Physical processes leading to surface inhomogeneities

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Outline

Outline

Surfaces inhomogeneities : what are they ?

1. Temperature patches
2. Chemical patches
3. Magnetic fields
4. Velocity fields
5. Radiation field

Example 1
Chemical patches

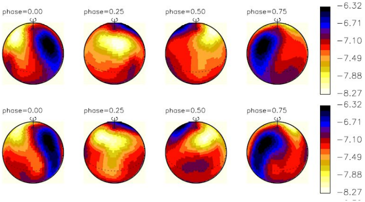


FIGURE: Chemical spots on HD 11753 (an HgMn star) from Kohronen et al. 2013.

Example 2
Magnetic fields

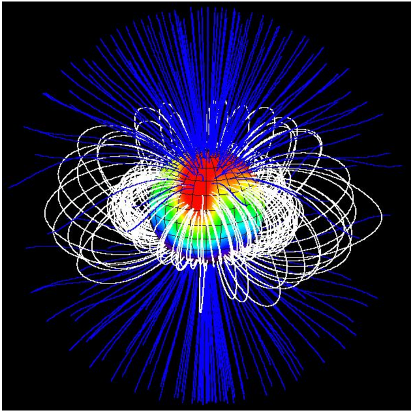


FIGURE: Extrapolation of the magnetic field of the star V374 Pegasi from spectropolarimetric observations (by M. Jardine & J.-F. Donati).

Example 3
Velocity patches

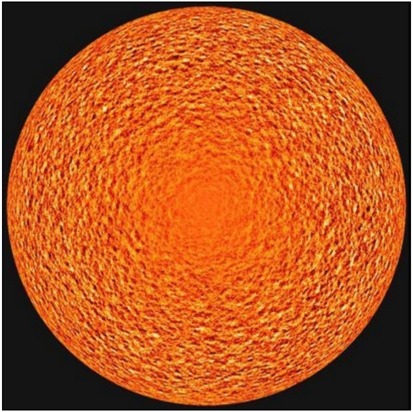
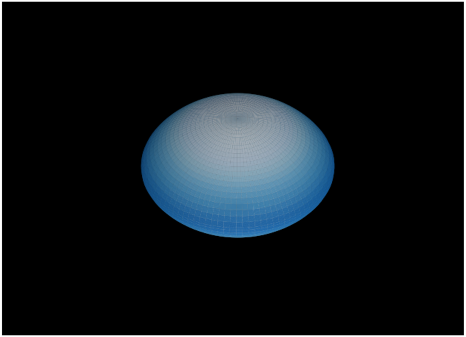


FIGURE: Solar supergranulation as viewed by SOHO/MDI.

Example 4 : Rotational effect

Gravity darkening of Achernar (α Eri)



Surfaces inhomogeneities : where do they come from ?

A phenomenon that breaks the spherical symmetry :

1. Rotation (or angular momentum)
2. Convection
3. Magnetic fields
4. Binarity

Outline

Old stuff : von Zeipel 1924

Let us assume the star is barotropic so that all thermodynamic quantities verify

$$\rho \equiv \rho(\Phi), \quad T \equiv T(\Phi), etc$$

Then

$$\vec{F}_{rad} = -\chi\vec{\nabla}T = -\chi(\Phi)T'(\Phi)\vec{\nabla}\Phi = K(\Phi)\vec{g}_{eff}$$

Hence von Zeipel law

$$T_{\rm eff} = Kg_{\rm eff}^{1/4} \qquad \text{on the surface}$$

What is wrong ?

With rotation T cannot be constant over an equipotential if in radiative equilibrium.

$$\left\{\begin{array}{l}\mathrm{Div}(\chi\vec{\nabla}T)=0\\ \Delta\Phi=4\pi G\rho+2\Omega^2\end{array}\right. \qquad (1)$$

which leads to

$$\mathrm{Div}(\chi(\Phi)T'(\Phi)\vec{\nabla}\Phi)=0\Longleftrightarrow 4\pi G\rho+2\Omega^2+(\ln(\chi T'))'g_{\mathrm{eff}}^2=0$$

On the surface $G\rho\ll\Omega^2$ while g_{eff} is not constant. **There is a contradiction ! !**

The model ELR11

Espinosa Lara & Rieutord 2011, AA533,A43

Can we make a simple model of the dependence of the flux with respect to latitude without computing the whole 2D structure of a rotating star ?
YES ! But not as simple as von Zeipel 1924 though !

The model ELR11 : the idea

In an evelope the flux just obeys :

$$Div\vec{F} = 0$$

namely energy is conserved and there are no energy sources. This is a first order partial differential equation, not enough to determine the flux, but if we add a constraint to the flux we may find it. We thus assume

$$\vec{F} = -f(r, \theta)\vec{g}_{\rm eff}$$

which reduces our system to a single unknown f. Thus it is solvable.

Comments

How strong is this hypothesis $\vec{F} \,//\, \vec{g}_{\rm eff}$?
In a convection zone the flux is driven by buoyancy so it looks reasonable.
In a radiative zone, the configuration is baroclinic so vectors are not aligned for sure. But how strong is the misalignment ? Let's have a look to a 2D model.

Baroclinic misalignment

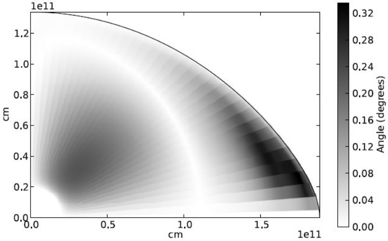


FIGURE: Misalignment between pressure gradient and flux, from ELR11.

Baroclinic torque

The baroclinic torque $(\vec{\nabla}P \times \vec{\nabla}\rho)/\rho^2$ thus involves a small misalignment of the vectors but strong gradients that make the driving. Moreover

$$\mathrm{Div}\vec{F} = 0 \quad \Longleftrightarrow \quad \mathrm{Div}(f\vec{\nabla}\Phi) = 0$$

Thus

$$\vec{g}_{\mathrm{eff}} \cdot \vec{\nabla} \ln f = -2\Omega^2$$

$$\frac{\partial \ln f}{\partial \xi} = -\frac{2\Omega^2}{g_{\mathrm{eff}}}$$

I shows that f has similar latitudinal variations than g_{eff}, which therefore will modify the von Zeipel law. So, again, $T_{\mathrm{eff}}/g_{\mathrm{eff}}^{1/4}$ cannot be constant.

The derivation of $f(r,\theta)$

f is given by the flux equation :

$$\mathrm{Div}(f\vec{g}_{\mathrm{eff}}) = 0 \Longleftrightarrow \vec{g}_{\mathrm{eff}} \cdot \vec{\nabla} f + f\mathrm{Div}\vec{g}_{\mathrm{eff}} = 0$$

To go further we need a model for $\vec{g}_{\mathrm{eff}}$. We shall use the Roche model which is not so bad if the star is centrally condensed.

Scaling the flux function f

Near the center of the star

$$\vec{F} = \frac{L}{4\pi r^2}\vec{e}_r \qquad \vec{g}_{\rm eff} = -\frac{GM}{r^2}\vec{e}_r$$

So that we may set

$$f(r,\theta) = \frac{L}{4\pi GM} F(r,\theta)$$

with

$$\lim_{r\to 0} F(r,\theta) = 1$$

The scaled PDE

We scale the gravity with GM/R_e^2 and the length scale with the equatorial radius R_e. The angular velocity is scaled by

$$\omega = \frac{\Omega}{\Omega_k} = \Omega\sqrt{\frac{GM}{R_e^3}}$$

thus we get

$$\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right)\frac{\partial F}{\partial r} - \sin\theta\cos\theta\frac{\partial F}{\partial\theta} = 2F$$

because in the Roche model $\mathrm{Div}\vec{g}_{\mathrm{eff}} = 2\Omega^2$. In addition $F(0,\theta) = 1$.

Solving the PDE

The probelm for F is well-posed and can be solved analytically. Let's do it !
First, we solve for $\ln F$ and remove the RHS. Namely,

$$\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right)\frac{\partial \ln F}{\partial r} - \sin\theta\cos\theta\frac{\partial \ln F}{\partial \theta} = 2$$

we set $\ln F = \ln G + A(\theta)$. We immediately find that

$$A'(\theta) = -2/\sin\theta\cos\theta \implies A(\theta) = -\ln(\tan^2\theta)$$

We still have to solve the homogeneous part

$$\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right)\frac{\partial \ln G}{\partial r} - \sin\theta\cos\theta\frac{\partial \ln G}{\partial \theta} = 0$$

Solving the PDE
Characteristics method

We look for places where $\ln G$ is constant. These curves are the characteristics of G. They are such that

$$\frac{\partial \ln G}{\partial r} dr + \frac{\partial \ln G}{\partial \theta} d\theta = 0$$

but G also verifies

$$\left(\frac{1}{\omega^2 r^2} - r \sin^2 \theta \right) \frac{\partial \ln G}{\partial r} - \sin \theta \cos \theta \frac{\partial \ln G}{\partial \theta} = 0$$

So eliminating $\frac{\partial \ln G}{\partial r}$ and $\frac{\partial \ln G}{\partial \theta}$ we get

$$\left(\frac{1}{\omega^2 r^2} - r \sin^2 \theta \right) d\theta + \sin \theta \cos \theta dr = 0$$

which is the equation of characteristics.

Solution of the characteristics equation

We first observe that we may multiply

$$\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right)d\theta + \sin\theta\cos\theta dr = 0$$

by any function $H(r,\theta)$ without changing anything. So we may solve

$$\left\{ \begin{array}{l} \frac{\partial h}{\partial r} = H\sin\theta\cos\theta \\ \frac{\partial h}{\partial\theta} = H\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right) \end{array} \right. \quad (2)$$

H needs to be chosen so that this system can be integrated. After try and fail, we find $H = \omega^2 r^2 \cos^2\theta / \sin\theta$.

Solution of the characteristics equation (bis)

Thus

$$\left\{ \begin{array}{l} \frac{\partial h}{\partial r} = \omega^2 r^2 \cos^3\theta \\ \frac{\partial h}{\partial \theta} = \frac{\cos^2\theta}{\sin\theta} - \omega^2 r^3 \cos^2\theta \sin\theta \end{array} \right. \quad (3)$$

and the solution is

$$h(r,\theta) = \frac{1}{3}\omega^2 r^3 \cos^3\theta + \cos\theta + \ln\tan(\theta/2)$$

Using characteristics

On the curves

$$\frac{1}{3}\omega^2 r^3 \cos^3\theta + \cos\theta + \ln\tan(\theta/2) = Cst$$

we know that $\ln G$ is constant. So $G \equiv G(h(r,\theta))$. Now we know that

$$\ln G(h) - \ln\tan^2\theta$$

is the solution of

$$\left(\frac{1}{\omega^2 r^2} - r\sin^2\theta\right)\frac{\partial \ln F}{\partial r} - \sin\theta\cos\theta\frac{\partial \ln F}{\partial\theta} = 2$$

or if we work with F

$$F = \frac{G(h(r,\theta))}{\tan^2\theta}$$

Using the boundary conditions

However we know $h(r, \theta)$ but we do not know G...
We need now to use the boundary condition : $F(0, \theta) = 1$. Thus we have to impose

$$\frac{G(h(0, \theta))}{\tan^2 \theta} = 1$$

or

$$\frac{G(\cos\theta + \ln\tan(\theta/2))}{\tan^2 \theta} = 1$$

or

$$G(\cos\theta + \ln\tan(\theta/2)) = \tan^2 \theta$$

Solving for G

Let's call $\cos\theta + \ln\tan(\theta/2) = h_0(\theta)$. Hence, we have

$$(G \circ h_0)(\theta) = \tan^2\theta$$

or

$$G \circ h_0 = \tan^2 \qquad \Longrightarrow \qquad G = \tan^2 \circ h_0^{-1}$$

so formally, the solution is

$$G(r,\theta) = \tan^2(h_0^{-1}(h(r,\theta)))$$

Solving for G (bis)

To make it more understandable, we set

$$\psi = h_0^{-1}(h(r,\theta))$$

so that

$$h_0(\psi) = \frac{1}{3}\omega^2 r^3 \cos^3\theta + \cos\theta + \ln\tan(\theta/2)$$

or

$$\cos\psi + \ln\tan(\psi/2) = \frac{1}{3}\omega^2 r^3 \cos^3\theta + \cos\theta + \ln\tan(\theta/2)$$

which is a transcendental equation for ψ not difficult to solve numerically (you know that when r or ω are small $\psi \simeq \theta$). So finally

$$F(r,\theta) = \frac{\tan(\psi(r,\theta))}{\tan^2\theta}$$

Some interesting latitudes : the pole

F seems to be singular at the pole ($\theta = 0$) and at the equator ($\theta = \pi/2$). What's going on there ?
At the pole : if $\theta \ll 1$ we get that

$$\cos\psi + \ln\tan(\psi/2) = \frac{1}{3}\omega^2 r^3 \cos^3\theta + \cos\theta + \ln\tan(\theta/2)$$

leads to

$$1 + \ln\tan(\psi/2) \simeq \frac{1}{3}\omega^2 r^3 + 1 + \ln\tan(\theta/2)$$

so that

$$\psi \simeq \theta e^{\omega^2 r^3/3} \qquad \Longrightarrow \quad F(r,0) = e^{2\omega^2 r^3/3}$$

Some interesting latitudes : the equator

More complicated ! We need to know that if $\varepsilon \ll 1$ then

$$\ln\left(\tan\left[\frac{\pi}{4}-\varepsilon\right)\right] = -\varepsilon - \frac{1}{6}\varepsilon^3 - \cdots$$

so that finally

$$F(r,\pi/2) = (1-\omega^2 r^3)^{-2/3}$$

The flux or the effective temperature

Back to the definitions we start with, we have

$$\vec{F} = -\frac{L}{4\pi GM} F(\omega, r, \theta)\vec{g}_{\text{eff}}$$

so that

$$T_{\text{eff}} = \left(\frac{L}{4\pi\sigma GM}\right)^{1/4} \sqrt{\frac{\tan\psi}{\tan\theta}}\; g_{\text{eff}}^{1/4}$$

so the function $\sqrt{\tan\psi / \tan\theta}$ shows the deviation from the von Zeipel law.

Comparison with full 2D models

Latitude dependence

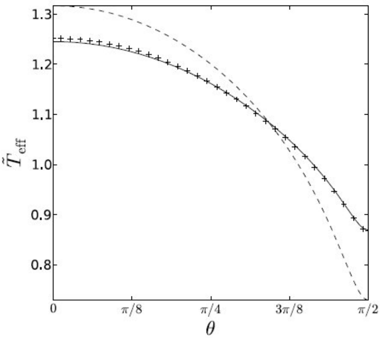


FIGURE: 3 $M_\odot$ model at $\Omega = 0.9\Omega_k$. Crosses=ESTER code, dashed= von Zeipel.

Comparison with full 2D models

Effective gravity dependence

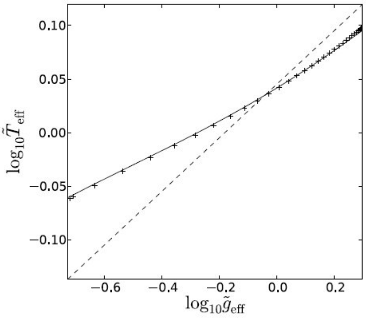


FIGURE: 3 $M_\odot$ model at $\Omega = 0.9\Omega_k$. Crosses=ESTER code, dashed= von Zeipel.

Comparison with full 2D models

Contrast of effective temperatures

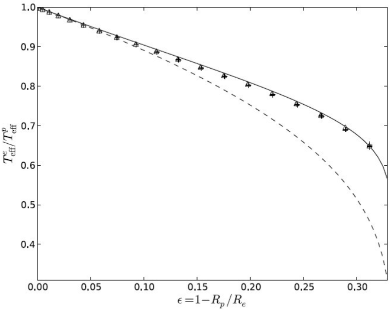


FIGURE: 3 $M_\odot$ model at $\Omega = 0.9\Omega_k$. Crosses=ESTER code, dashed= von Zeipel.

β-model

Let's investigate what is the behaviour of the β-exponent defined as

$$T_e = T_p \left(\frac{g_e}{g_p}\right)^\beta$$

From the polar and equatorial expression of the flux :

$$F_e = (1-\omega^2)^{-2/3} g_e \qquad F_p = e^{2\omega^2 r_p^3/3} g_p$$

while

$$\frac{g_e}{g_p} = r_p^2(1-\omega^2) \quad \text{with} \quad r_p = \frac{1}{1+\omega^2/2}$$

where r_p is the polar radius.

β-model

So we find

$$\left(\frac{T_e}{T_p}\right)^4 = \frac{(1-\omega^2)^{1/3}}{(1+\omega^2/2)^2} e^{-2\omega^2 r_p^3/3}$$

and

$$\beta = \frac{1}{4} - \frac{1}{6}\frac{\ln(1-\omega^2)+\omega^2 r_p^3}{\ln(1-\omega^2)-2\ln(1+\omega^2/2)}$$

For small values of ω, this leads to

$$\beta = \frac{1}{4} - \frac{1}{6}\omega^2 + \mathcal{O}(\omega^4) \qquad \text{or} \qquad \beta = \frac{1}{4} - \frac{1}{3}\varepsilon + \mathcal{O}(\varepsilon^2)$$

β-model
Comparisons

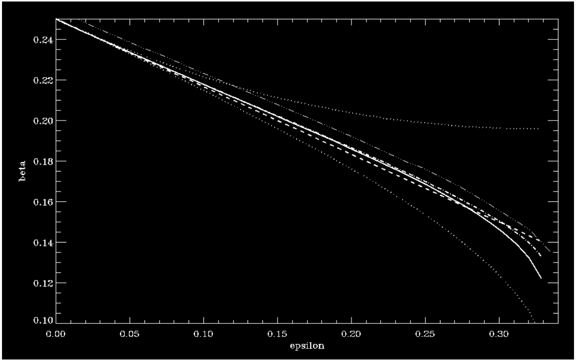


FIGURE: β-values from various models (full ESTER model in dashed dot), dashed = linear law $\beta = 0.25 - \varepsilon/3$.

β-model
Observations

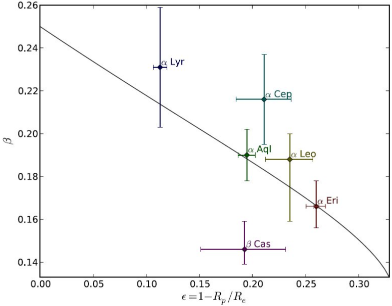


FIGURE: Observationnaly derived β-values for various stars (Domiciano et al. 2014).

Lucy's problem

In 1967 it was realized that gravity darkening was very important for interpretation of the light curves of contact binaries (like W UMa).
But most of these stars are low mass, thus with a convective envelope so that using von Zeipel results was doubtful.
So Lucy asked : "What is the gravity-darkening law appropriate for late-type stars whose subphotospheric layers are convective ?"

Lucy's answer

Lucy reasoning was the following : let us consider models of solar type stars and let us see how their effective temperature changes with their gravity.
He used five models (3 with M=1$M_\odot$, 2 with M=1.26$M_\odot$) and found that for these models

$$0.069 \leq \beta \leq 0.088$$

So Lucy adopted $\beta = 0.08$ as a representative value.

The origin of Lucy's result

Shape of a surface convection zone

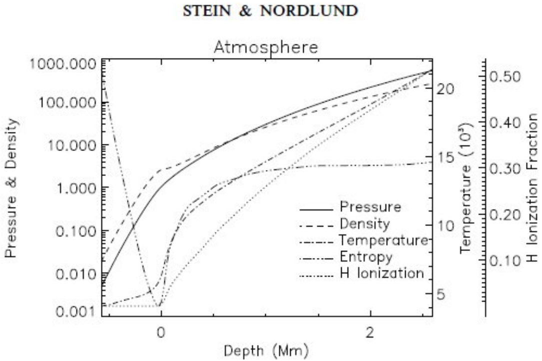


FIGURE: Thermodynamic profile of the Sun according to Stein & Nordlund 1998.

Lucy's results

With the simulations of SN98, we see why it is allowable to use the pressure boundary condition (adapted to radiative case)

$$P = \frac{2g}{3\kappa}$$

at the top of an outer convective zone. 1D codes always use this. It comes from the hydrostatic equilibrium :

$$\frac{\partial P}{\partial z} = -\rho g \quad \Longleftrightarrow \quad \frac{1}{\rho\kappa}\frac{\partial P}{\partial z} = -\frac{g}{\kappa} \quad \Longleftrightarrow \quad \frac{\partial P}{\partial \tau} = \frac{g}{\kappa}$$

which is integrated from the optical depth $\tau = 2/3$ upwards.

Lucy's results

Now in convective envelopes, the variation of pressure and densities are related to temperature through

$$P \propto T^{n+1} \qquad \text{and} \qquad \rho \propto T^n \; .$$

Even where convection is no longer efficient these polytropic laws are approximately true as shown by numerical simulations of SN98.
Thus we get

$$g \propto T^{n(\mu+1)+1-s}$$

leading to the gravity darkening exponent :

$$\beta = \frac{1}{n(\mu+1)+1-s} \tag{4}$$

Lucy's results

In some range of temperatures and density, opacity may be written

$$\kappa = \kappa_0 \rho^\mu T^{-s}$$

For instance Christensen-Dalsgaard uses $\mu = 0.408$ and $s = -9.283$ for models of helioseismology. Using Christensen-Dalsgaard's solar values and $n = 3/2$ yields

$$\beta \simeq 0.0807$$

So we clearly see that β in that case depends on the chemical properties of the surface as they control the opacities.

Lucy's results and gravity darkening

Can Lucy's law represent a gravity darkening effect ?
Obviously, it contradicts our model (β never reaches such low values). Our model assumes

$$\mathrm{Div}\vec{F} = 0 \qquad \text{and} \qquad \vec{F} = -f\vec{g}_{\rm eff}$$

It is based on the hydrostatics of surface layers :

$$\frac{dP}{d\tau} = \frac{g}{\kappa} \; .$$

while in rotating (possibly binary) the mechanical balance is rather :

$$\vec{v}\cdot\vec{\nabla}\vec{v} = -\frac{1}{\rho}\vec{\nabla}P - \vec{\nabla}\Phi$$

where $\vec{v}$ is the fluid velocity in some inertial frame.

Lucy's results and gravity darkening

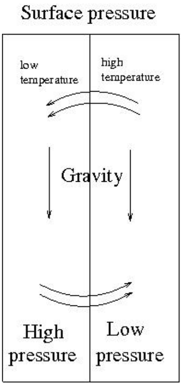


FIGURE: Schematic representation of the generation of a baroclinic flow

Lucy's results and gravity darkening

So Lucy's result does not apply because it juxtaposes 1D equilibria that cannot be.

We therefore expect higher values of β even for low mass stars.

How can that be tested : likely on β Cas. and on close binaries.

Outline

The case of binary stars

From ELR12, AA, 547, A32

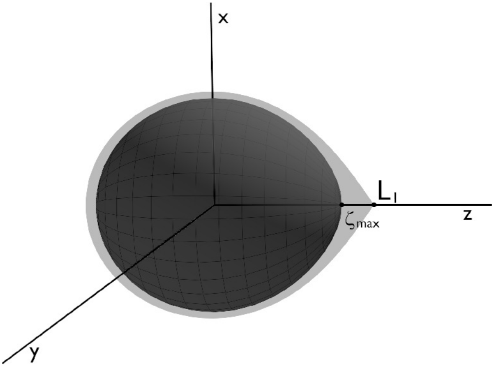


FIGURE: Schematic representation of the primary star with filling factor 0.8. The position of the Lagrange point L_1 is indicated.

The case of binary stars

The equations are the same,

$$\mathrm{Div}\vec{F} = 0 \qquad \text{and} \qquad \vec{F} = -f\vec{g}_{\mathrm{eff}}$$

but now the effective gravity comes from the 3D potential :

$$\begin{aligned}\phi = \; & -\frac{GM_1}{r} - \frac{GM_2}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} \\ & -\frac{1}{2}\Omega^2 r^2 (\sin^2\theta \sin^2\varphi + \cos^2\theta) + a\frac{M_2}{M_1 + M_2}\Omega^2 r\cos\theta \,,\end{aligned} \tag{5}$$

The equation for f

Let us write $\mathrm{Div}(f\vec{g}_{\mathrm{eff}}) = 0$ as

$$\vec{n}\cdot\nabla\ln f = \frac{\nabla\cdot\vec{g}_{\mathrm{eff}}}{g_{\mathrm{eff}}}\ , \qquad (6)$$

Let us build a trajectory $C(\theta_0, \varphi_0)$ that starts at the centre of the star with an initial direction indicated by (θ_0, φ_0) and is tangent to $\vec{n}$ at every point. $C(\theta_0, \varphi_0)$ is therefore a field line of the effective gravity field.

The equation for f

The value of f at a point $\vec{r}$ along the curve can be calculated as a line integral

$$f(\vec{r}) = f_0 \exp\left(\int_{C(\theta_0,\varphi_0)} \frac{\nabla\cdot\vec{g}_{\rm eff}}{g_{\rm eff}}\,{\rm d}l\right) \qquad \text{for } \vec{r}\in C(\theta_0,\varphi_0)\ . \qquad (7)$$

Despite much efforts no analytical solution could be found. We integrated the equations numerically.

Some results

$T_{\rm eff}$ distribution

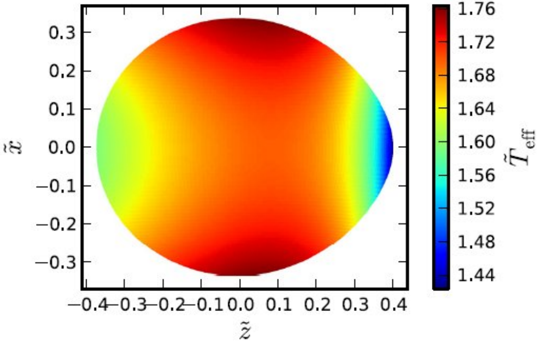


FIGURE: $q = M_2/M_1 = 1$, $\rho = 0.8$ (filling factor).

Some results

$T_{\rm eff}$ distribution

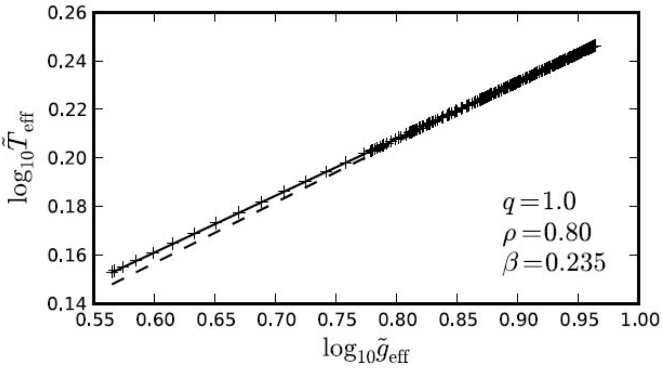


FIGURE: Dashed=von Zeipel, solid=linear fit (β-model), pluses=data of our model).

Some results

$T_{\rm eff}$ distribution

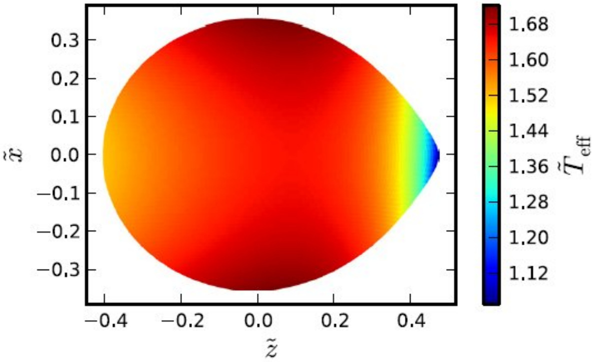
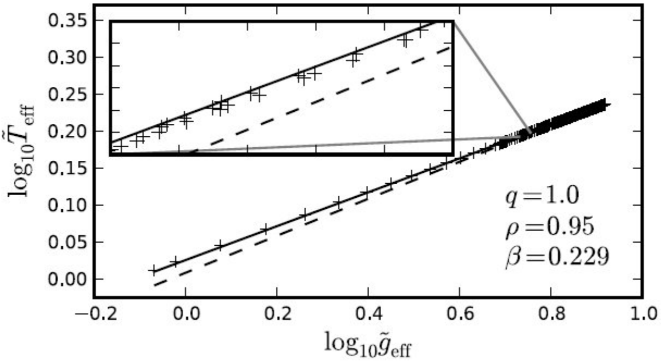


FIGURE: q=1, $\rho = 0.95$

Some results

$T_{\rm eff}$ distribution



Some results

$T_{\rm eff}$ distribution

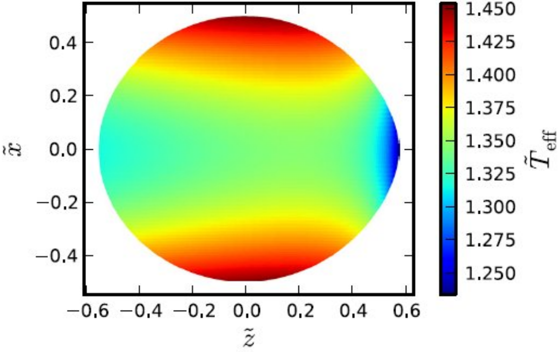
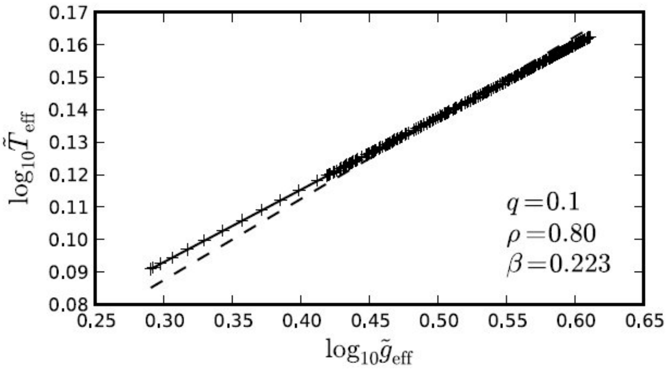


FIGURE: q=0.1, $\rho = 0.8$

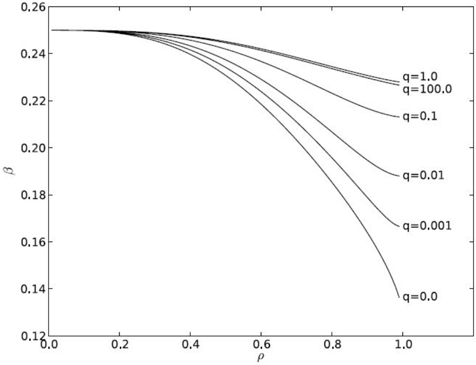
Some results

$T_{\rm eff}$ distribution



Some results

β values



Observations

Djurasevic et al. 2003, 2006

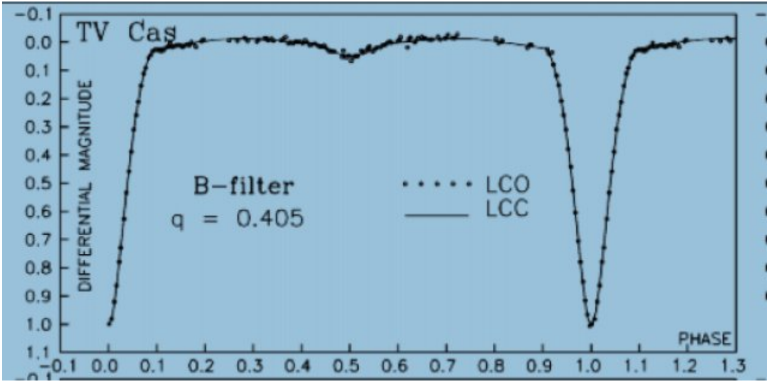


FIGURE: The light curve.

Observations
Djurasevic et al. 2003, 2006

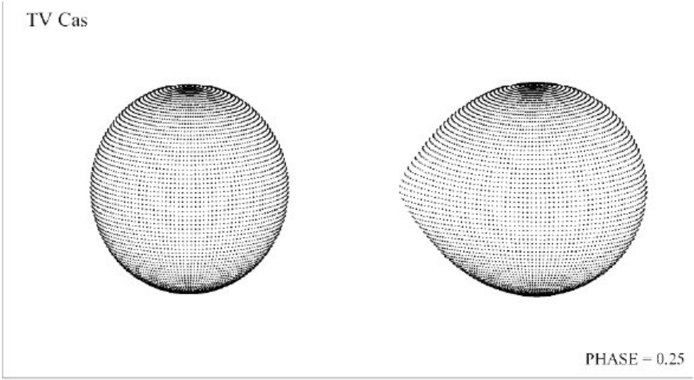


FIGURE: The first model that leads to $\beta = 0.15$.

Observations

Djurasevic et al. 2003, 2006

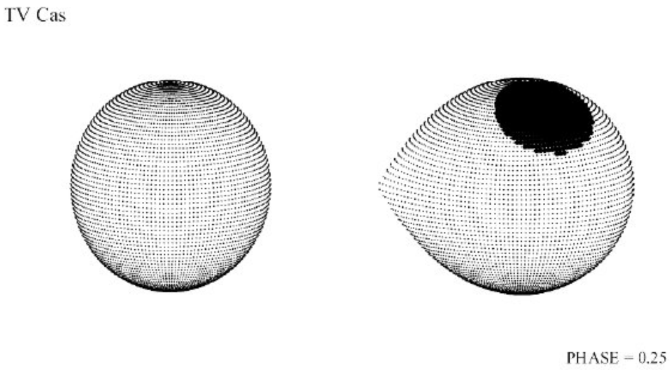


FIGURE: The second model with a spot....

The effects of magnetic fields

- Important in convective zone : a strong magnetic field inhibits convection (it raises the threshold of the instability)
- So expected to be important for the brightness of low mass stars (spots) but unimportant for intermediate-mass and massive stars.
- But here they might structure the distribution of chemical elements by inhibition of horizontal transport (roAp stars).

Outline

Some conclusions

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The End