi}{2}\right)\right) \qquad C\left(\bar{\theta}\right)=\left(1+\pi\tan\bar{\theta}\right)^{-1/2}$$

$$E_1(t)=\exp\left(-\frac{2}{\pi}\cot\bar{\theta}\cot t\right) \qquad E_2(t)=\exp\left(-\frac{1}{\pi}\cot^2\bar{\theta}\cot^2 t\right)$$

Alternative model of the photometric function?

- Lommel-Seeliger law :

$$D_{LS} = \cos i / (\cos i + \cos e).$$

"Lunar-Lambert function" (McEwen, 1996):

$$D_{LZ+L}(\alpha,\beta,\gamma) = L(\alpha)\frac{2\cos\gamma}{\cos(\gamma-\alpha)+\cos\gamma} + (1-L(\alpha))\cos\beta\cos(\gamma-\alpha)$$

Balance factor $L(\alpha)$ decreases from 1 to 0:

$$L(\alpha) = 1 + A\alpha + B\alpha^2 + C\alpha^3$$

$A = -1.9\ 10^{-2}$, $B = 2.42\ 10^{-4}$, and $C = -1.4\ 10^{-6}$

α, β, γ – phase angle, luminance longitude, luminance latitude

Alternative model of the photometric function

Photometric function of planetary surface F:

$F(\alpha,\beta,\gamma)=f(\alpha)\cdot D(\alpha,\beta,\gamma)$,

α,β,γ – phase angle, luminance longitude, luminance latitude,

$f(\alpha)$ - phase function

$D(\alpha,\beta,\gamma)$ - disk function

• $f(\alpha) \rightarrow$ complexity of the structure of light scattering surface

• $D(\alpha,\beta,\gamma) \rightarrow$ global brightness trend from the limb to terminator on the planetary disk (sphericity of the planet)

Alternative model of the photometric function

<u>Relation of (α,β,γ) to (i,e,φ),</u>

i -incidence, ε -*emergence* ,φ -azimuth angle

$$\left.\begin{aligned}
&\cos\alpha = \cos i \cos e + \sin i \sin e \cos\varphi \\
&\cos\beta = \sqrt{\frac{(\sin(i+e))^2 - \left(\cos\frac{\varphi}{2}\right)^2 \sin 2e \sin 2i}{(\sin(i+e))^2 - \left(\cos\frac{\varphi}{2}\right)^2 \sin 2e \sin 2i + (\sin e)^2 (\sin i)^2 (\sin\varphi)^2}} \\
&\cos\gamma = \frac{\cos e}{\cos\beta}
\end{aligned}\right\}$$

Alternative model of the photometric function

Semi-empirical equation:

[Akimov, Soviet Astron., 1976;19(3):385–88;
Akimov, Kinem Phys Celest Bodies, 1988;4(1):3–10;
Shkuratov et al., Icarus 1994;109:168-190;
Shkuratov et al., JOSA 2003;20(11):2081-92]

$$A(\alpha,\beta,\gamma) = A_{eq}\,\frac{e^{-\mu_1\alpha} + m e^{-\mu_2\alpha}}{1+m}\cos\frac{\alpha}{2}\cos\left(\frac{\pi}{\pi-\alpha}\left(\gamma-\frac{\alpha}{2}\right)\right)\frac{(\cos\beta)^{\alpha/2(\pi-\alpha)}}{\cos\gamma}$$

Four parameters:

μ_1 - associated with surface roughness,
m and μ_2 - the amplitude and the width of the opposition peak,
A_{eq} –equigonal albedo (at standard geometry)

Alternative model by Akimov & Shkuratov

Disk function:

$$D(\alpha,\beta,\gamma) = \cos\frac{\alpha}{2}\cos\left(\frac{\pi}{\pi-\alpha}\left(\gamma-\frac{\alpha}{2}\right)\right)\frac{\left(\cos\beta\right)^{\alpha/2(\pi-\alpha)}}{\cos\gamma}$$

Phase function:

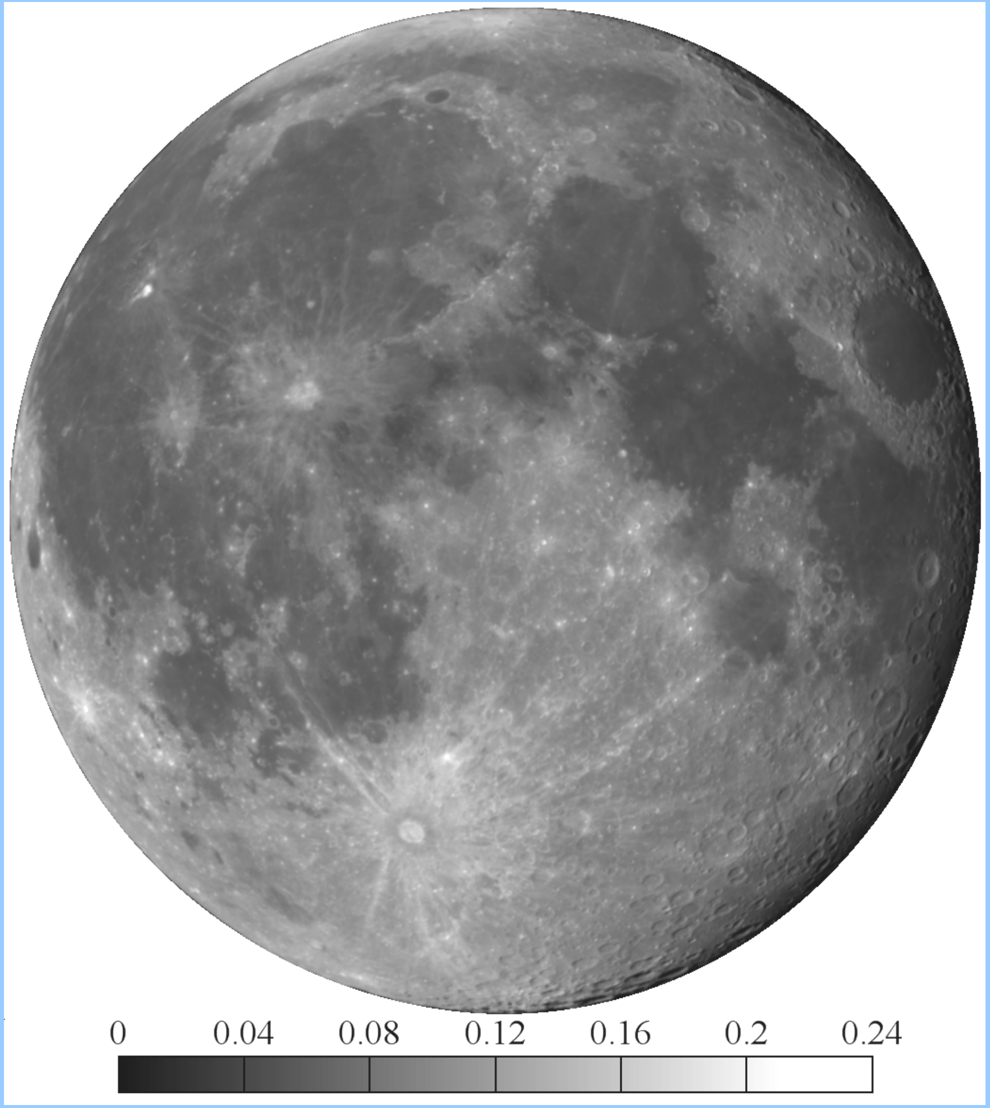
$$f(\alpha) = \frac{e^{-\mu_1\alpha} + me^{-\mu_2\alpha}}{1+m}$$

Practical importance of H-model

Test base:
set of 30 maps of the lunar apparent albedo $A(a,i,e)$
λ = 603 nm, a = 1.7–73°.

Velikodsky et al., LPSC 41-st 2010;1760 LPI Houston USA.
Velikodsky et al. Electromagnetic and Light Scattering XII, 2010, Finland, Conf. Proc. pp. 302-305.

http://astrodata.univer.kharkov.ua/moon/albedo/

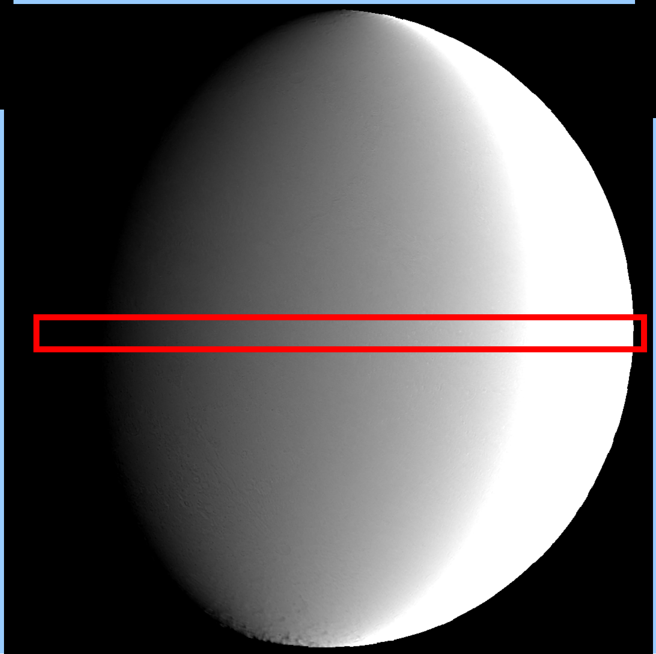


Apparent albedo, α=22.2°

Practical importance of H-model

Correction of the ratio of albedos $A(-16.2°)/A(+16.2°)$

$A(-16.2°)/A(+16.2°)$



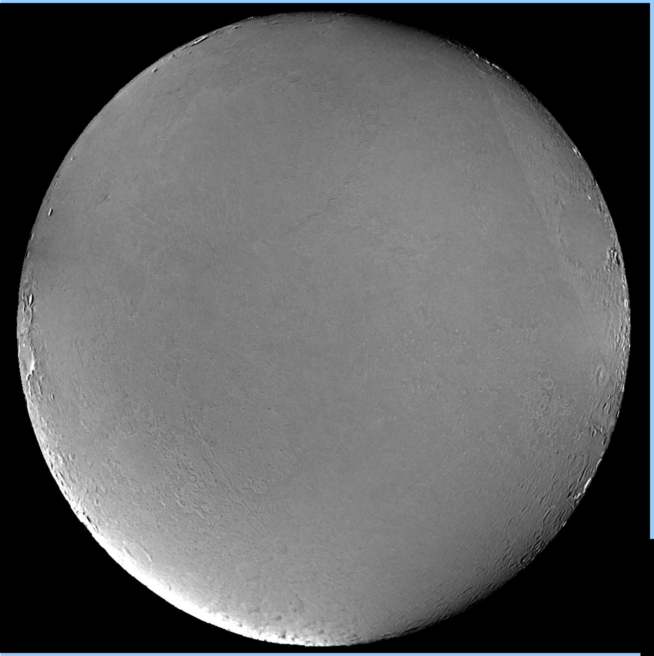
$A(-16.2°)$

$A(+16.2°)$

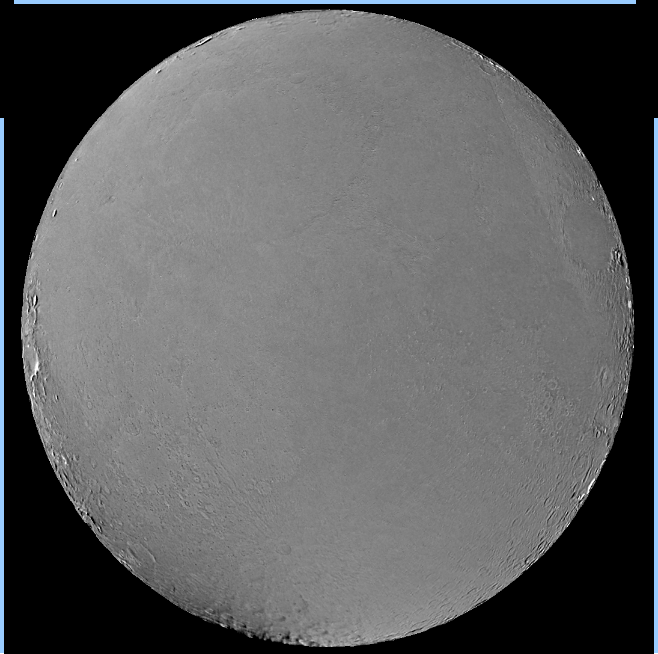
Effect of luminance longitude

Practical importance of H-model

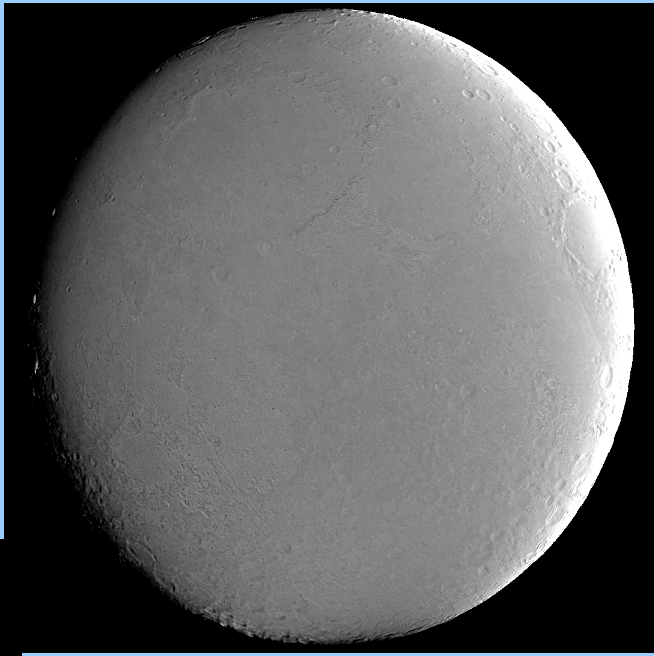
Correction of the phase ratio: results



Hapke



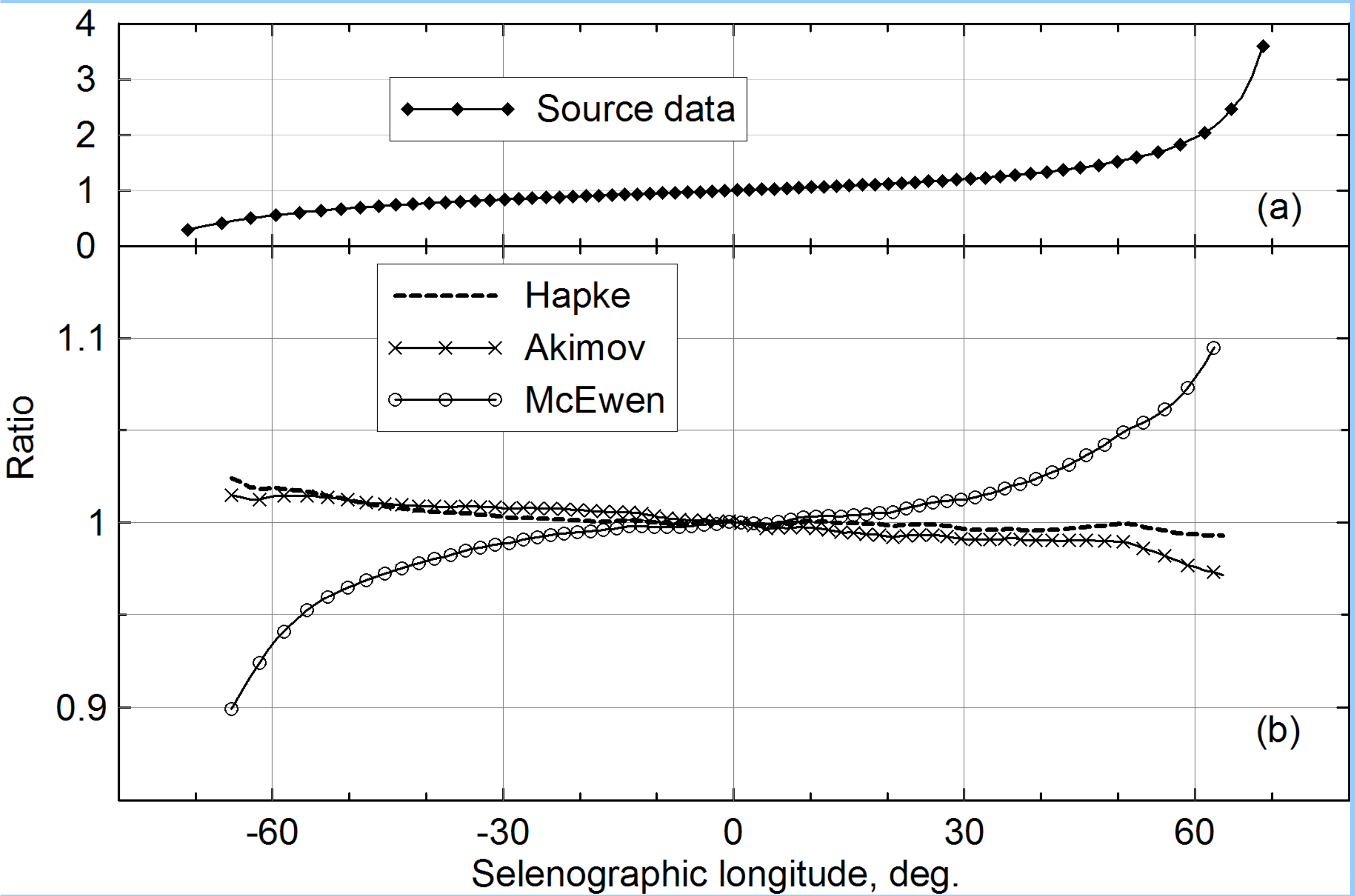
Akimov



McEwen

Practical importance of H-model

Correction of the phase ratio: results

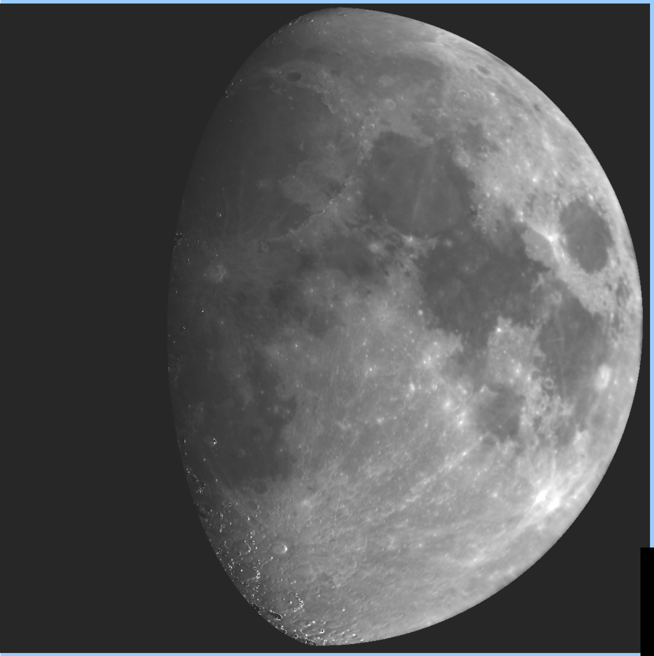


Equatorial profile of the ratio

H-model parameters: ω = 0.4, h = 0.06, B_0 = 1, and $\bar{\theta}$ = 25.5º (Helfenstein et al., Icarus 1997;128:2–14, Hillier et al., 1999;141:205–25), typical for the Moon

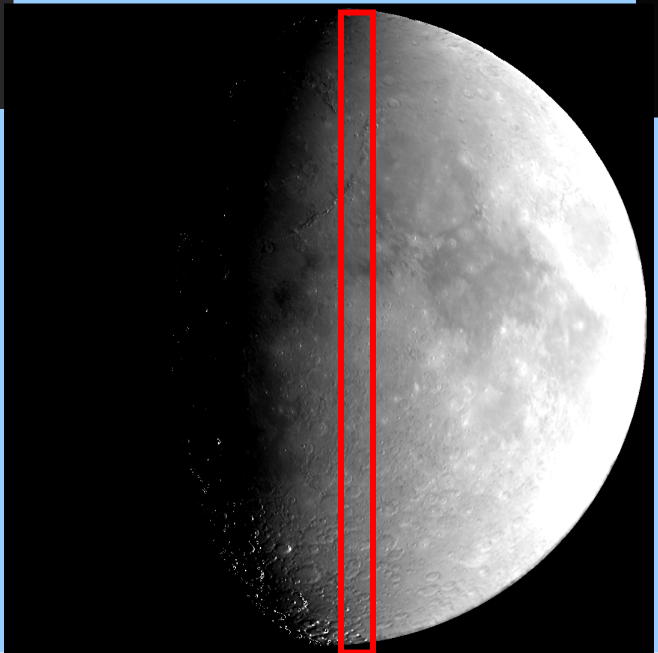
Practical importance of H-model

Correction of the ratio of albedos $A(63.3^{\circ})/A(2.7^{\circ})$



$A(63.3^{\circ})$

$A(63.3^{\circ})/A(2.7^{\circ})$

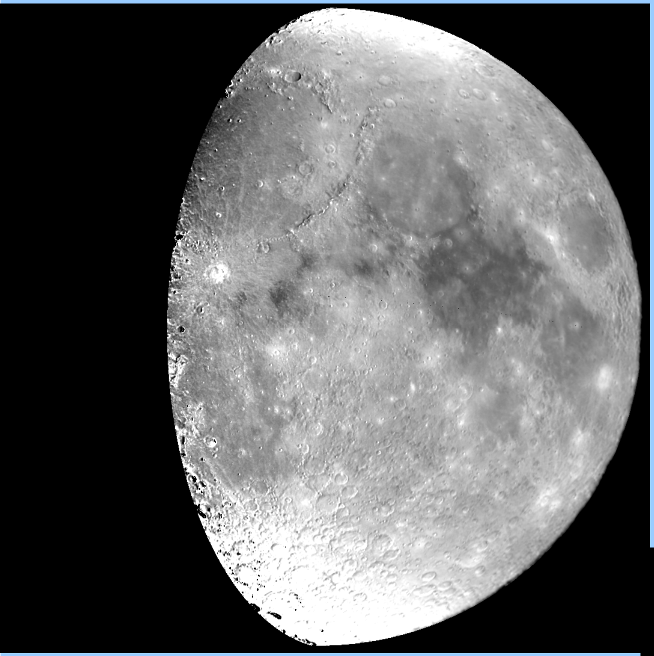


$A(2.7^{\circ})$

Effect of
luminance latitude

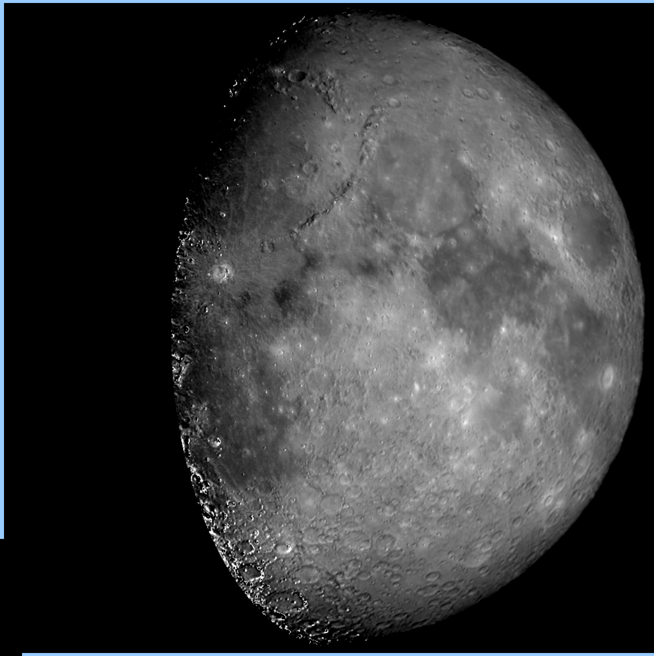
Practical importance of H-model

Correction of the phase ratio: results



Hapke

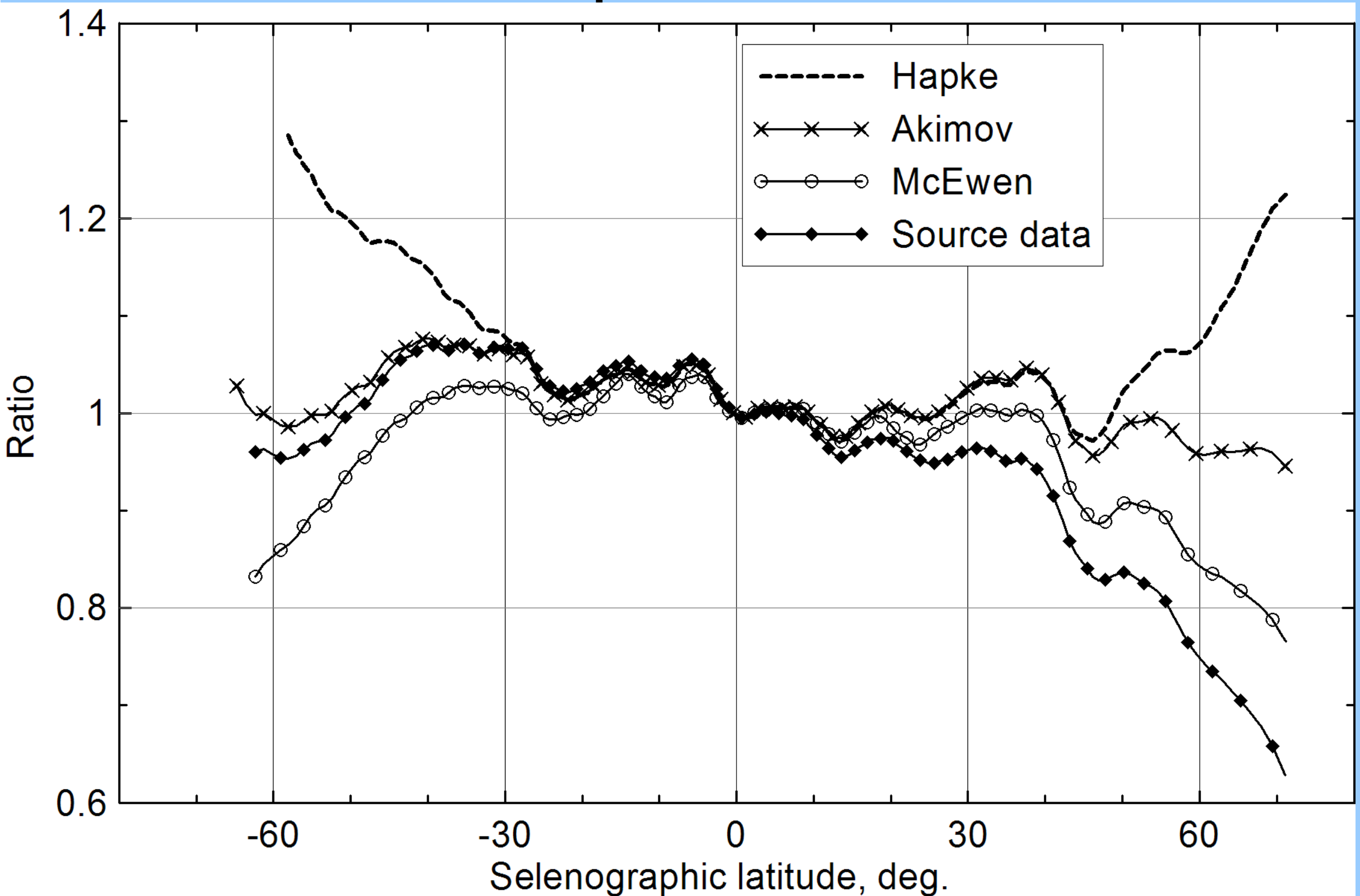
Akimov



McEwen

Practical importance of H-model

Correction of the phase ratio: results



Meridional profile of the ratio

H-model parameters: ω = 0.4, h = 0.06, B_0 = 1, and $\bar{\theta}$ = 25.5º (Helfenstein et al., Icarus 1997;128:2–14, Hillier et al., 1999;141:205–25), typical for the Moon

Mapping the parameters of H-model

Set of 30 maps of the lunar apparent albedo $A(\alpha,i,e)$
at 603 nm, $\alpha = 1.7–73°$.

↓

Fitting a theoretical curve to the observed phase dependence for each point of the lunar disk

↓

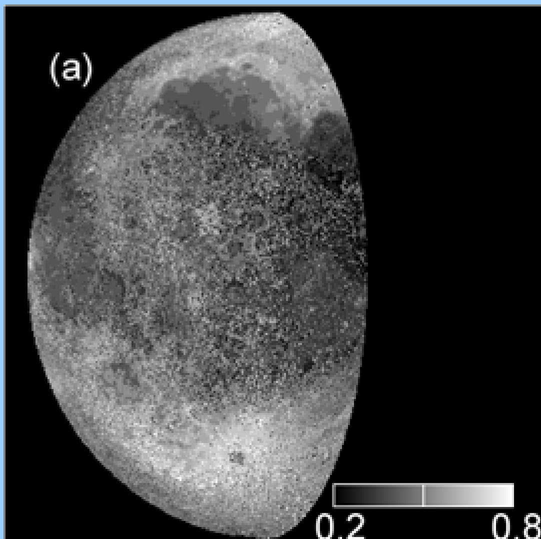
Space of 6 dimensions with homogeneous grid:

$0<\omega<1$ $0<\theta<50°$ $0<B_0<1$ $0<h<1$ $0<b<1$ $-1<c<1$

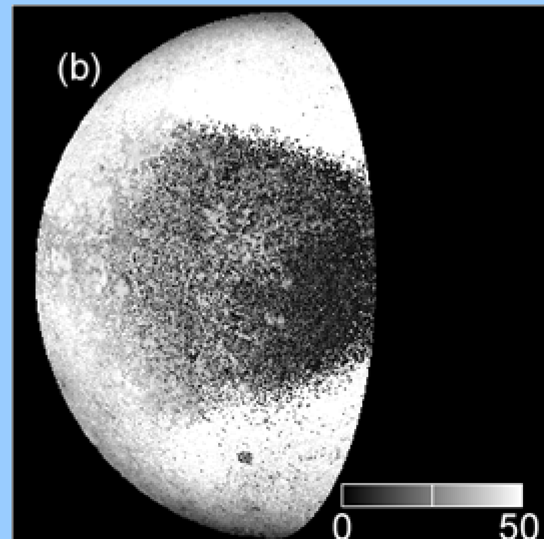
↓

Maps of model parameters,
best fitted the source data

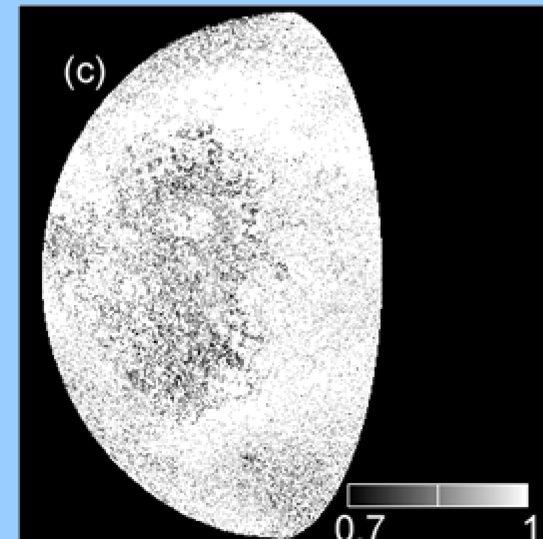
Mapping the parameters of H-model



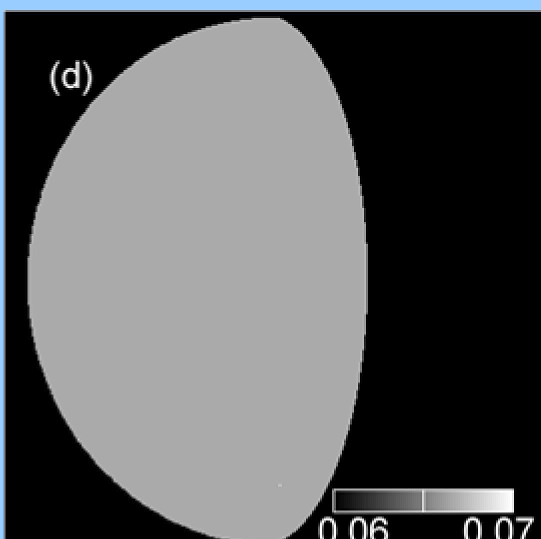
ω



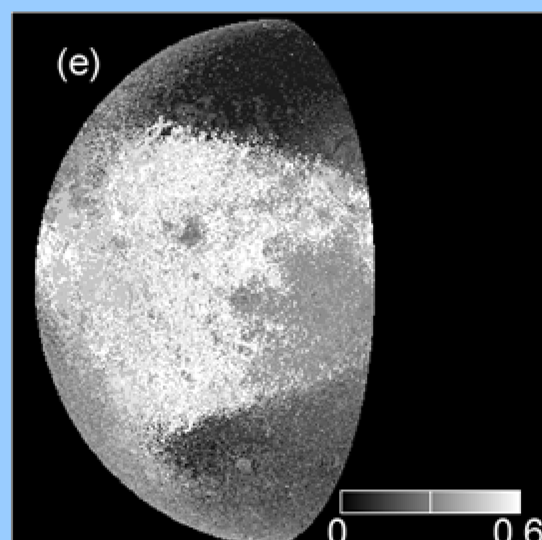
θ



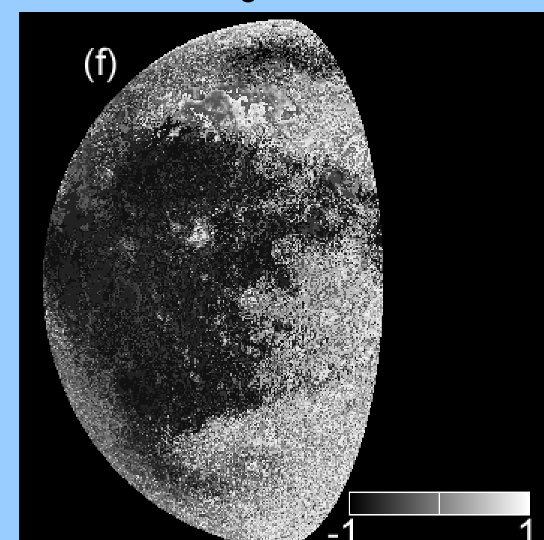
B_0



h

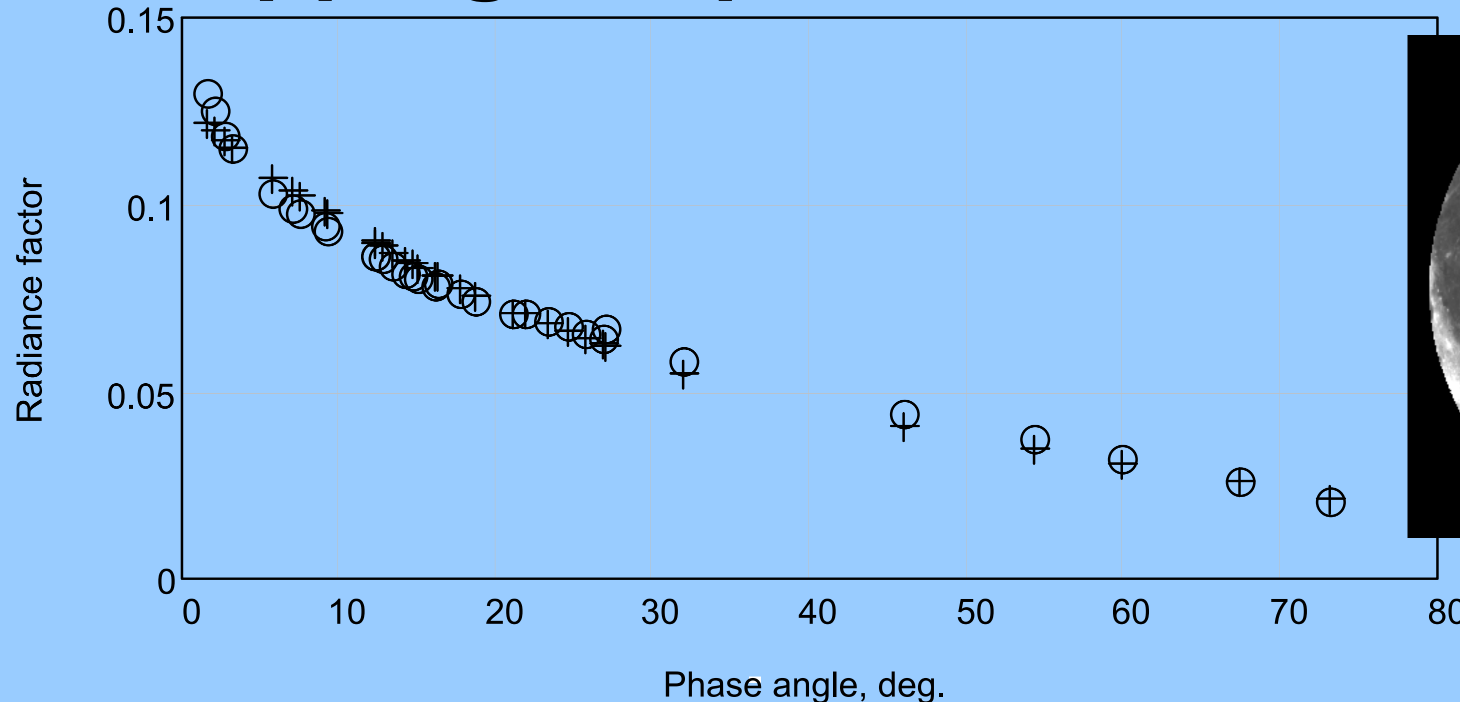


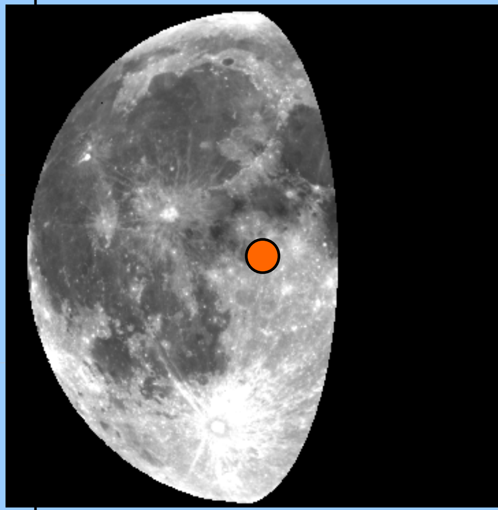
b



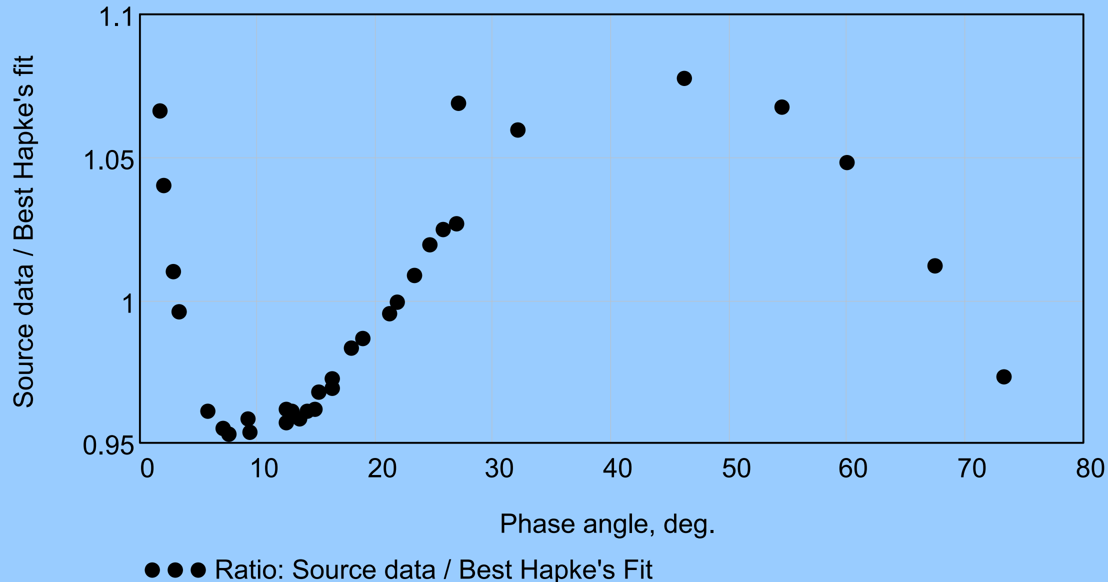
c

Mapping the parameters of H-model

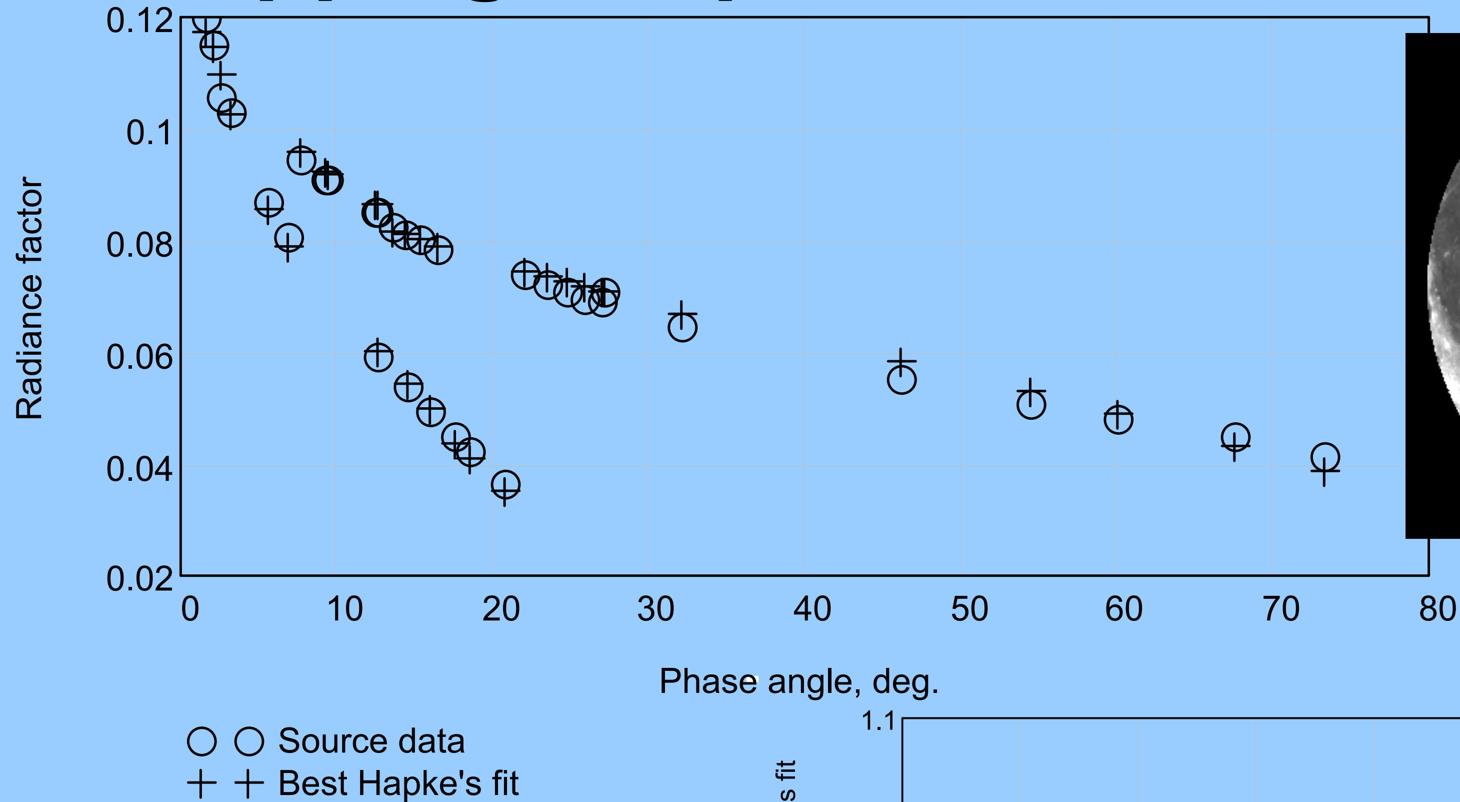


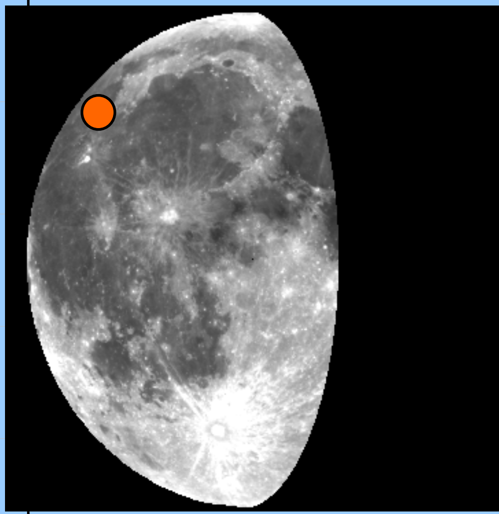


Test area:
center of nearside

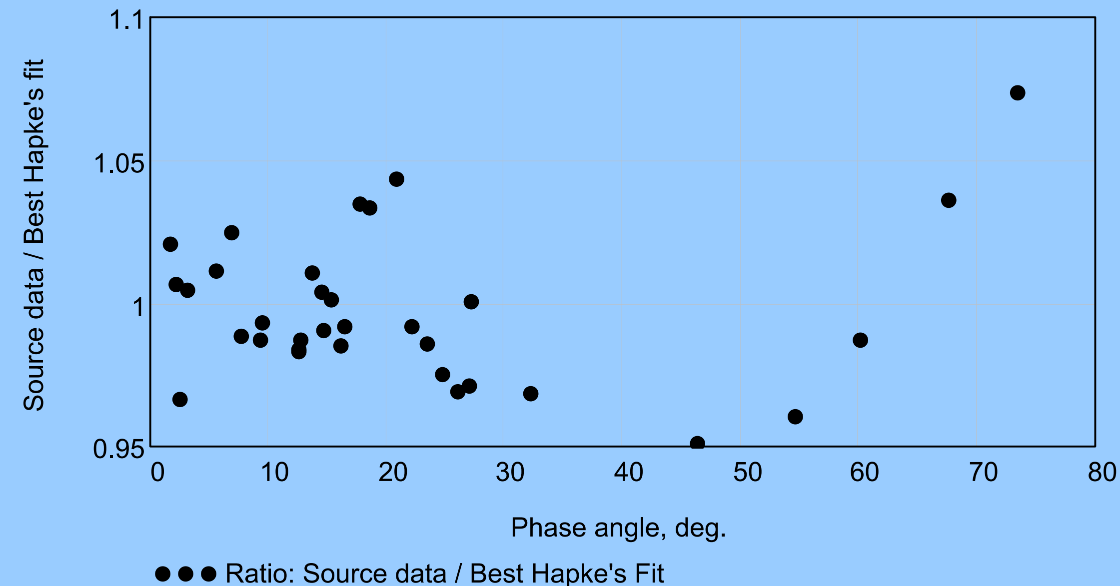


Mapping the parameters of H-model





Test area:
north-western limb
of lunar disk



Alternative model of the photometric function (A&S)

Semi-empirical equation:

[Akimov, Soviet Astron., 1976;19(3):385–88;
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Shkuratov et al., Icarus 1994;109:168-190;
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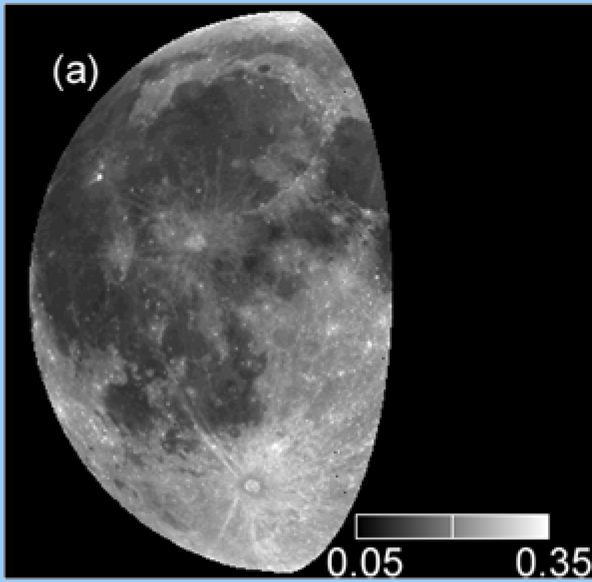
$$A(\alpha,\beta,\gamma) = A_{eq}\,\frac{e^{-\mu_1\alpha} + m e^{-\mu_2\alpha}}{1+m}\cos\frac{\alpha}{2}\cos\left(\frac{\pi}{\pi-\alpha}\left(\gamma-\frac{\alpha}{2}\right)\right)\frac{(\cos\beta)^{\alpha/2(\pi-\alpha)}}{\cos\gamma}$$

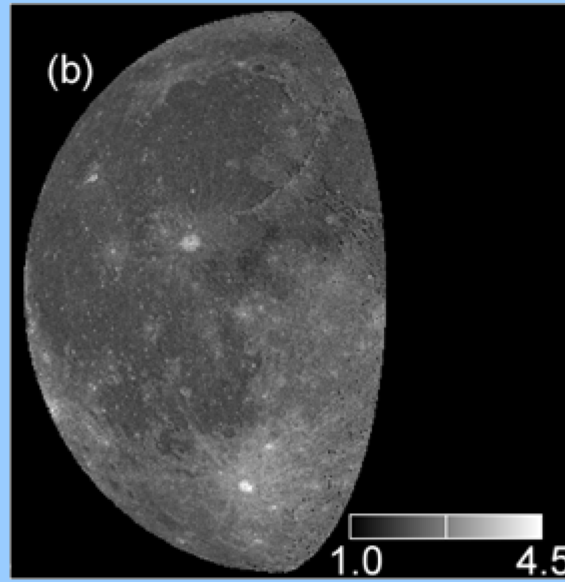
Four parameters:

μ_1 - associated with surface roughness,
m and μ_2 - the amplitude and the width of the opposition peak,
A_{eq} –equigonal albedo (at standard geometry)

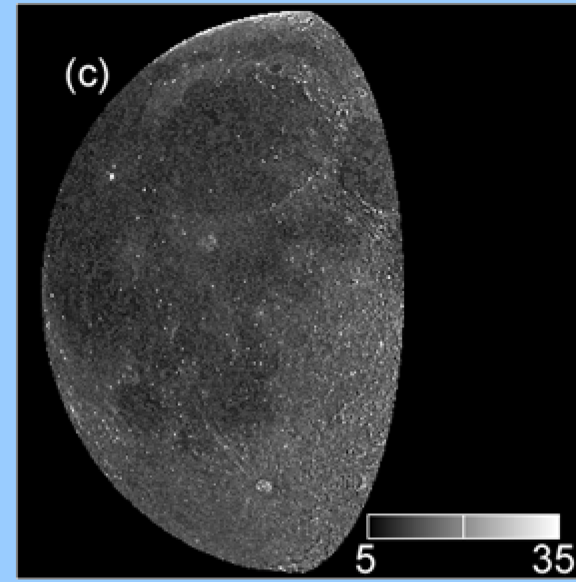
Mapping the parameters of A&S model



A_{eq}

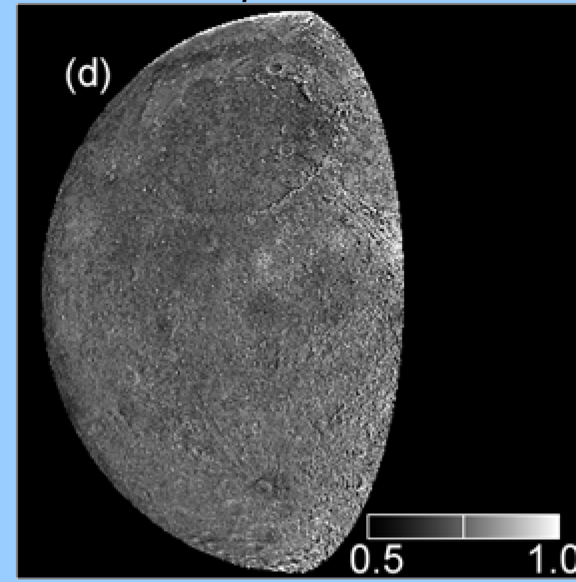


m



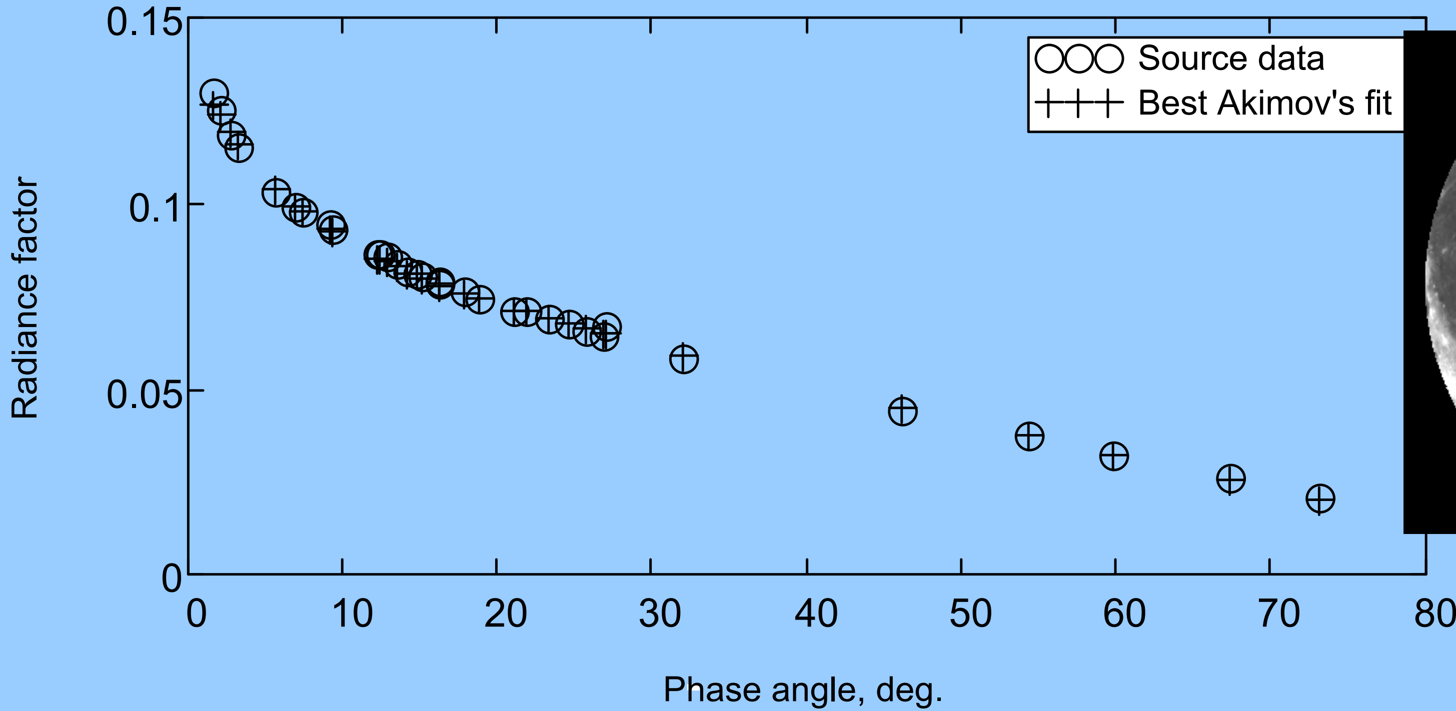
μ_1

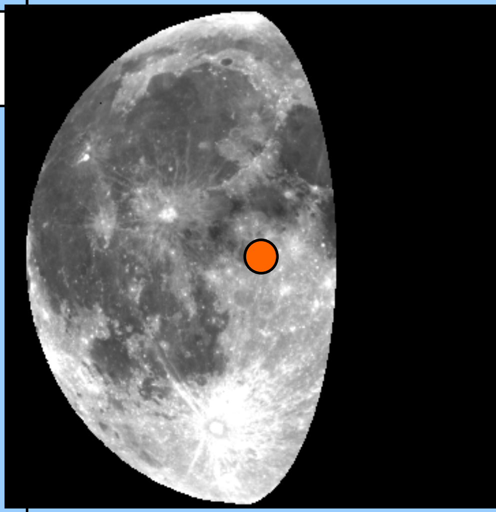
μ_1 - surface roughness,
m and μ_2 - the amplitude and the width of the opposition peak,
A_{eq} –equigonal albedo at standard geometry

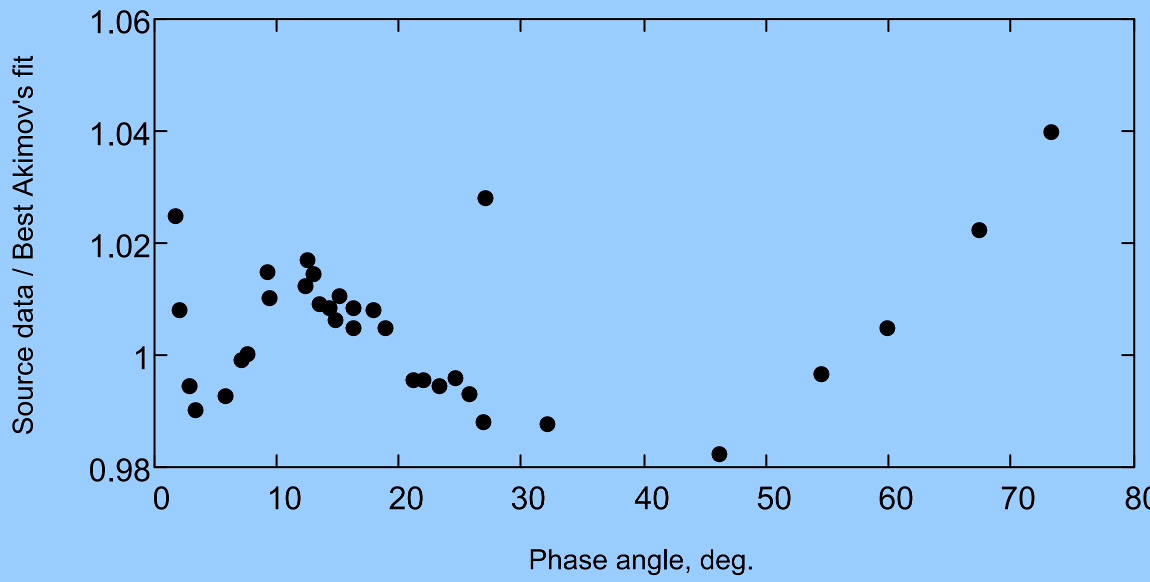


μ_2

Mapping the parameters of A&S model

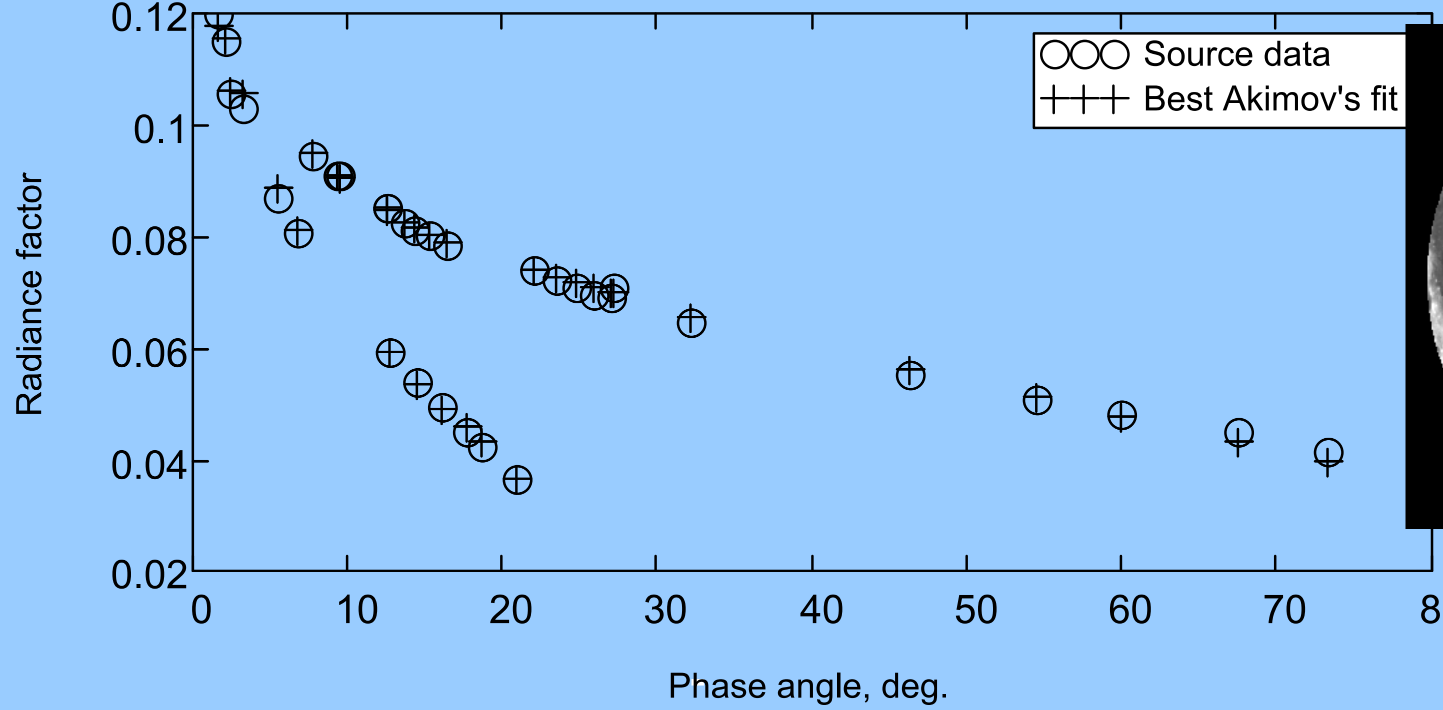


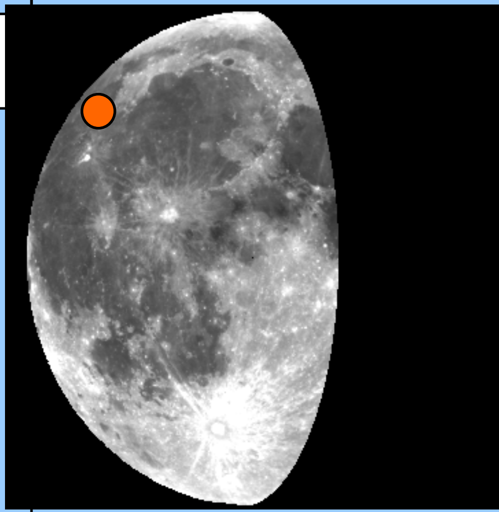




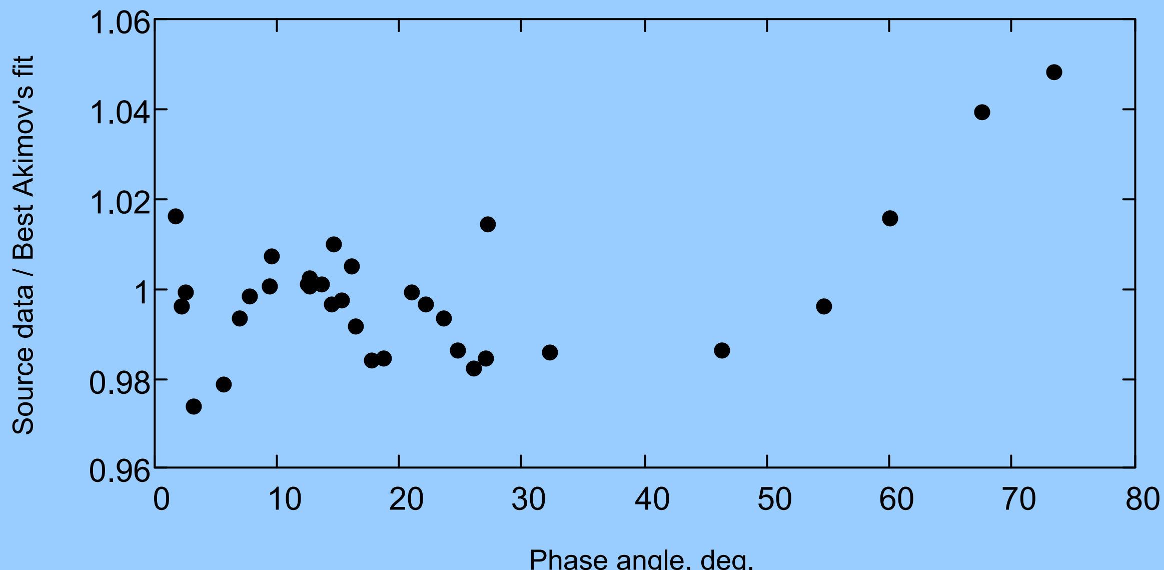
Test area:
center of nearside

Mapping the parameters of A&S model





Test area:
north-western limb
of lunar disk



Conclusion

- H-model poorly describes the latitude brightness trend

- H-model does not suggest a physically meaningful distribution of the model parameters, excepting only single-particle albedo

- Hapke parameters are mutually dependent; some of them are empirical.

- In the case of the Moon, we found very close anticorrelation between the parameters of the single-particle indicatrix *b* and *c*.