Shocks waves and discontinuities

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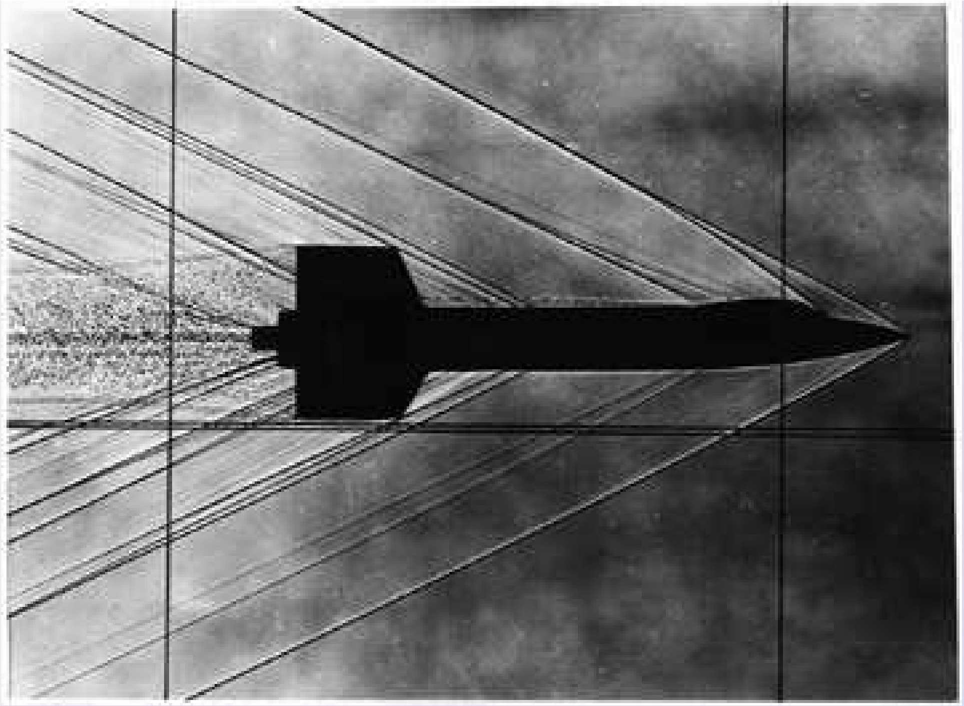
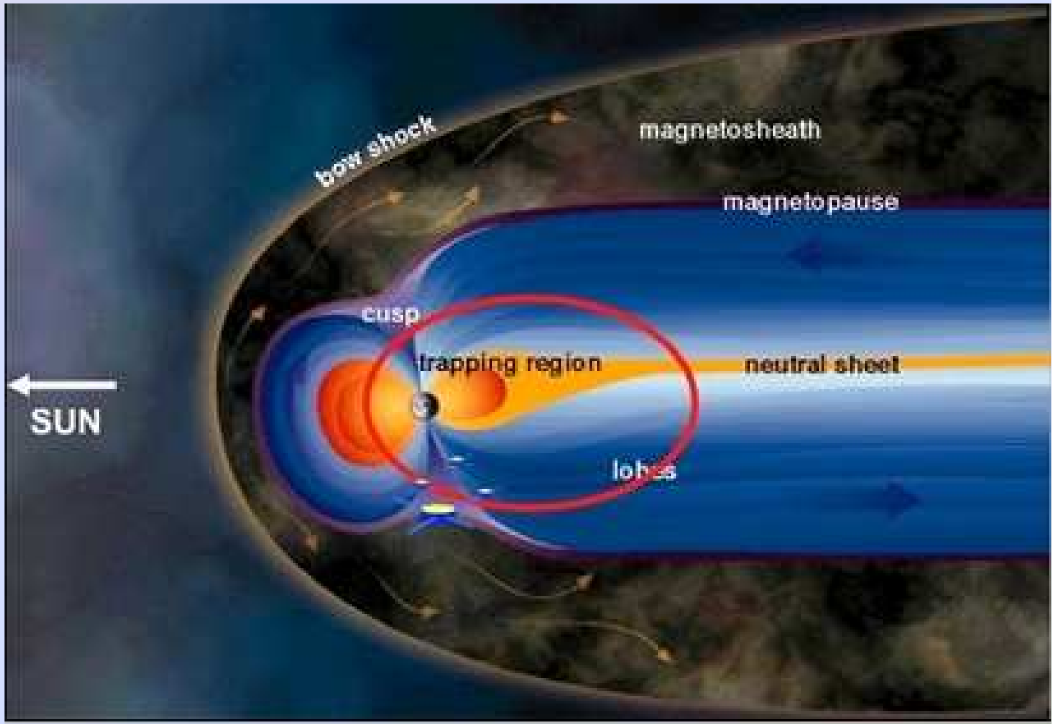
OUTLINE

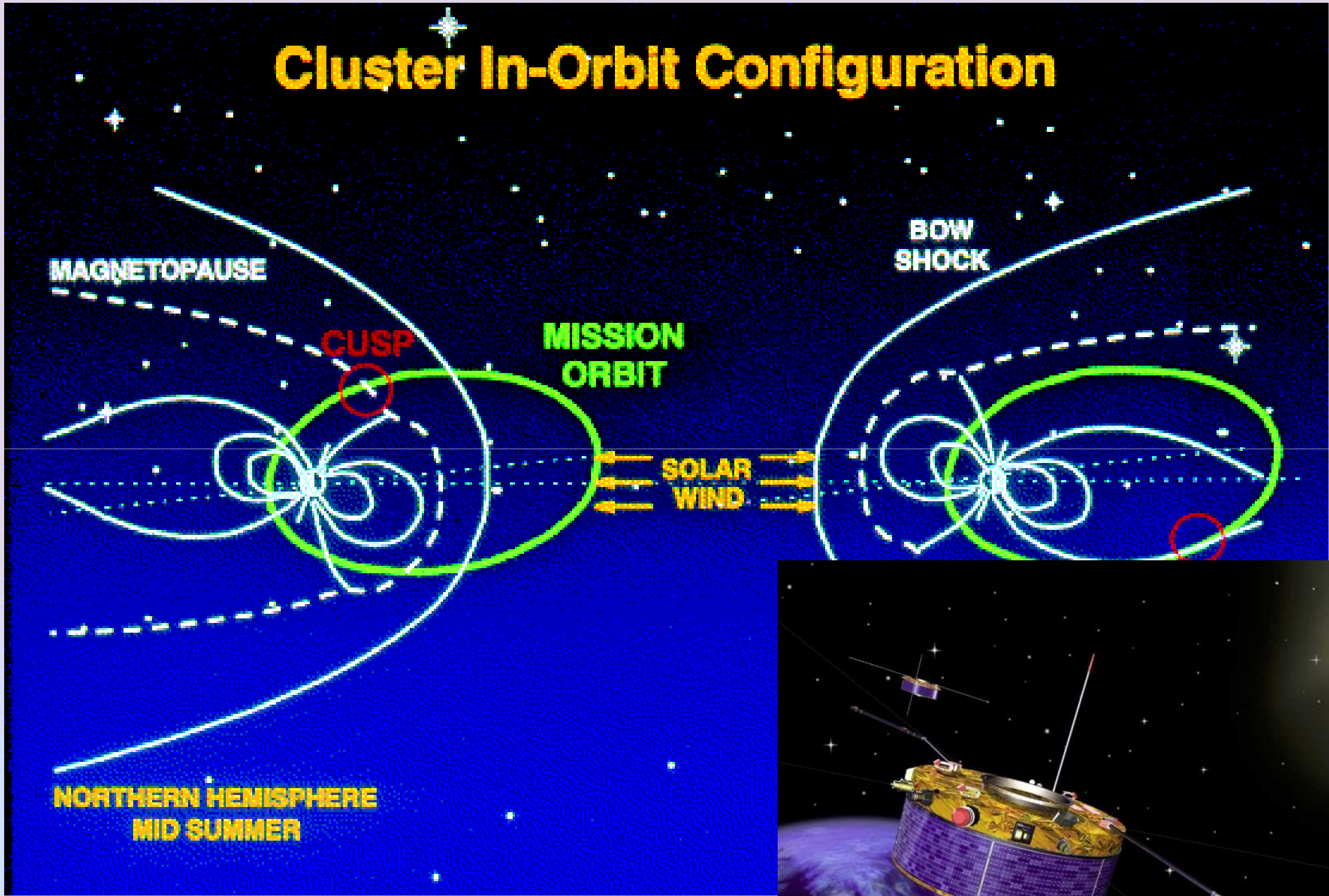
- Observations of discontinuities
- Jump conditions at the boundary
- Different kinds of discontinuities
- What about the boundaries around the planets ?

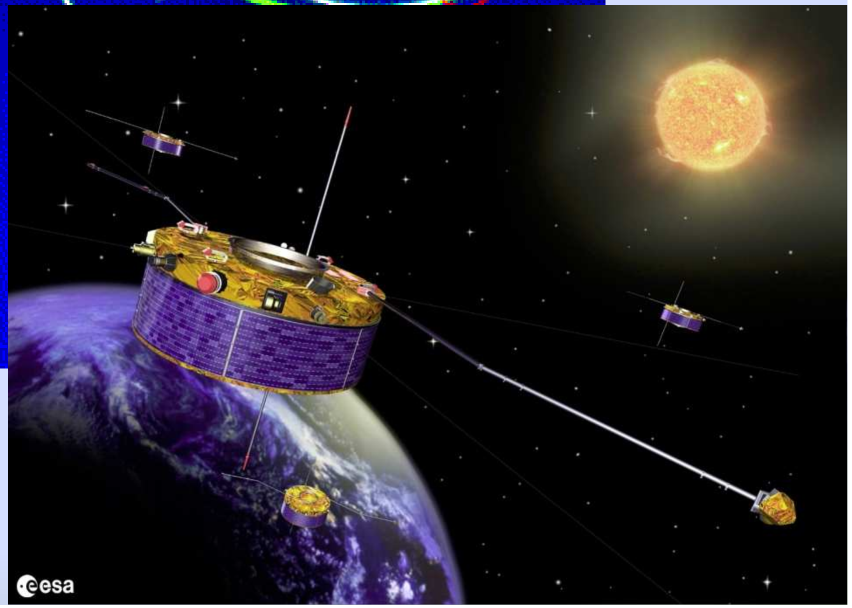
Discontinuities are observed in plasmas



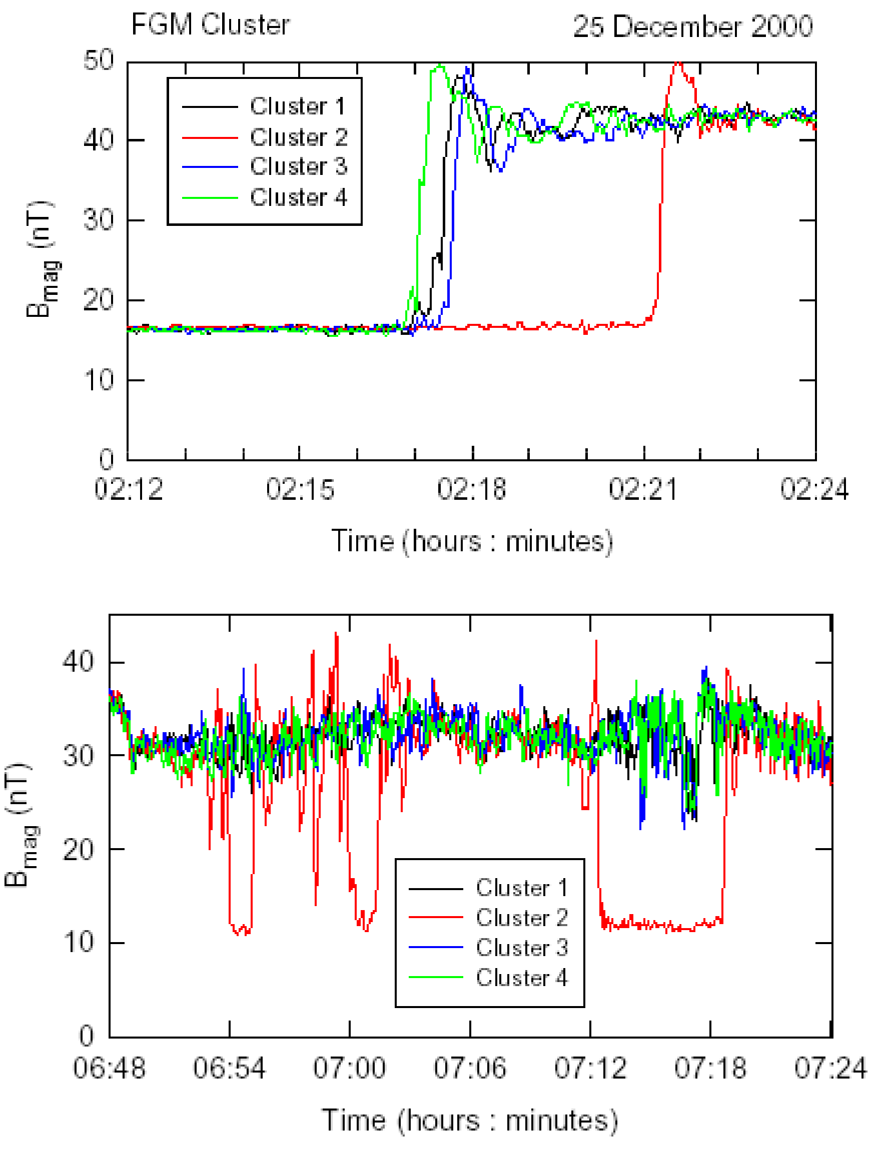

bow shock
magnetosheath
magnetopause
cusp
trapping region
neutral sheet
SUN

CLUSTER orbits





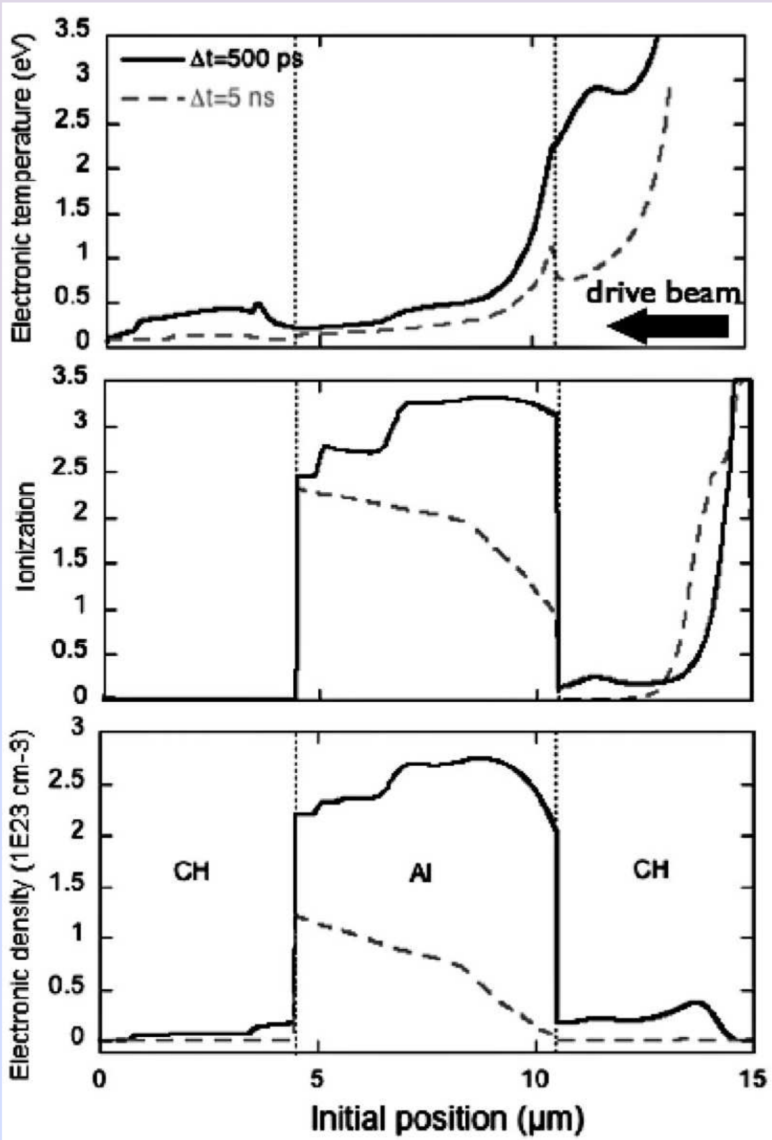
Earth Shock crossing by CLUSTER



Magnetometer data
FGM, (A. Balogh)


Orion Nebula
Image crédit: Hubble (NASA)

Shock generated by a laser pulse on a target



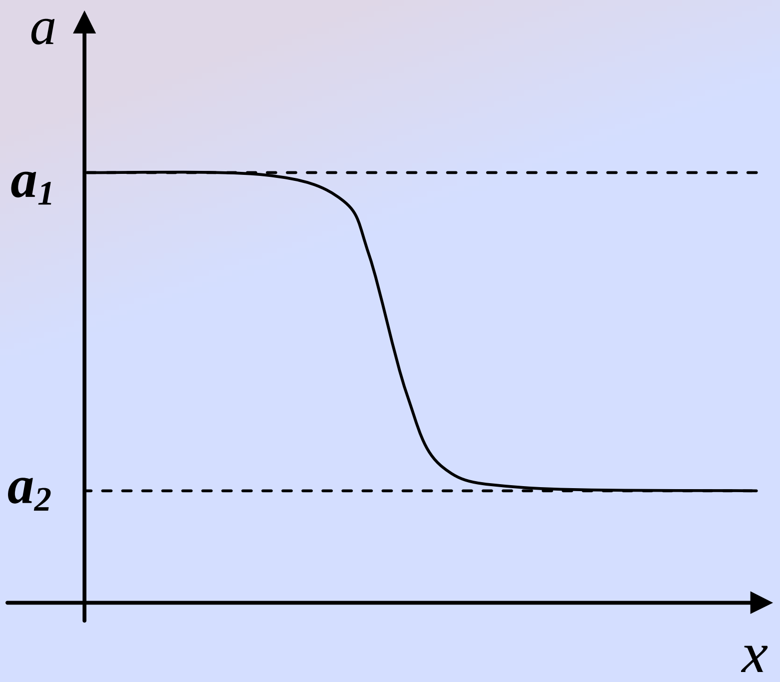
Ravasio et al, PRL, 2007

Jump conditions at the boundary

Meaning of the jump conditions

We assume:

- The problem is 1D
- The frame is chosen to have the discontinuity stationary
- The parameters are constant on each side



Defines a jump $\Delta a = a_2 - a_1$

Setting of the jump conditions

MHD equation in a conservative form : $\partial_t(a) + \nabla.(\mathbf{b}) = 0$

Stationary problem : $\nabla.(\mathbf{b}) = 0$

1D problem, equation integrated along the normal : $\mathbf{n}.\Delta(\mathbf{b}) = 0$

For instance : $\nabla.(\mathbf{B}) = 0$ means $B_{n2} = B_{n1}$

The resulting equations are called **Rankine-Hugoniot equations**

MHD equations

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \mathbf{v}) = 0$$

Conservation of momentum

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \vec{\nabla}.(\rho \mathbf{v}\mathbf{v}) = -\vec{\nabla} p + \mathbf{J} \times \mathbf{B} = -\vec{\nabla} p + \frac{1}{\mu_0} \vec{\nabla}.\left(\mathbf{B}\mathbf{B} - \frac{B^2}{2} \mathbf{I} \right)$$

Conservation of total energy (fluid + fields)

$$\frac{\partial}{\partial t}\left(\rho \frac{v^2}{2} + \frac{3}{2} p + \frac{B^2}{2\mu_0} \right) + \vec{\nabla}.\left(\left(\rho \frac{v^2}{2} + \frac{5}{2} p \right) \mathbf{v} + \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right) = 0$$

Rankine-Hugoniot equations:
invariant quantities across the boundary

$$B_{2n} = B_{1n} = B_n$$

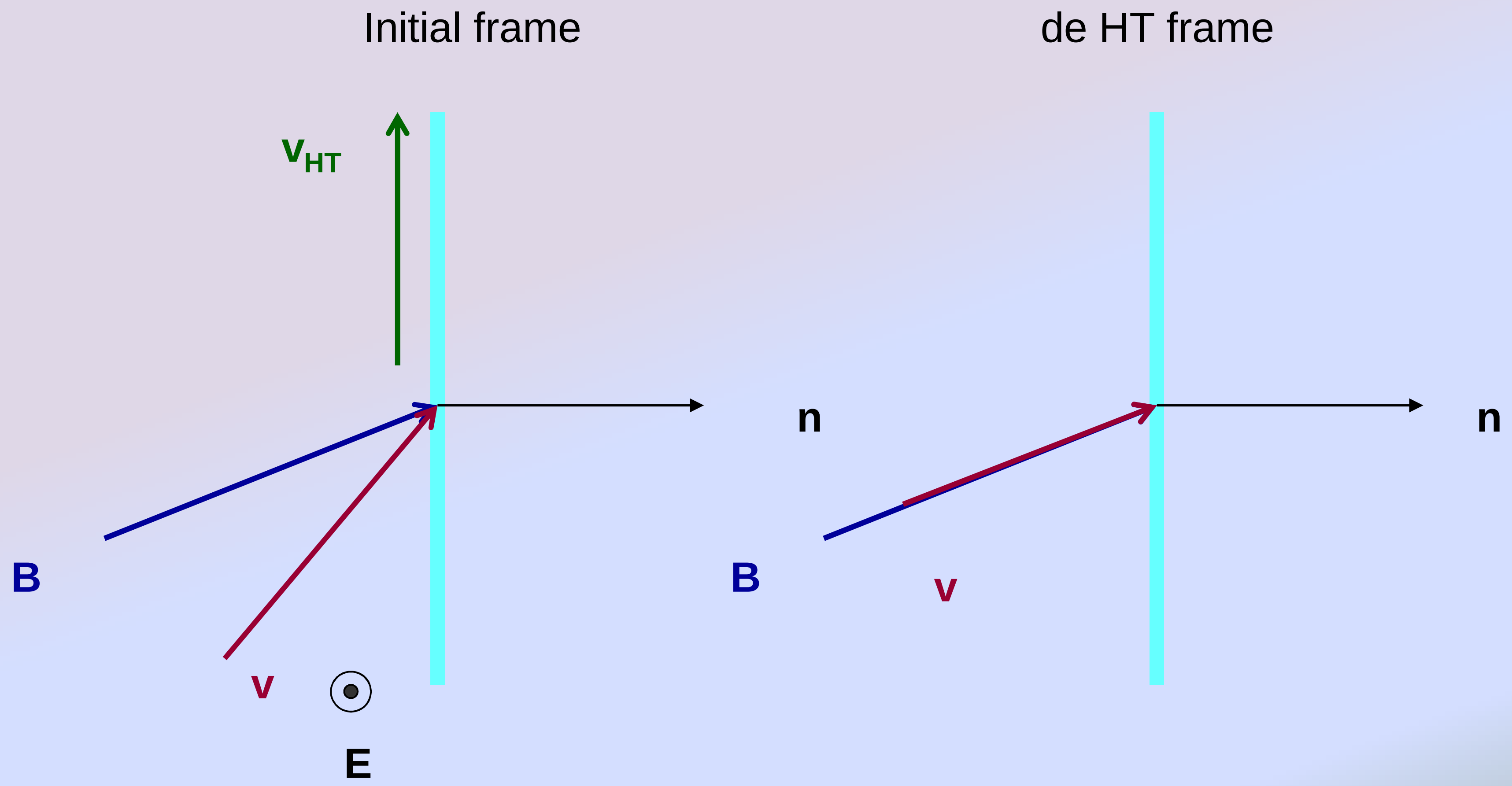
$$\rho_2 v_{2\mathrm{n}} = \rho_1 v_{1\mathrm{n}} = \boldsymbol{\Phi}_m$$ Mass

$$\rho_2 v_{n2} \mathbf{v}_2 + \left(p_2 + \frac{B_2^2}{2\mu_0} \right) \mathbf{n} - \frac{B_n \mathbf{B}_2}{\mu_0} = \ldots = \mathbf{\Phi}_i$$ Impulse

$$\frac{1}{2} \rho_2 v_2^2 v_{n2} + \frac{5}{2} p_2 v_{n2} - \frac{1}{\mu_0} \left[B_n \left(\mathbf{B}_2 . \mathbf{v}_2 \right) - B_2^2 v_{n2} \right] = \ldots = \Phi_e$$ Energy

$$v_{n2} \mathbf{B}_{T2} - B_{n2} \mathbf{v}_{T2} = \mathbf{v}_{n1} \mathbf{B}_{T1} - B_{n1} \mathbf{v}_{T1} = \mathbf{n} \times \mathbf{E}_T$$ Ohm’s law

de Hoffmann Teller frame



Drawn in the **B**, **v plane**

The frame exists if Bn ≠ 0
In the de HT frame: **E** = **0**

Different kinds of discontinuities

Conservation of impulse in the de HT frame

$$v_n \mathbf{B}_T - B_n \mathbf{v}_T = \mathbf{n} \times \mathbf{E}_T = 0$$

Then the perpendicular component of the conservation of impulse

$$\rho_2 v_{n2} \mathbf{v}_2 + \left(p_2 + \frac{B_2^2}{2\mu_0} \right) \mathbf{n} - \frac{B_n \mathbf{B}_2}{\mu_0} = \ldots = \mathbf{\Phi}_i$$

becomes:

$$\left(v_n - \frac{B_n^2}{\mu_o \Phi_m} \right) \mathbf{B}_T = \left(v_{n1} - \frac{B_n^2}{\mu_o \Phi_m} \right) \mathbf{B}_{T1}$$

Conservation of impulse in the de HT frame

$$\left(v_n - \frac{B_n^2}{\mu_o \Phi_m} \right) \mathbf{B}_T = \left(v_{n1} - \frac{B_n^2}{\mu_o \Phi_m} \right) \mathbf{B}_{T1}$$

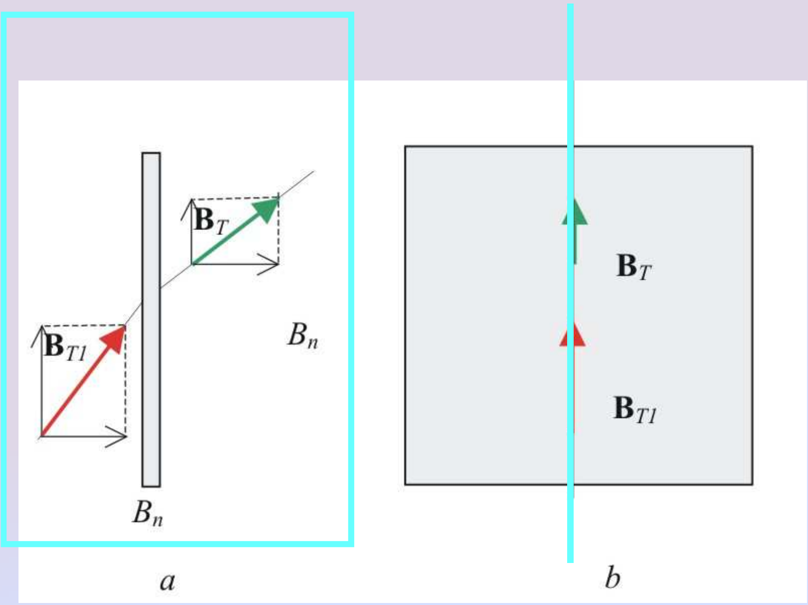
Either:

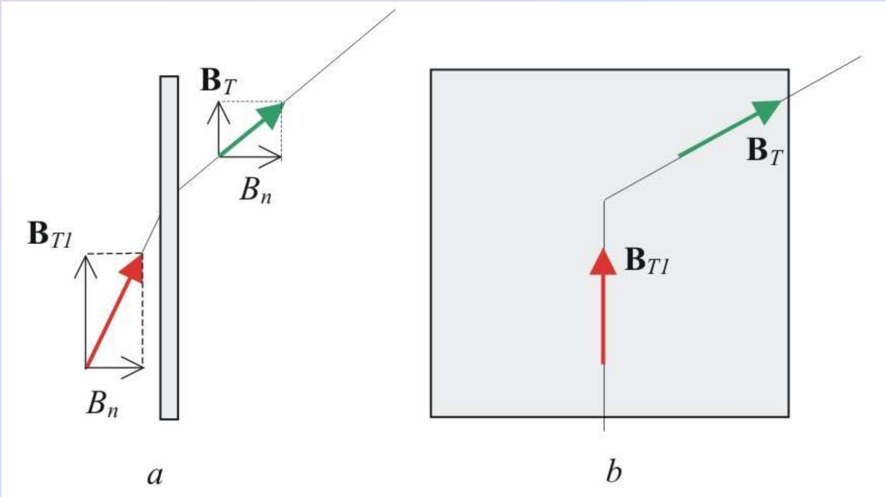
- $\mathbf{B}_T$ // $\mathbf{B}_{T1}$, **B** stays in the same plane $\Rightarrow$ coplanar discontinuity
- v_n is conserved and $v_n = \dfrac{B_n^2}{\mu_o \Phi_m}$ $\Rightarrow$ non coplanar discontinuity

Behavior of the magnetic field at the crossing

Coplanar discontinuity

Non-Coplanar discontinuity





Perpendicular to the plane

In the plane of the discontinuity

Perpendicular to the plane

In the plane of the discontinuity

Shock

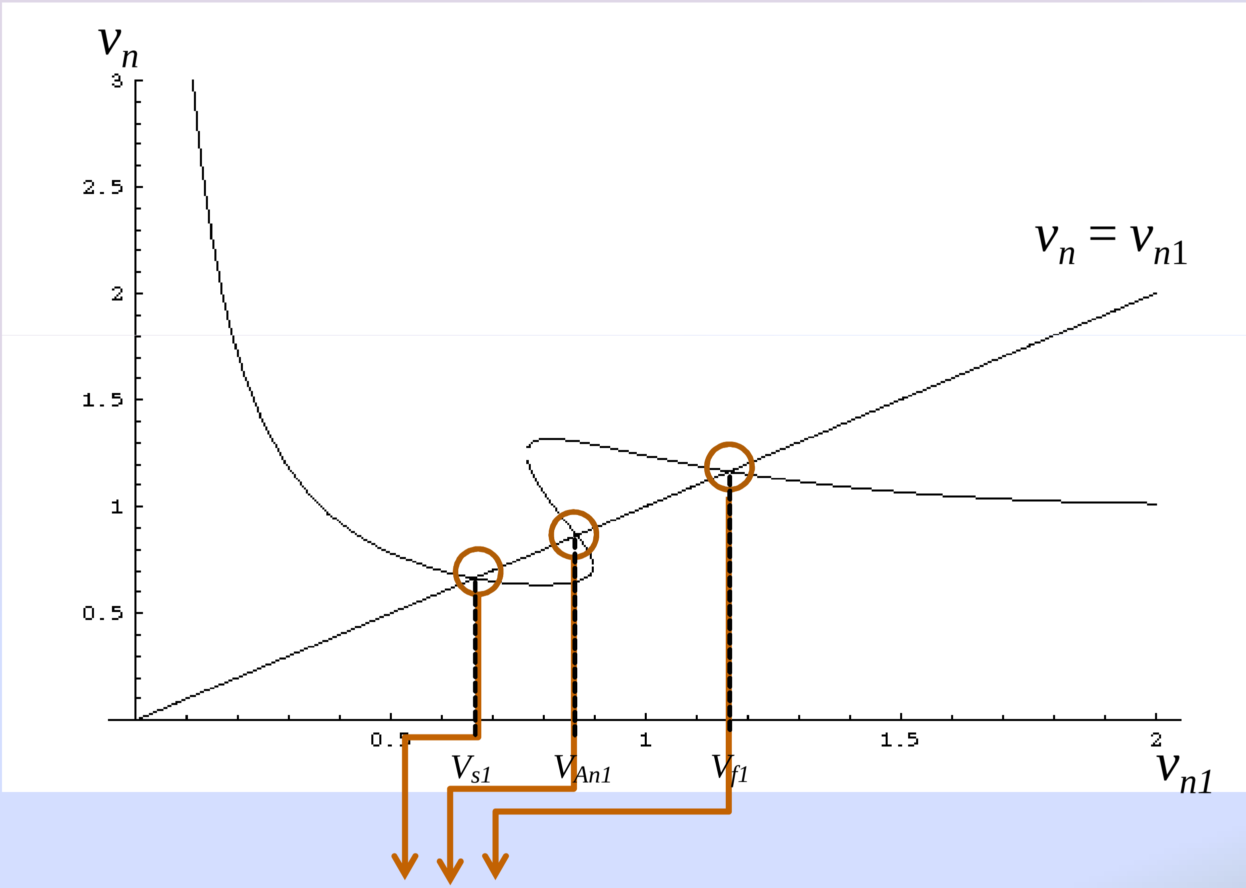
Rotational discontinuity

The rotational discontinuity

- v_n invariant $\Rightarrow \rho$ invariant $\Rightarrow$ no compression, no shock
- p and B invariant
- B_T and v_T invariant $\Rightarrow$ pure rotation
- Whalen relation : $\mathbf{v} = \dfrac{\mathbf{B}}{\sqrt{\mu_0 \rho}} = \mathbf{V}_A$ **Alfven velocity**

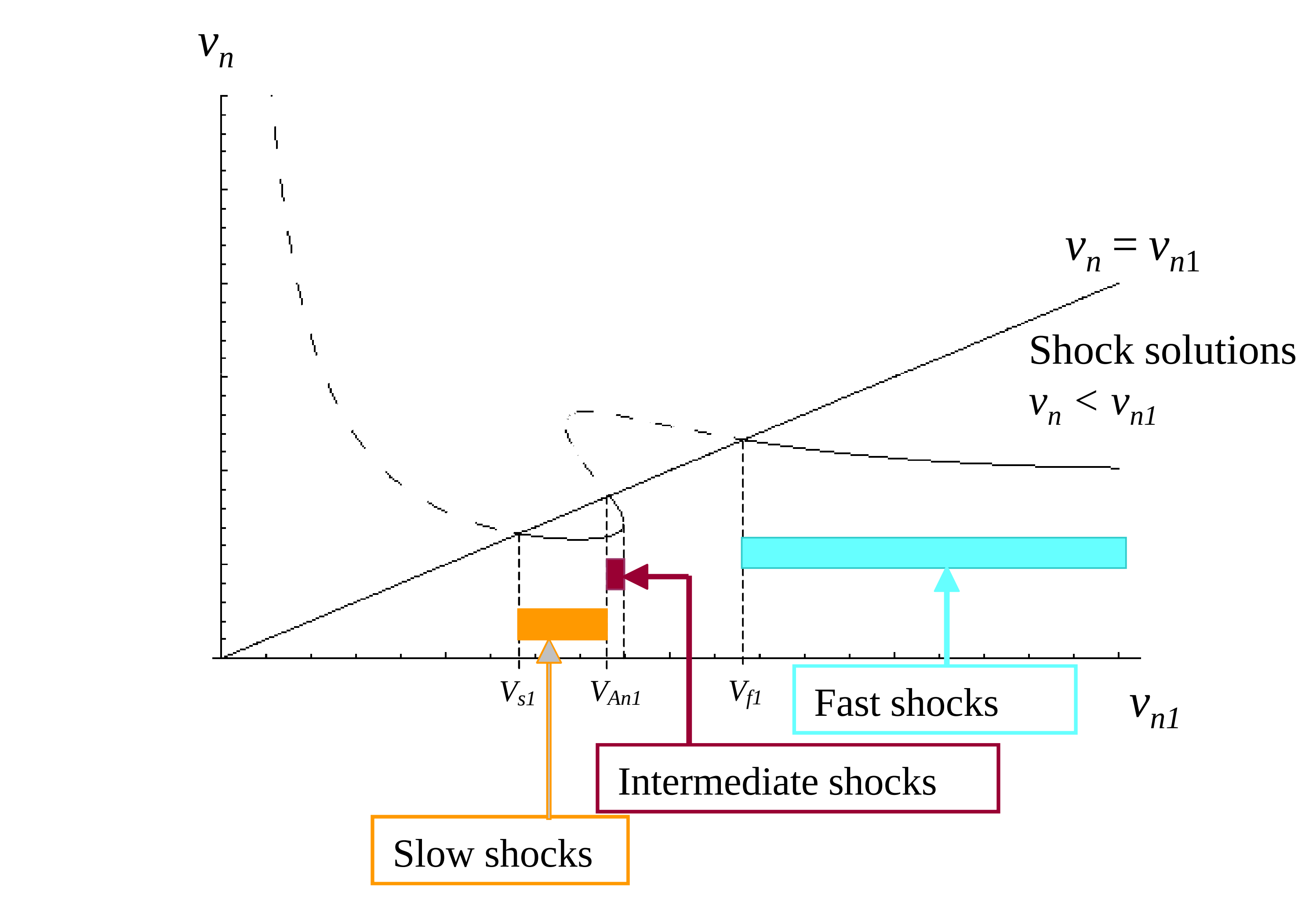
The different kinds of shocks

Full resolution → $v_n = f(v_{n1})$



Linear solutions of MHD equations → phase velocities of linear wave modes : slow, Alfven and fast

The different kinds of shocks



The different kinds of shocks

Slow shock	Intermediate shock	Fast shock
v_n ↘	v_n ↘	v_n ↘
ρ, p ↗	ρ, p ↗	ρ, p ↗
B_T ↘	B_T ↘ And changes sign	B_T ↗

The tangential discontinuity

- $v_n = B_n = 0$
- no de HT frame
- no relations between the two sides
- except invariance of $p + \dfrac{B^2}{2\mu_0}$

Summary

Coplanar discontinuity			Non-Coplanar discontinuity	
Slow shock	Intermediate Shock	Fast shock	Rotational discontinuity	Tangential discontinuity

And what about the boundaries around planets?

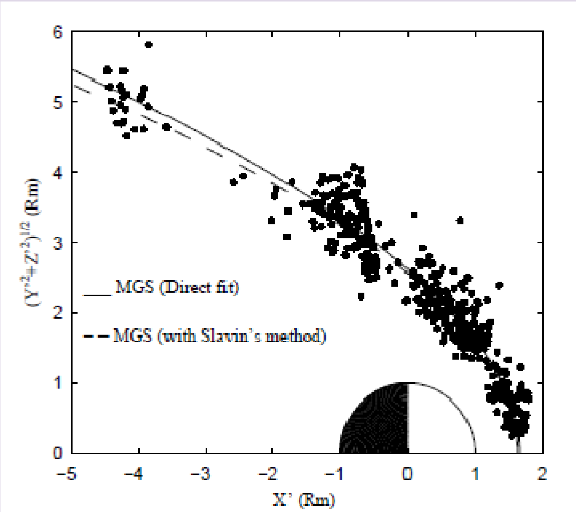
Nature of the obstacle

- The planet itself — Venus, Mars and *Pluton*
- The magnetic field of the planet — Earth, Mercury, Jupiter, Saturn, Uranus and Neptune

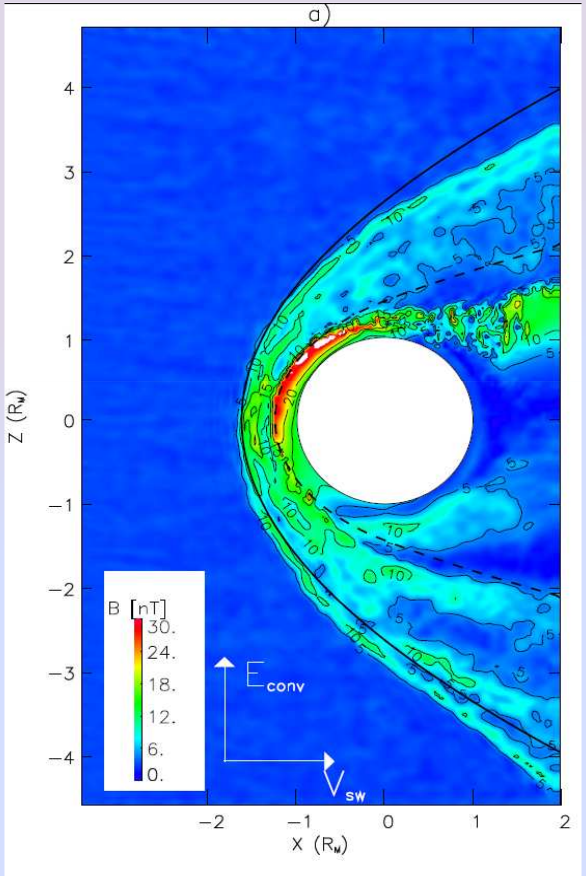
Mercury has a very small magnetic field

All the planets have been explored (more or less), except Pluton

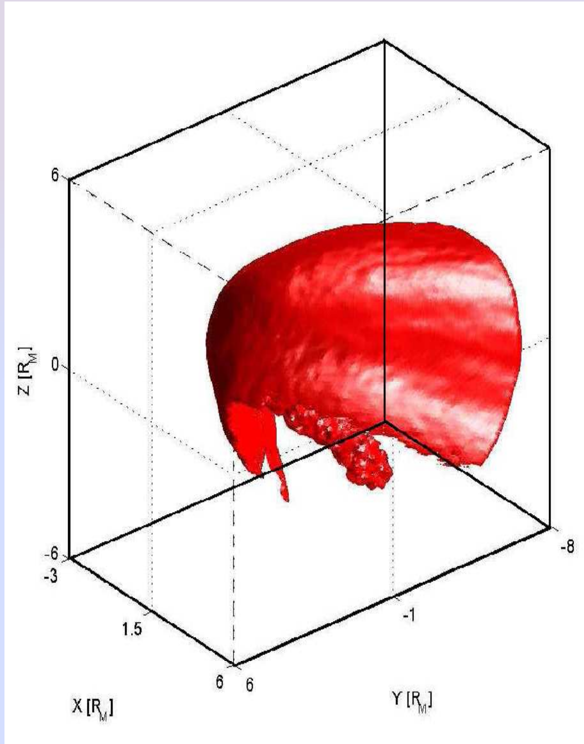
Mars



Martian shock fits from MGS observations in an aberrated MSO coordinate (Vignes et al, 2000)

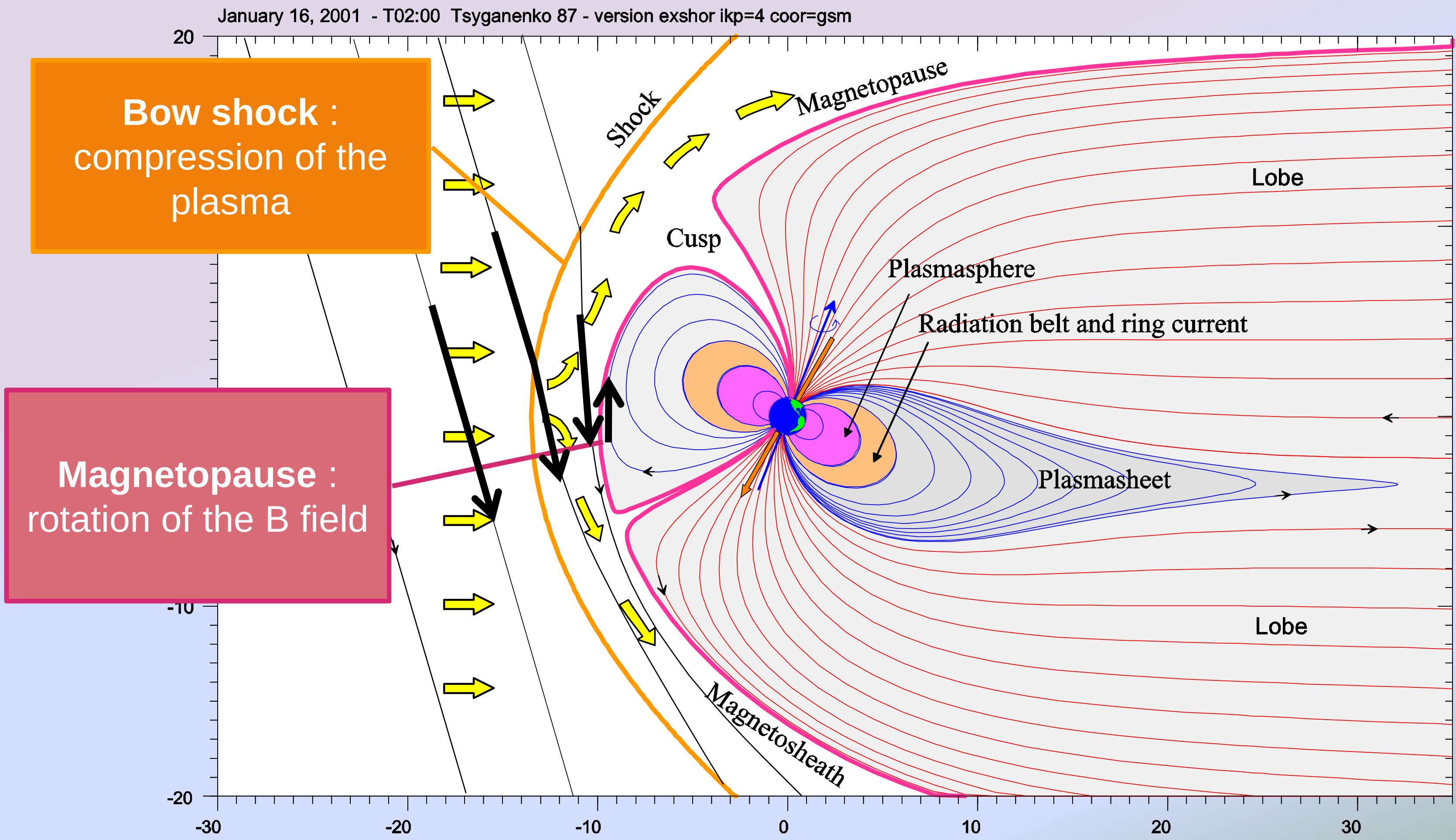


Intensity of the magnetic field [Modolo et. al, 2005]

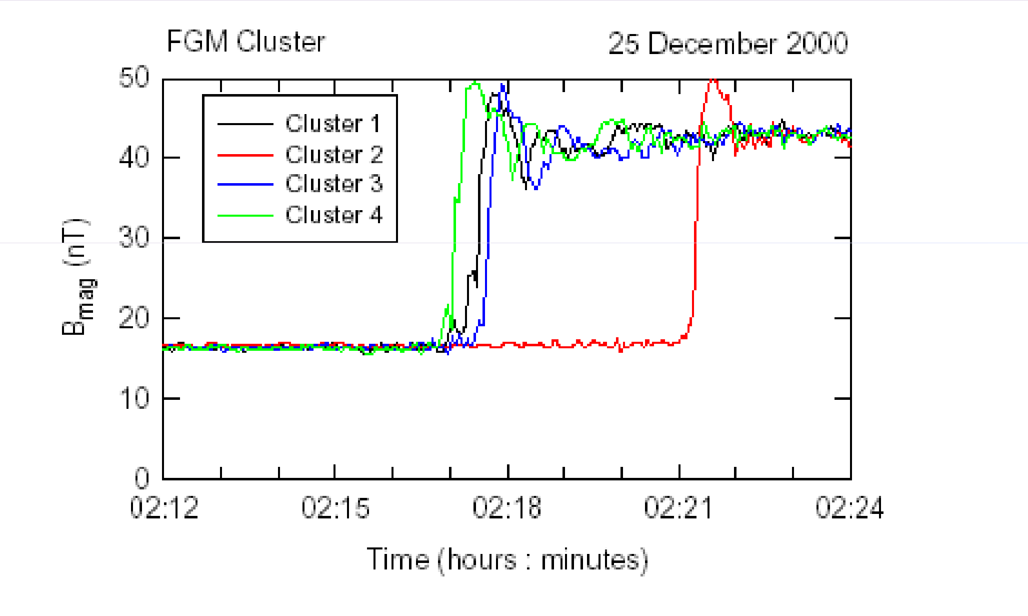


3D hybrid simulation of the shock wave:
iso-value of the B field strength (Emilie Richer)

2 discontinuities are observed in front of a magnetosphere



The Earth bow shock is a fast shock

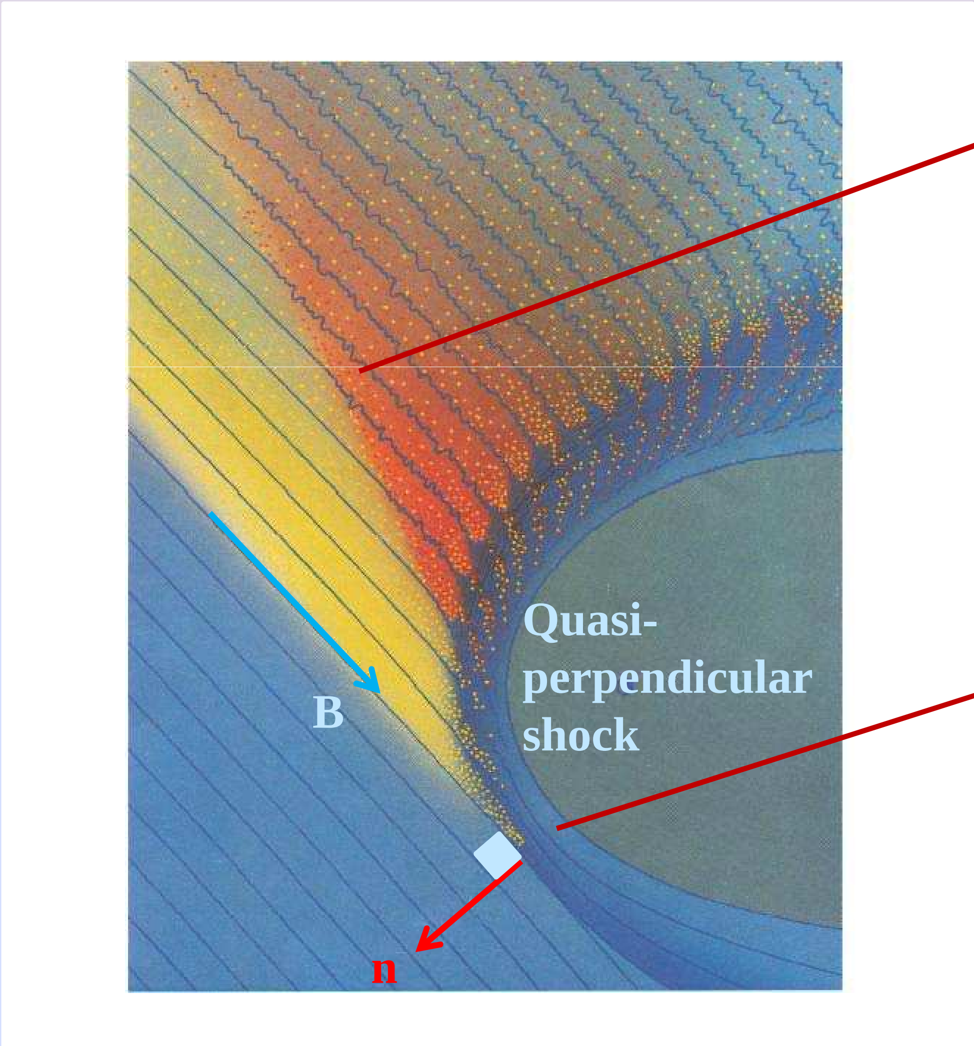


- Strong increase in B_T
- $V >> V_A$

Average parameters: $V \approx$ 450-700 km s^{-1}

$B \approx 3$ nT, $n \approx 10$ cm$^{-3} \Rightarrow V_A \approx 20$ km s^{-1}

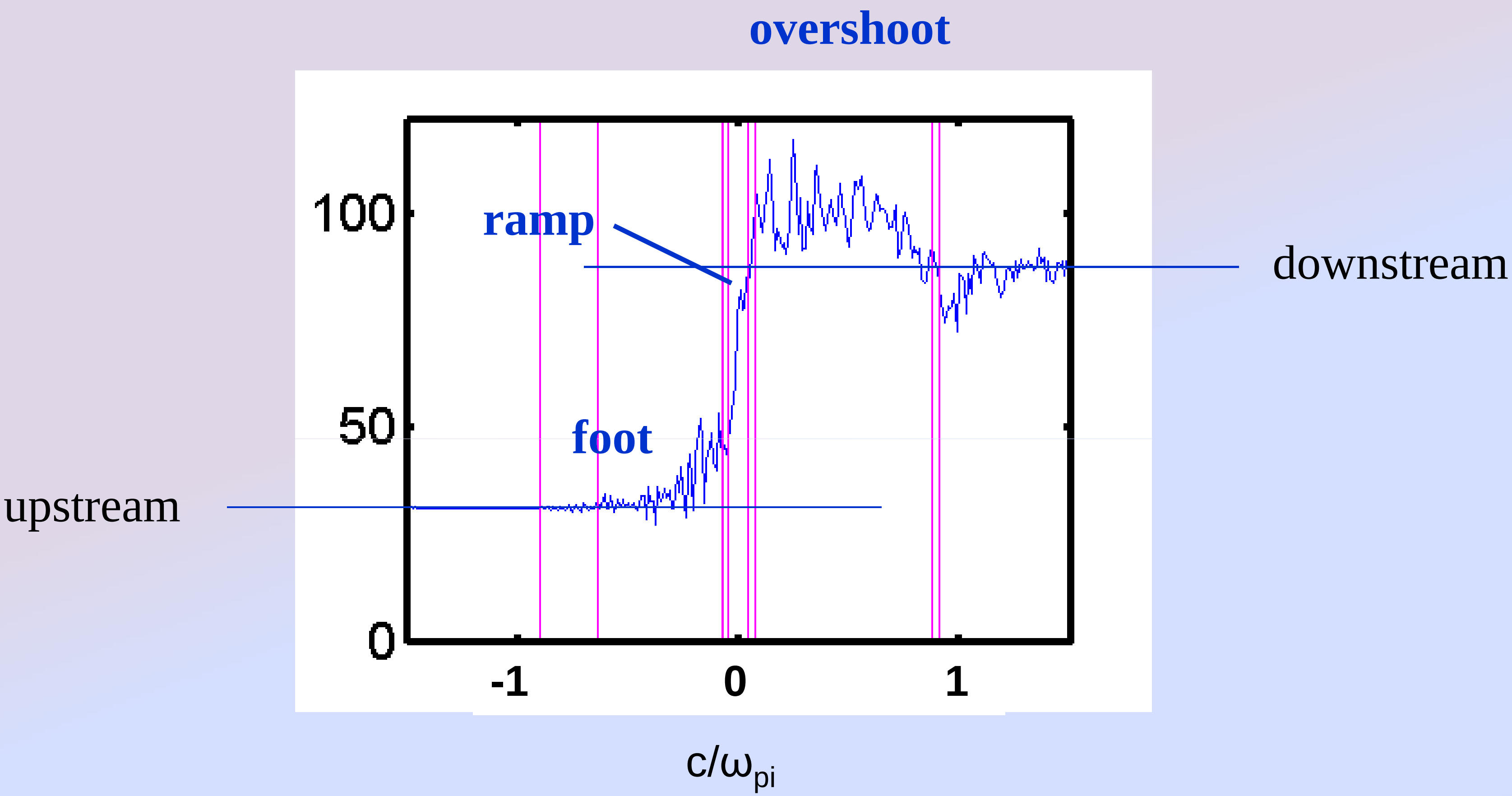
The shock is more complicated than in fluid theory



There is a foreshock: a backward influence of the magnetosphere, non-fluid effect

The physics strongly depend on the geometry

Detailed structure of the Earth bow shock



Multipoint measurements (CLUSTER) allow:

- Determination of the normal
- Determination of the thickness
- Determination of the velocity

[Mazelle et al, 2010]

Results on the properties of quasi-perpendicular shocks

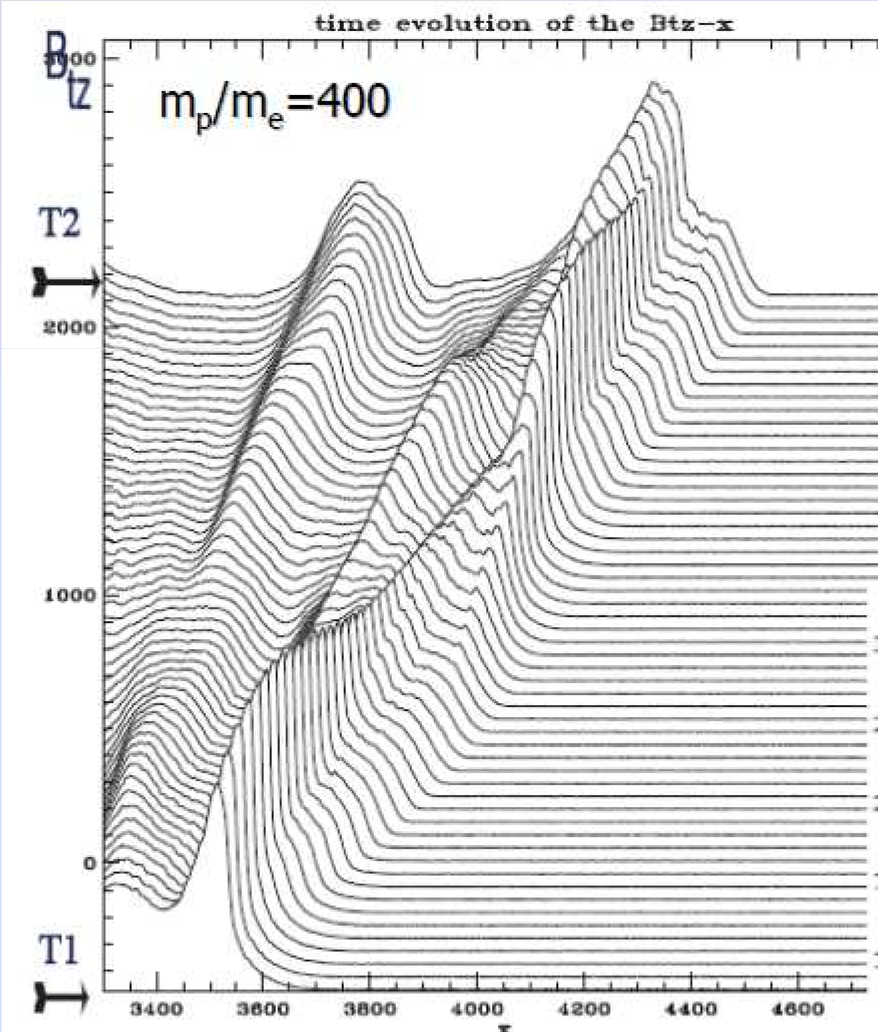
•The ramp is often very thin (electron scale) at least for 75°< $\Theta_{\mathbf{Bn}}$ <90°

•$L_{foot} < \rho_{i, upstream}$

•Signatures of cyclic self-reformation as predicted by 1D/ 2D PIC simulations

•No stationarity

[Mazelle, 2010, Lembège and Lebugle, 2005]



What is the nature of the magnetopause ?

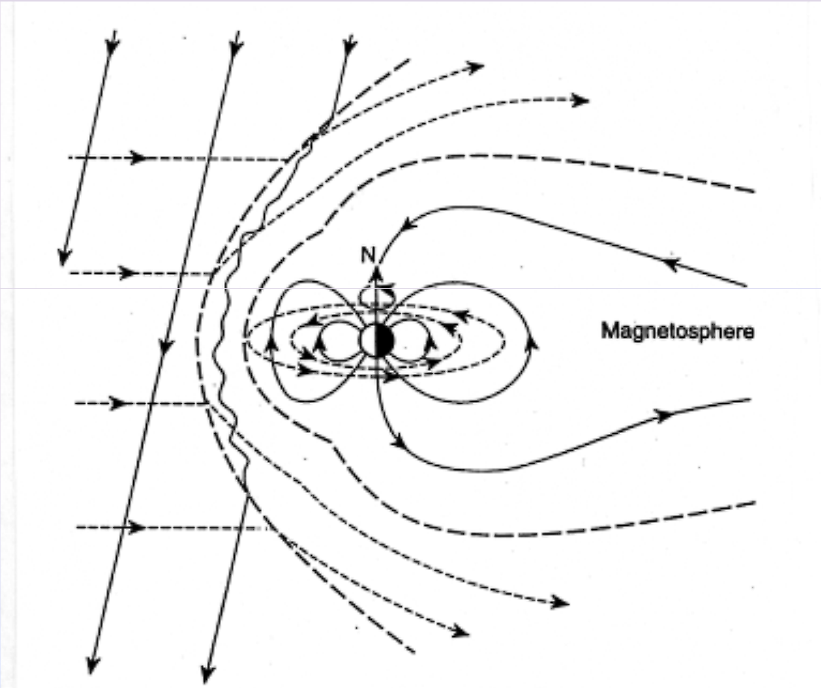
• A rotation of the field is observed : from the Solar wind direction to the Earth magnetic field direction $\Rightarrow$ it is not a coplanar discontinuity, it is not a shock

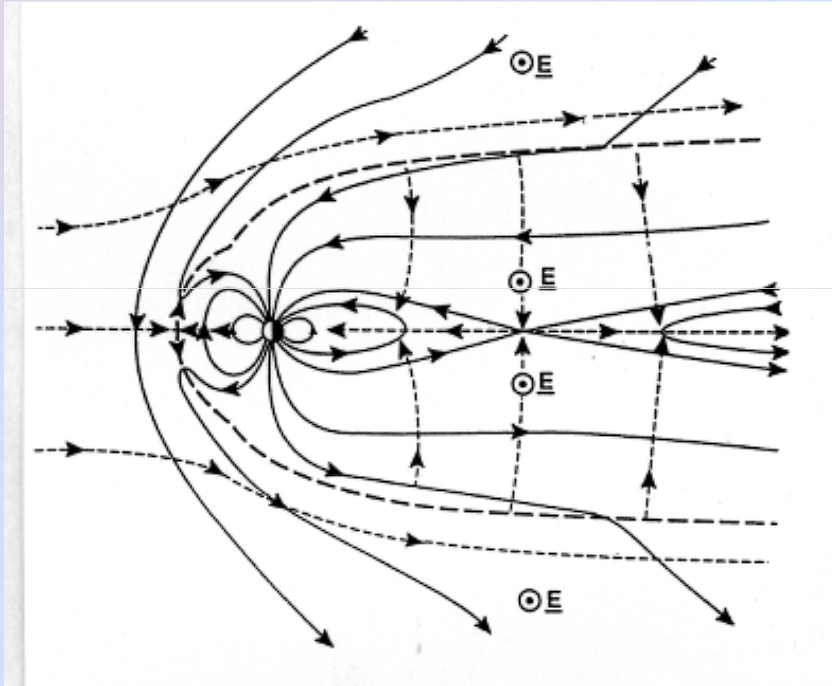
• It can be a **tangential discontinuity** or a **rotational discontinuity (**or something more **complicated) ?**

The physical consequences of the two situations are very different $\Rightarrow$ the diagnostic tools to distinguish between the two are crucial

Tangential or rotational ?

Is it closed or open ?





i.e. $B_n = 0$

i.e. $B_n \neq 0$

Experimental diagnostics of the magnetopause

1) Is the magnetopause connected ➔ $B_n \neq 0$ (reconnection happens somewhere) ?

otherwise B = B_T (tangential discontinuity)

Measurement is difficult because B_n always small

B_n calculated with Minimum Variance Analysis : precision problem

2) Is the magnetopause a "discontinuity" ⇔ 1-D (planar), stationnary, with a small thickness ?

Test : does the deHoffman-Teller frame exist $\mathbf{E} = 0$ ($\mathbf{v} = \mathbf{v}_{//}$) ?

3) Is the magnetopause a rotational discontinuity ?

Walen test $\mathbf{v} = \mathbf{V}_A + \mathbf{V}_{HT}$

Conclusion

- Different kinds of discontinuities are observed in the solar system
- The space measurements give lots of informations about the physics
- Especially since multi-point measurements are available and there is still a lot to be done
- Wave instruments show that these boundaries are home to an important turbulence