# The design of a radiotelescope: A multidisciplinary game

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**The "design" of a simple radio telescope is proposed as a multidisciplinary exercise, bringing together radioastronomy, electronics, and the theory of structures. The telescope can be built by relatively inexperienced students and used to observe Jupiter bursts as well as the quiet sun (under favorable conditions).**

## INTRODUCTION

As a lecturer in astronomy, I sometimes take my students on a tour of the radio astronomical observatory at Nançay, France. A type of question which crops up remarkably often is "Why was the telescope/antenna built this particular way, rather than any other? Why are structural elements the size they are, and not smaller or larger?" Such questions are astonishingly difficult to answer, since the design of a radio telescope brings into play elements from various formally separate domains of physics.

The exercise I shall outline (one might perhaps more accurately call it a group project . . . it cannot be done in a single session) attempts to lead the student to a better feeling of the various parameters involved in building a radio telescope; it is a multidisciplinary exercise, in that elements from quite diverse subjects (radioastronomy, electronics, mechanics of structures, etc.) are fused together to create a specific object. In effect, the student is led to design a simple radio telescope, subject to a number of "realistic" constraints; the student learns the art of finding an optimal solution to a particular problem.

The exercise is most suitable for reasonably advanced students who have already mastered (or have the basic knowledge to acquire readily) the essential notions which will be used. Less advanced students can play the same game, but will have to assume some of the rules. This is not really a drawback: in practice, an engineer *uses* standard formulas. Since the exercise is a multidisciplinary one, it need not be restricted to astronomy students; for example, an electronics class can thereby learn some astronomy and appreciate the problems of radio astronomers. Finally, if there is enough money available, the telescope can even be built: pedagogically, this is a most valuable end result, since it moves the game from a purely theoretical level to practice.

In this exercise, one important choice has been deliberately taken out of the hands of the student—"build or buy." Almost suitable antennas are apparently available in the U.S. and can be easily retuned for radio-astronomy purposes, but I am unable to make specific recommendations since I do not know the American market—interested parties will have to consult the appropriate ham radio shops. While buying such an antenna would of course enable one to obtain Jovian "squiggles" sooner, I believe that it would detract from the initial motivation of the exercise. Only by "getting one's hands dirty" can one really learn about the constraints imposed by physics, electronics, and mechanics.

The exercise can be readily divided into a number of sections: (a) the basic problem of how a radio telescope works; (b) definition of the source we wish to observe, and an investigation of how its properties define the basic electronic constraints; (c) choice of an antenna type which best satisfies these constraints, and the translation of (b) into antenna parameters; (d) search for a viable mechanical realization of (c).

To add spice to the game, we impose a certain number of "realistic" constraints: we use a standard communications receiver; we "buy" our material off the shelf; we can build or buy any supplementary electronic equipment needed; we can "spend" only up to some fixed sum of money (I spent $400 in France) for an antenna and its support.

Different students will, of course, compute different configurations, most of which will turn out to exceed the "budget" or to be insufficently sensitive. They should be allowed to go through with their analysis—this exercise is a kind of simulacrum of the world of research!

Many of the basic concepts used in this paper are explained in considerable detail in Kraus[1,2] and in Schelkunoff and Friis.[3]

## BASIC RADIO-TELESCOPE CONCEPTS

This part of the exercise is best presented by analogy with the optical case. One might begin by asking how an optical astronomer "sees" the world—the immediate answer is, of course, via a photographic plate, a television camera, or some suchlike device. In short, the optical astronomer obtains first an extended image, which he thereafter analyses.

Is this true of the radio astronomer? Why not? The discussion can be made in terms of parabolic radio antennae—this is not the kind we shall "build," but the same concepts apply and the discussion is much more straightforward.

The radio astronomer does not "see" an image directly. The parabolic mirror does, of course, produce an image; however, "radio-photographic plates" do not exist and the radio astronomer is limited to sampling the energy coming to him from different parts of the image, which he reconstructs a posteriori in the manner of a mosaic. The mirror-antenna system is just a device for absorbing energy from finite regions of the sky: it is analogous to an optical photometer.

How big are these regions? Here again, we return to the optical analogy—is a point source reproduced as a point image?

The Airy disc of a lens or mirror ($\approx\lambda/D$ rad, $D$ being the

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aperture diameter) represents the smallest angular extent of the sky one can resolve. An optical image is essentially just a "mosaic" of regions each of which has angular extent $\approx\lambda/D$ (leaving aside atmospheric effects). Consequently, in the case of a radio telescope, the basic "mosaic" size is also $\approx\lambda/D$ rad; energy is absorbed at a given observation only from such "mosaic" elements. This is simply the antenna "lobe."

It is interesting to note that image reconstruction from "mosaic" elements has its counterpart in modern optical astronomy. Photographic plates are often analyzed by electronic devices, whose scanning elements have of course a finite size. The electronically reconstructed image is then a mosaic of tiny squares. This is also true generally in satellite astronomy, where the image, projected on some suitable surface, is electronically dissected and transmitted to earth by radio. In these cases, it is usual to refer to "picture elements" or pixels.

Using simple radio telescopes, a planet such as Jupiter, or even a body as large as the Sun, has a much smaller angular extent than the basic pixel so that we absorb energy not only from the given source, but also from surrounding regions of the sky. If the latter emits energy, the contribution from the source is likely to be a small fraction of sky signal—how can we improve the "contrast"? (to use an optical term).

The notion of antenna gain can now be introduced via the optical analogy. The solid angle of the Airy disc is $\Omega$, $\Omega \approx \pi\lambda^2/D^2 \approx \pi\ ^2\lambda^2/4S$, where $S$ is the surface area of the aperture.

This type of relation holds for any radiation absorbing device; in the radio domain, it is usual to define a main lobe solid angle and an effective collecting surface through $\Omega S = \lambda^2$.

This relation is used even if the antenna has no "aperture" in the optical sense, for example, a dipole. $S$ is then just a measure of the efficiency with which the antenna absorbs incident energy (one might compare it to the "cross-section" used in nuclear physics) and can be calculated for any given system. In all calculations, we can use $S$ wherever an aperture area would have been appropriate.

Consider now a small source observed against a uniform background. The smaller the value of $\Omega$, the smaller the contribution from the sky, and therefore the larger the relative contribution from the source (so long as its solid angle remains less than $\Omega$). Consequently, the ratio $4\pi/\Omega$ is a measure of how good the "contrast" is between the source and the background. This is known as the antenna gain $G$. Therefore, $G = 4\pi/\Omega = 4\pi S/\lambda^2$. We emphasize here that the antenna gain is a dimensionless quantity. Its value is a function of wavelength and (through $S$) of antenna design.

At this stage, it is useful to note that $G$ can be made arbitrarily large for a given $\lambda$ simply by increasing $S$, but this is expensive, and hence the need to find an optimal solution for a given application.

Students often do not easily grasp the important idea that the energy received from the source has to exceed a certain finite amount before it can be seen against the background; this is because they are not accustomed to signals which arrive in the form of noise. It is necessary to show chart recordings of celestial radio sources, such as the quiet Sun, galactic emission, and so on; a good technique is to play with computer generated noise to show that if the signal is too small with respect to the random fluctuations, it cannot be distinguished. The recipe then emerges in a natural way: the signal must be several times larger than the rms amplitude of the background fluctuations.

What is the rms amplitude of the background fluctuations? This is given by Nyquist's theorem and some statistical reasoning which the students can be asked to demonstrate or, at a lower level, it can be given them.

$$W = 2kT/(\Delta\nu\Delta t)^{1/2},$$

where $W$ is the rms noise amplitude; $\Delta\nu$ is the bandwidth of the apparatus; $\Delta t$ is the period of time over which a given measurement will be taken; $T$ is the temperature of the background signal.

Here again, optimization will rear its head; clearly, the noise can be reduced to any desired extent by increasing $\Delta\nu$ or $\Delta t$ but we then lose information about the frequency or time structure of the source. Worse still, if $\Delta\nu$ is too large, we risk picking up locally generated radiofrequency interference.

To compare the source with the background, one must express both in terms of equivalent parameters; since astronomers have different ways of describing different types of source, one should first understand their language and we shall do this in an intuitive way.

Consider an extended black body source of surface area $\Sigma$ and temperature $T$. In this context, "extended" signifies merely that the source solid angle at the telescope is larger than the telescope main lobe solid angle $\Omega$ (or the Airy disc, in optical terms). According to Planck's radiation law, the source radiates per unit bandwidth at the frequency $\nu$ a power $W_\nu$:

$$W_\nu = 2\pi h\ \Sigma\nu^3/c^2(e^{h\nu/KT} - 1)\ \ \mathrm{W\,Hz^{-1}},$$

where $h$ is Planck's constant and $K$ is Boltzmann's constant.

In the radio domain, $h\nu \ll KT$, which gives the Rayleigh-Jeans law

$$W_\nu = 2\pi\Sigma\nu^2KT/c^2.$$

Now, each unit area of surface radiates uniformly outwards into a solid angle $2\pi$; since the telescope receiving area subtends a solid angle $S/d^2$ at the source, $d$ being its distance, the telescope can only intercept a fraction $S/2\pi d^2$ of the energy radiated by unit area.

Moreover, through its lobe, the telescope can "see" only a fraction $\Omega d^2/2\Sigma$ of the emitting surface; therefore, the power absorbed by the telescope $P_\nu^{\mathrm{ext}}$ is given by

$$P_\nu^{\mathrm{ext}} = \frac{S}{2\pi d^2}\frac{\Omega d^2}{2\Sigma}W_\nu\ \ \mathrm{W\,Hz^{-1}},$$

where ext stands for extended. Substituting the expressions for $\Omega$ and $W_\nu$ one finds immediately $P_\nu^{\mathrm{ext}} = KT/2$.

In this estimate we have ignored such details as the shape of the emitting surface; assuming Lambert's law and using the proper photometric formulae (Kraus[2]), one obtains the exact expression $P_\nu^{\mathrm{ext}} = 2KT$.

The quantity measured at the antenna terminals is $P_\nu$. We note that in the case of an extended source, a measurement of the absorbed power gives directly the temperature of the source whatever the antenna size. This quantity is called the brightness temperature of the extended source.

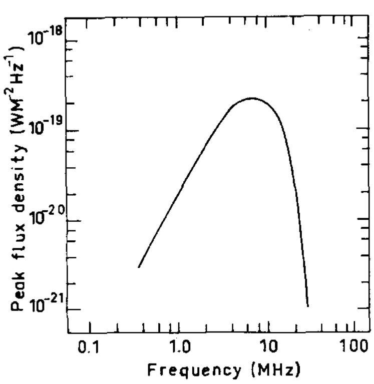


Fig. 1. Jovian peak flux density as a function of frequency [based on a figure in Carr and Desch (Ref. 4)].

Now, sources do not necessarily radiate as black bodies; in such a case, the above relation has no particular physical significance. However, astronomers tend to *define* the quantity $P_\nu^{\text{ext}}/2K$ as the brightness temperature: it is then just a convenient mnemonic for absorbed power, and its identification with a thermodynamic temperature depends on the source itself. For example, sky emission (with which we shall be concerned) is usually parametrised in terms of sky brightness temperature.

Consider now a source having solid angle $\omega \ll \Omega$. In this case, the telescope intercepts energy radiated by half of the emitting surface, whence

$$P_\nu^{\text{small}} = (S/2\pi d^2)(1/2)W_\nu \ \text{W Hz}^{-1}$$
$$= (\lambda^2/4\pi)(2\pi\Sigma/d^2c^2)\nu^2 KT$$
$$= (1/2)(\omega/\Omega)KT.$$

Again, a rigorous calculation leads to $P_\nu^{\text{small}} = 2(\omega/\Omega)KT$. In this case, the power absorbed is not a direct measure of the temperature (even if the source has a black body spectrum); one must also know the source size and the antenna lobe. Knowing these quantities, the temperature one obtains is called the source brightness temperature; again, its identification with a thermodynamic temperature depends on the physics of the source.

One often finds the received power parametrised in terms of the *flux density* $F_\nu$, this is defined as the power per unit surface area of telescope $F_\nu = P_\nu/S$. For a small source, this is quite useful, being a function of source parameters only

$$F_\nu^{\text{small}} = 2\omega KT\lambda^2 \ \text{W Hz}^{-1}\,\text{m}^{-2}.$$

For extended sources, the notion is less useful $F_\nu^{\text{ext}} = 2KT/S$.

A final definition used by astronomers is the antenna temperature: whatever the source, the antenna temperature $T_a$ is *defined* by $P_\nu = 2KT_a$. Again, this is just another way of expressing the power absorbed by the antenna (the rigorous definition is in terms of the operations carried out in order to measure this quantity—however, for our purposes, the outcome is the same). For an extended source $T_a \equiv$ brightness temperature. For a small source $T_a = (\omega/\Omega) \times$ brightness temperature

One can now pick up the threads of the basic discussion. We are choosing to observe small sources against an extended source (the sky); all parameters must be expressed in the same form. Tables and graphs usually quote the strength of small sources in terms of flux densities while sky emission is given as an antenna temperature (see Fig. 2). To save unnecessary computing, it is convenient to express the latter as a flux density, consequently $F_{\text{back}} = 2KT/S$, where back stands for background. Consequently, the background radiation will have an rms flux density variation given by

$$F_{\text{rms}} = 2KT/S(\Delta\nu\Delta t)^{1/2}.$$

To distinguish the source from the background, $F_{\text{sour}}/F_{\text{rms}}$ (which is the signal to noise ratio) must exceed some critical number, say five, whence,

$$F_{\text{sour}} > 5[2KT/S(\Delta\nu\Delta t)^{1/2}]$$
$$= [40\pi KT/G\lambda^2(\Delta\nu\Delta t)^{1/2}].$$

The student is now ready to calculate all the basic antenna parameters, for a given source and background.

## CHOICE OF SOURCE AND FREQUENCY

A good procedure is to let the student discover for himself which, out of a certain selection of sources, is the most viable. Some students will try to "build" a telescope for impossibly faint quasars!

We shall fix our ideas on a Jovian eruption receiver; it will turn out that the device we build can be used to observe solar outbursts as well as the quiet Sun (under favorable conditions). The reasons are clear. These are by far the strongest celestial sources. Data on Jupiter can be found in Carr and Desch[4] (Fig. 1 shows the peak flux density as a function of frequency) and data on the radio sun in, for example, Erickson *et al.*[5] or Gibson[6]; Kraus[2] gives a graph of the sky temperature as a function of frequency, which is reproduced schematically in Fig. 2. The student will now be confronted by another problem of optimization. The background flux drops with increasing frequency but so does the flux emitted by Jupiter. What is a reasonable upper limit to the frequency? The lower limit is troublsome too. The flux rises up to 18 MHz but so does the importance of man-made interference. In short, in which frequency range will Jupiter be reasonably strong, and free from man made and galactic noise?

It turns out that 25–30 MHz is a good range. Here, the maximum flux density emitted by Jupiter is $\approx 10^{-21}\ Wm^{-2}$ $\text{Hz}^{-1}$. On the other hand, the sky temperature at 25–30 MHz is in the range $4 \times 10^4$–$5 \times 10^5$ K. Consequently, taking the most unfavorable case, the telescope parameters have to satisfy

$$10^{-21} > 40\pi KT/G\lambda^2(\Delta\nu\Delta t)^{1/2},$$

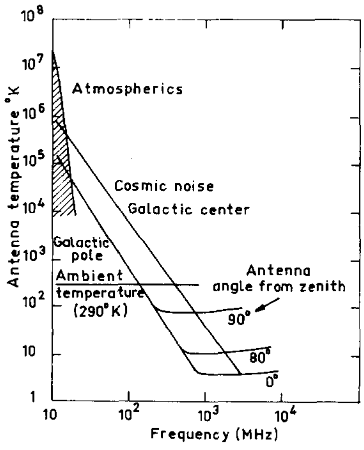


Fig. 2. Sky temperature as a function of frequency [based on a figure in Kraus (Ref. 2)].

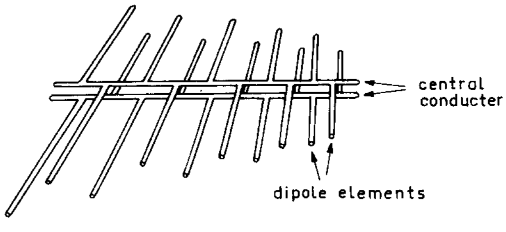


Fig. 3. Schematic diagram of a Log Periodic antenna.

whence

$$G(\Delta\nu\Delta t)^{1/2} > 2.6 \times 10^3. \qquad (1)$$

Any configuration will have to satisfy this criterion in order to see Jovian bursts with any certitude. The definition of the telescope comes next.

## THE TELESCOPE

It is clear from Eq. (1) that if $\Delta\nu$ and $\Delta t$ are sufficiently large, $G$ can be quite small. However, we were "given" a standard receiver; a typical maximum bandwidth for such a device is about 20 kHz. With a chart recorder, one can manually integrate for several seconds. To fix our ideas, let us take $\Delta t = 5$ sec. With these figures $G \approx 9$.

It is instructive to compute the effective surface area needed: it turns out to be of the order of 20 $m^2$. Is it meaningful to build a 20-$m^2$ dish for $\nu \approx 30$ MHz?

A simple form of antenna is the dipole. This is essentially two colinear sections of a conducting material placed close together; the receiver is connected to the center. If the total length is $\lambda/2$, the dipole will produce a particularly strong current for an incident signal of wavelength $\lambda$. Can one such dipole give the necessary gain? Advanced students can be asked to calculate the gain of a dipole from its radiation pattern—it turns out to be 3/2, much too small. However, it is worthwhile noting that were we to build a special receiver, with a sufficiently large $\Delta\lambda$ and $\Delta t$, a simple dipole *could* work (once again a simulacrum of the world of modern research—do we prefer to invest in special electronics or special hardware?). An intuitive solution for small $\Delta\nu$ and $\Delta t$ immediately springs to mind. The gain is a function of effective aperture area $S$; one might therefore connect together several dipoles, and these as a whole must have a larger gain than a single unit.

The single dipole has another property which one must consider rather carefully. A resonator responds principally to one frequency $\nu_0$; however, energy losses damp out the oscillation on a time scale $\tau$: $\tau = E/(dE/dr)$, where $E$ is the energy stored in the system and $dE/dt$ is the rate of loss of energy. This means that oscillations will be maintained even if the frequency of the exciting signal is $\nu_0 + \Delta\nu$, providing that $\Delta\nu \lesssim 1/\tau$. $\Delta\nu$ is the bandwidth of the resonator; it is often expressed as $\Delta\nu/\nu = (dE/dr)/\nu E$ or in terms of the angular frequency $w$: $\Delta w/w = 2\pi(dE/dr)/\omega E$. The quantity $wE/(dE/dr)$ is called the "$Q$" of the system; when the energy loss is small, $Q$ is high and the frequency response is narrow.

Now, we have seen that the signal/noise ratio depends on the bandwidth of the received signal. Using the bandwidth of a communication receiver—20 kHz—the telescope must have a gain of at least nine. Consequently, the telescope bandwidth must not be less than 20 kHz.

Advanced electronics students may be made to find the bandwidth of a simple dipole; it is given by Schelkunoff and Friis[8] $\Delta w/w = 46.5/Z_0$, where $Z_0$ is the characteristic impedance of a dipole and is equal to 120 [ln $(2l_{1/2}/D) - 1)$], $2l_{1/2}$ is the total length of the dipole, and $D$ is the external diameter of the dipole. An apparently more accurate expression for $Z_0$, which we use in practise, is given by Carrel[7]

$$Z_0 = 120[\ln(2l_{1/2}/\mathrm{D}) - 2.25]. \qquad (2)$$

At 30 MHz, $2l_{1/2} = 6$ m. Taking D = 2 cm as a typical diameter, the characteristic impedance is about 414 Ω; the relative bandwidth of this kind of dipole is thus given by $\Delta\nu/\nu = 46.5/414 \approx 0.1$. Therefore, a single dipole will certainly respond uniformly over the bandwidth accepted by the receiver.

However, our telescope should satisfy another condition. We have chosen to work in the 25–30 MHz range, but we have not specified the exact frequency. This was deliberate—local interference is difficult to forsee in advance and may vary from time to time, so it is desirable to have the freedom to tune to a "quiet" part of the spectrum after the telescope has been installed. Therefore, the instrument must respond uniformly over the entire band in which we have chosen to work. The single dipole, with its 3-MHz bandwidth, satisfies this condition only marginally. The situation will be much worse if one chooses to work in a broader frequency range, such as 20–30 MHz, in order to benefit, when interference is low, from the increased Jovian flux density at low frequencies.

It follows that our telescope should consist of several dipoles, arranged so as to have a broad frequency response. This can be done by noting that if the dimensions of a dipole which resonates at frequency $\nu$ are scaled by a factor $\tau$, it will have its original electrical properties but at the new frequency $\tau\nu$. Now, a set of dipoles, whose lengths vary from $L_{\min}$ to $L_{\max}$, will as a whole be sensitive to a wavelength range $2L_{\min} - 2L_{\max}$; moreover if the lengths $L_n$ and separations $d_n$ of intermediate dipoles follow the simple relation:

$$L_n = \tau L_{n-1}, \quad d_n = \tau d_{n-1}, \qquad (3)$$

where $\tau$ is a number $<1$, the response of the dipole system is essentially constant in the wavelength range $2L_{\min} - 2L_{\max}$.

This is called the "log-periodic" antenna, and is the type we shall design. It is shown schematically in Fig. 3: the dipoles feed a common waveguide in the form of two parallel conductors. By its very nature it is rather insensitive to small errors in construction and, as we shall see, its impedance is easy to adjust.

The most recent description and explanation of this type of antenna may be found in Smith[8]; other useful references are Carrel[7] and Jasik.[9]

In a nutshell, the structure of log periodic antennae, drawn schematically in Fig. 4, is defined by $L_{\min}$, $L_{\max}$, $\tau$,

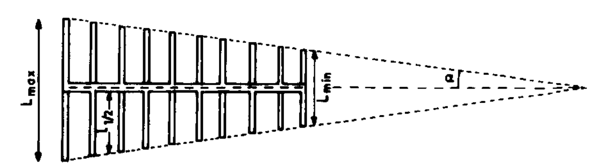


Fig. 4. Basic parameters of a Log Periodic antenna.

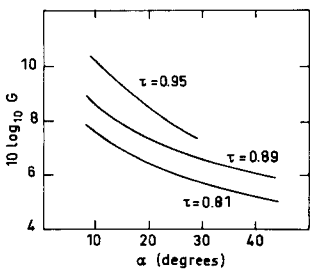


Fig. 5. Gain of a Log Periodic antenna as a function of basic antenna parameters.

and $\alpha$ (the angle by which the ends of the dipoles converge towards the central conductors). Now, the grain of such a system is a function of the number of dipoles which are not too far from resonance at a given frequency, and this is a function of both $\tau$ and $\alpha$: the closer $\tau$ is to 1, the larger the number of dipoles between $L_{max}$ and $L_{min}$; the smaller the value of $\alpha$, the larger the number of dipoles having almost the same length. Therefore the gain is a rising function of $\tau$ and $1/\alpha$.

Exact calculations have been made, and are summarized in Fig. 5.

The number of dipoles $N$ for a given frequency range is easily shown to be approximately

$$N \approx 1 + \frac{\log(\lambda_{min}/\lambda_{max})}{\log(1/\tau)}.$$

A more accurate expression, which takes into account antenna end corrections is

$$N = 1 + \frac{\log(0.5\,\lambda_{min}/0.4\,\lambda_{max})}{\log(1/\tau)}.$$

We have already estimated how much gain is needed. The student now has to play about, to get a feel for the results. Is it better to have a low $\tau$ (and so relatively few dipoles) but with a very small $\alpha$ (which therefore gives a very long and unwieldy telescope), or is there some convenient optimum? There is, in fact, no unique solution; the answer depends on purely technical constraints and this is the whole point of the exercice.

A particular solution in the 25–30 MHz range is $L_{min} =$ 3.9 m, $L_{max} = 5.9$ m, $\tau = .955$; $\alpha = 13°$, $N = 10$. This gives an antenna which is just over 4 m long with a gain of 10.

The electrical parameters of the antenna are now fixed. How do we translate into hardware?

## TECHNICAL REALIZATION OF THE TELESCOPE

A technically viable and electrically advantageous way of constructing the telescope has already been indicated in Fig. 3.

The waveguide is a parallel conductor, from which emerges 1/2 of a complete dipole. The dipoles are spaced along the parallel conductor according to Eq. (3).

A formal operation consists of computing the positions of the dipoles along the central conductor.

A more subtle problem is in the choice of the materials from which to construct the antenna. Metallic rods immediately spring to mind but should they be solid or hollow? Steel or aluminum? What should their cross section be?

This is where the student learns about cantilevers and bending moments. A tube of length $L$, internal diameter $d$ and outside diameter $D$, fixed at one end, bends under its own weight by an amount $f$,[10] $f = PL^4/8EI$, where $P$ is the weight/meter of tube, $E$ is the modulus of elasticity, and $I$ is the $\pi(D^4 - d^4)/64$.

Playing about with this formula, for different values of $P$, $E$, $D$, and $d$ soon teaches the student that steel is out of the question, and that hollow tubes are more efficient than solid ones (they are also cheaper!). An important point to hammer home is that, from an electrical point of view, hollow tubes are just as good as rods (skin effect). The student must also learn not to be too fussy; after all, an element can be allowed to bend by a few centimeters, since this is small with respect to the length and wavelength.

The diameter of the central conductors poses an interesting problem. Antennae can be designed so that a heavy nonconducting supporting member holds all of the dipoles. The central conductors can then be small since they have no structural importance. However, impedance matching (dipoles to conductor, conductor to receiver) is then somewhat subtle and requires an electrical solution. A simple procedure, which simultaneously solves the mechanical and electrical problems, consists of forcing the dipoles into the conductors themselves. The conductors are then also supporting elements.

This introduces a mechanical constraint. The conductor must not bend under the combined weight of the dipoles, must be significantly thicker than any of the dipoles while not being too heavy, etc. With this solution, an additional electrical constraint appears for the dipole elements. The impedance presented by each dipole to the central waveguide is a function of dipole length and diameter, and for optimal operation each dipole should present the same impedance. The impedance is given by Eq. (2); we can see that since the dipole elements are of different lengths, their diameters will be different also.

We have finally obtained the mechanical parameters of a possible antenna. Were we to manufacture the tubes ourselves, these could even be considered as the working numbers.

Unfortunately, we "buy" our material from a manufacturer, who supplies tubes only in certain diameters and lengths, which the instructor can find by obtaining a catalogue. Almost certainly, most of the diameters we have just calculated simply do not exist. One has to make do with the nearest approximation. For the central conductor, should one take the upper or lower available diameter? How should one group the diameters of the dipoles, so that their impedances do not vary too much from their average value?

The lengths of the tubes can also cause headaches. Standard lengths are certainly not simple multiples of the numbers we need, and if one is not careful, one ends up throwing away as much as a half of what has been acquired. One must not forget to add to the lengths of the dipole elements the diameter of the central conductor. The dipole elements have to be held somehow! How to group the lengths so as to fit the standard modules? Can one, just by changing the initial electrical specification by an insignificant amount, reduce by a large fraction the amount of material to be bought? One begins to see how the original choice really does affect the final design.

Finally, we have a "working" design. Different students will, of course, have found different parameters, as a consequence of errors, different starting conditions, etc. Let them fight it out: which design is the better one? Table I

Table 1. Dimensions of a log periodic antenna for the 25–30 MHz range.

| Dipole No. | Distance from longest dipole (in m) | $l_{1/2}$ (in m) | External diameter (in m) |
|---|---|---|---|
| 1 | 0 | 2.94 | 0.025 |
| 2 | 0.577 | 2.808 | 0.025 |
| 3 | 1.128 | 2.731 | 0.025 |
| 4 | 1.654 | 2.561 | 0.02 |
| 5 | 2.157 | 2.445 | 0.02 |
| 6 | 2.637 | 2.335 | 0.02 |
| 7 | 3.095 | 2.230 | 0.02 |
| 8 | 3.533 | 2.13 | 0.016 |
| 9 | 3.95 | 2.034 | 0.016 |
| 10 | 4.35 | 1.943 | 0.016 |

shows a representative set of figures, for material available in France.

## ELECTRICAL CONNECTION TO THE RECEIVER

We have now, in principle, the mechanical description of an antenna which can deliver a signal such that the chosen source is distinguishable from sky and galactic noise. This signal must now be detected by a receiver, and recorded.

Standard communication receivers have an input impedance of 75 Ω. To prevent losses, the parallel conductor must also present, at the input terminals, an impedance of 75 Ω. This is where we see one of the principal advantages of this type of antenna: the impedance $Z_0$ of two parallel conductors, each of diameter $D$ separated by a centre to center distance $S$ is given by

$$Z_0 = 120/\cosh(S/D)\ \Omega.$$

Once again, electronics students may be made to find or prove this formula.

For $Z_0 = 75\ \Omega$, this gives $S/D = 1.2$. Clearly, such an antenna can easily be "tuned" to any input impedance one likes, simply by changing the separation of the parallel conductors.

The next problem is analogous to our original design constraint. The power from the source had to be greater than the fluctuations due to background. In the present case, the signal at the receiver terminals must exceed the fluctuations due to receiver noise.

The noise of a reasonably good communications receiver corresponds to an rms voltage of about 1 $\mu$V at the input terminals. Consequently, the power delivered up by the antenna has to correspond to an rms voltage of at least, say, 2 $\mu$V.

Now, if the antenna absorbs a power $W$, one has $W = \langle V^2 \rangle / 2R$ where $\langle V^2 \rangle^{1/2}$ is the rms voltage and $R$ is the antenna impedance.

The antenna has been optimized for Jovian bursts; using the design data

$$W = 10^{-21}\, S\, \Delta\nu/2 = \frac{10^{-21}\, \lambda^2 G\, \Delta\nu}{8} \approx 0.8 \times 10^{-15}\ \text{W}.$$

Note the extra factor of 1/2 which has been introduced. A dipole antenna responds to only one polarization vector, which reduces by one-half the power absorbed from an unpolarized source.

Consequently,

$$\langle V^2 \rangle = 0.8 \times 10^{-15} \times 75 \times 2 = 1.2 \times 10^{-13}$$

$$\langle V^2 \rangle^{1/2} \approx 4 \times 10^{-7}\ \text{V}.$$

Therefore, our antenna with its gain of 10 cannot alone give a singal which exceeds the receiver noise—the power is too small by a factor of about 16. Some students will react by redesigning a monstrous antenna with a gain of 160; let them, it will be a good lesson. In fact, all we need to do is to attach a good pre-amplifier at the antenna output, to boost the signal by about 20 before entering the receiver.

The type of pre-amplifier chosen will in principle be determined by noise considerations. The thermal noise generated by an amplifier stage may be parametrised in terms of a noise temperature $T_N$ which represents the equivalent power fluctuations of a resistance connected across the antenna terminals. The power delivered by the antenna has to be sufficiently smaller than the rms value of these fluctuations.

Manufacturers generally quote the noise temperature in terms of the noise figure $F$ (in dB):

$$F = 10 \log(1 + T_N/290).$$

It is quite easy to obtain high gain pre-amplifiers with a noise figure of less than 10. It is easy to show that the noise power passed on to the receiver from such an amplifier is negligible with respect to the power absorbed from Jupiter.

## MOUNTING THE ANTENNA

We have now a viable radio-astronomy antenna. How should we mount it? This is another good example of how the original constraints define the end result.

One can think up quite complicated adjustable equatorial mountings but is this necessary? The antenna has a gain of about 10. Therefore, its lobe extends to about $4\pi/10$ rad$^2$, which amounts to a resolving power of about 1 rad, or about 60° (in fact, the resolving power is not the same parallel and perpendicular to the dipole plane; however, this hardly affects the argument). Therefore, for practical purposes, a fixed antenna inclined at 45° and in the plane of the meridian, will be quite adequate!

## FINAL REMARKS

Many astronomy courses dismiss telescope design in a few words. At the very best, one might find a tedious discussion of various kinds of optical aberrations, or the different places around a telescope where one might find a focal plane. Radio telescopes are generally treated even less well; indeed for many practising radio astronomers, their instrument is just a heap of wire and a black box. This is rather sad. Radio-telescope design is not just a technical problem, but brings into play much good physics, and in particular obliges one to use a variety of different skills to solve a single problem.

Electronics students often suffer from the converse problem. They know the theory, but the applications are always rather contrived and pedantic. This exercise brings electronics and the theory of structures into the astronomy classroom, and astronomy to the budding engineer.

*Note added in proof.*

In an interference-free region of France, and using a 27 dB preamplifier, I was able to measure the continuous decametric emission of the sun with the antenna described in this paper, and so deduce the temperature of the upper corona. The result is so surprising ($>10^6$ K) for many students (the photospheric temperature being only $\approx 6000$ K, measurable by a simple bolometer consisting of a piece of blackened brass), that the experiment is worth doing for its own sake.

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[1]J. D. Kraus, *Antennas* (McGraw-Hill, New York, 1950).

[2]J. D. Kraus, *Radioastronomy* (McGraw-Hill, New York, 1966).

[3]S. A. Schelkunoff and H. T. Friis *Antenna Theory and Practise* (Wiley, New York, 1952).

[4]T. D. Carr and M. D. Desch, *Jupiter* (University of Arizona, Arizona 1976).

[5]W. C. Erickson and T. E. Gergely, Sol. Phys. **54** 57, (1977).

[6]E. G. Gibson, NASA Publication No. SP-303 (1973) (unpublished).

[7]R. Carrel, IRE Int. Conv. Rec. **9**, 61, (1961).

[8]C. E. Smith, *Log Periodic Antenna Design Handbook,* Smith Electronics Inc., 8200 Snowville Road, Ohio 44141.

[9]H. Jasik, *Antenna Engineering Handbook* (McGraw-Hill, New York, 1961).

[10]J. E. Gordon, *Structures, or Why Things Don't Fall Down* (Pelican, London, 1978).