Stellar Winds, MHD and Disks

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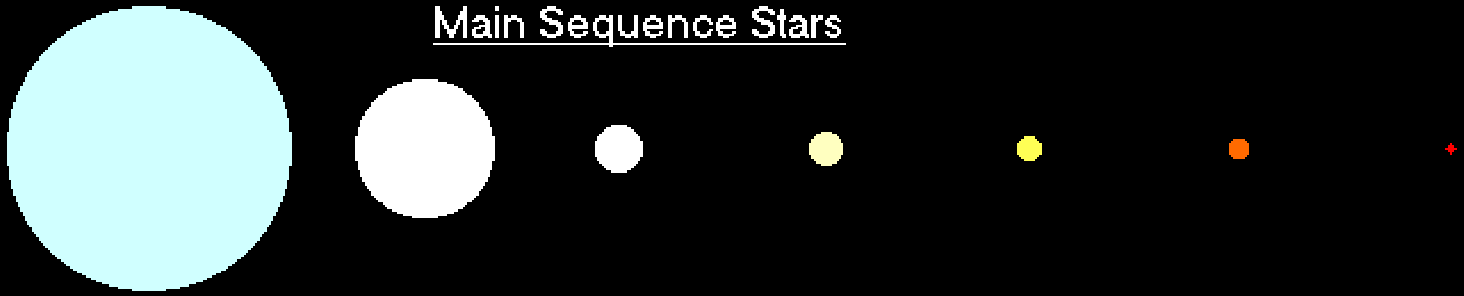
Outline

- Stellar Winds
- Gas pressure driven winds
- Radiative driving
- Magnetic Confinement
- Rotation/Disks/Spindown

Main Sequence Stars



Spectral Type:	O	B	A	F	G	K	M
Temperature:	40 000K	20 000K	8500K	6500K	5700K	4500K	3200K
Radius (Sun=1):	10	5	1.7	1.3	1.0	0.8	0.3
Mass (Sun=1):	50	10	2.0	1.5	1.0	0.7	0.2
Luminosity (Sun=1):	100 000	1000	20	4	1.0	0.2	0.01
Lifetime (million yrs):	10	100	1000	5000	10 000	50 000	100 000
Abundance:	0.00001%	0.05%	0.3%	1.5%	4%	9%	80%

Giant Stars

Low mass stars near the end of their life.

Spectral Type:	G, K or M
Temperature:	4000K
Radius (Sun=1):	20
Mass (Sun=1):	1.2
Luminosity (Sun=1):	200
Lifetime (million yrs):	10
Abundance:	0.5%

White Dwarfs

Dying remnant of an imploded star.

Spectral Type:	D
Temperature:	Under 50 000K
Radius (Sun=1):	Under 0.01
Mass (Sun=1):	Under 1.4
Luminosity (Sun=1):	Under 0.01
Lifetime (million yrs):	–
Abundance:	5%

Supergiant Stars

High mass stars near the end of their life.

Spectral Type	O, B, A, F, G, K or M
Temperature:	4000 to 40 000K
Radius (Sun=1):	30 to 500
Mass (Sun=1):	10 to 70
Luminosity (Sun=1):	30 000 to 1000 000
Lifetime (million yrs):	10
Abundance:	0.0001%

r powell

Whirlpool Galaxy

Massive stars dominate the light from galaxies

Mass Loss from Stars

All stars lose mass, the **continuous** outflow is called the **Stellar Wind**

Stars like the sun lose very little mass ($\sim 10^{-14}$ M_{Sun}/yr)
Solar wind is driven by **gas pressure gradient**

Hot stars (O and B type) lose enormous amount of material ($10^{-9} \sim 10^{-5}$ M_{Sun}/yr)
Hot star winds are driven by scattering of **radiation** by resonance lines of heavy ions.

Sound speed; thermal pressure has little significance.

Solar corona & wind

- Solar corona
  - high T => high P_{gas}
  - scale height $H \leq R$
  - breakdown of hydrostatic equilibrium
  - pressure-driven solar wind expansion
- How does magnetic field alter this?
  - closed loops => magnetic confinement
  - open field => coronal holes
  - source of high speed solar wind

Hydrostatic Scale Height

Hydrostatic equilibrium:

$$-\frac{GM}{r^2} = \frac{1}{\rho}\frac{dP}{dr} \equiv \frac{a^2}{H}$$

$$P = \rho a^2$$

Scale Height:

$$\frac{H}{R} = \frac{a^2 R}{GM} \approx \frac{T_6}{14}$$

solar photosphere: $T_6 = 0.006$

$$\frac{H}{R} \approx \frac{1}{2000}$$

solar corona: $T_6 = 2$

$$\frac{H}{R} \approx \frac{1}{7}$$

Failure of hydrostatic equilibrium for hot, isothermal corona

hydrostatic equilibrium:

$$0 = -\frac{GM}{r^2} - \frac{a^2}{P}\frac{dP}{dr}$$

$$\frac{P(r)}{P_o} = \exp\left[-\frac{R}{H}\left(1 - \frac{R}{r}\right)\right] \rightarrow \exp\left[-\frac{R}{H}\right] \quad for\ r \rightarrow \infty$$

decades of pressure decline:

$$\log\left(\frac{P_o}{P_\infty}\right) = \frac{R}{H}\log e \approx \frac{6}{T_6}$$

observations :

$$\log\left(\frac{P_0}{P_{ISM}}\right) = 12$$

Solar corona T_6 ~ 2, pressure too high => corona must expand!

Spherical Expansion of Isothermal Solar Wind

Momentum and Mass Conservation:

$$\mathrm{v}\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{GM}{r^2} - \frac{a^2}{\rho}\frac{d\rho}{dr} \qquad \frac{d\ \ \rho \mathrm{v} r^2}{dr} = 0$$

Combine to **eliminate density**:

$$\left(1\text{-}\frac{\mathrm{a}^2}{\mathrm{v}^2}\right)\mathrm{v}\frac{\mathrm{dv}}{\mathrm{dr}} = \frac{2\mathrm{a}^2}{r} - \frac{GM}{r^2}$$

RHS=0 at "critical" radius:

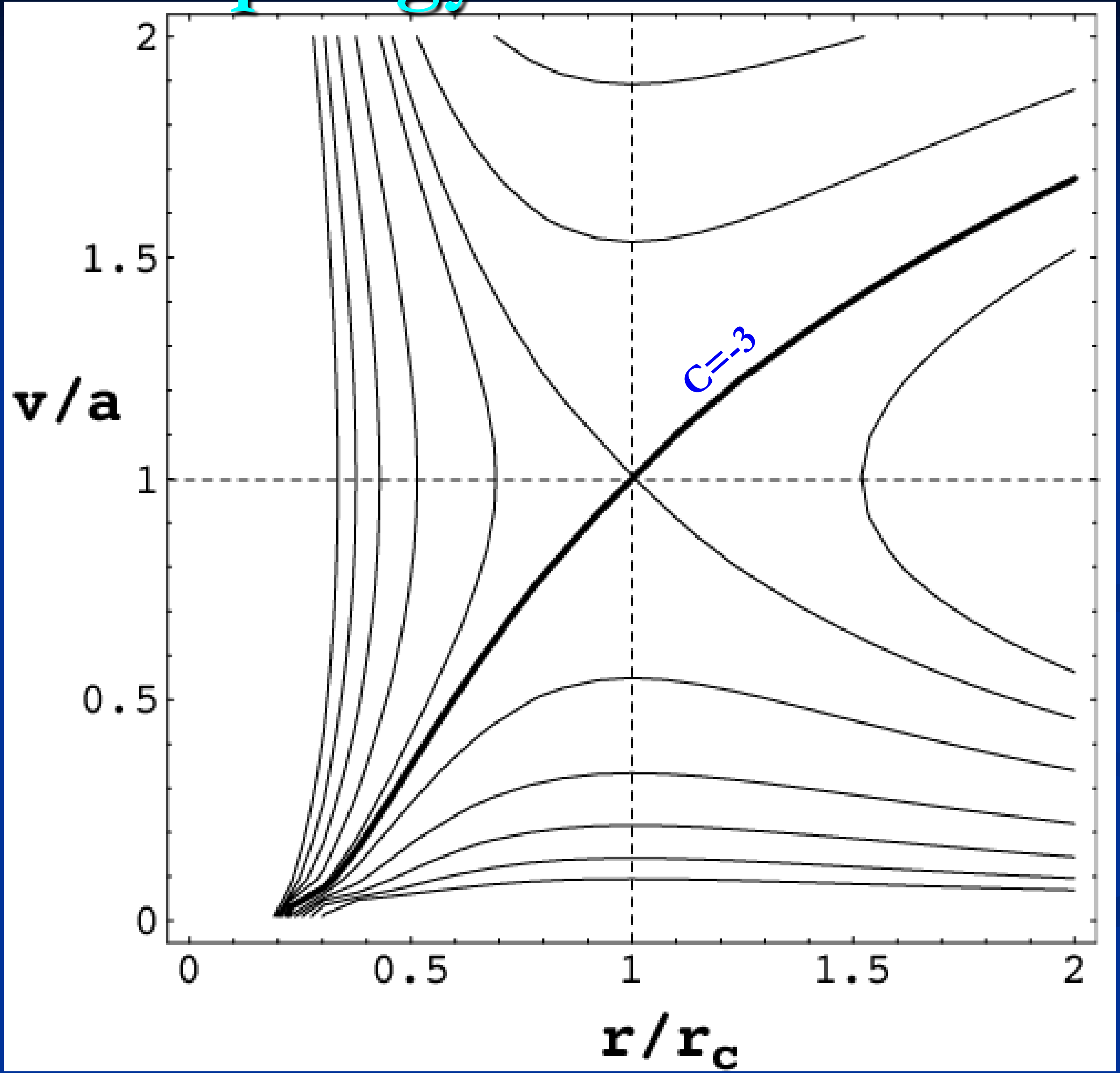
$$r_c = \frac{GM}{2a^2}$$

Integrate for transcendental soln:

$$\frac{\mathrm{v}^2}{a^2} - \ln\frac{\mathrm{v}^2}{a^2} = 4\ln\frac{r}{r_c} + \frac{4r_c}{r} + C$$

$C = -3$ => Transonic soln: $\mathrm{v}\ \ r_c \equiv a \qquad r_c = r_s$ **sonic** radius

Solution topology for isothermal wind



How about massive star wind?

- Stellar Photosphere
  - high luminosity, UV heating => isothermal
  - low T => low sound speed
  - line-driven stellar wind expansion

Light's Momentum

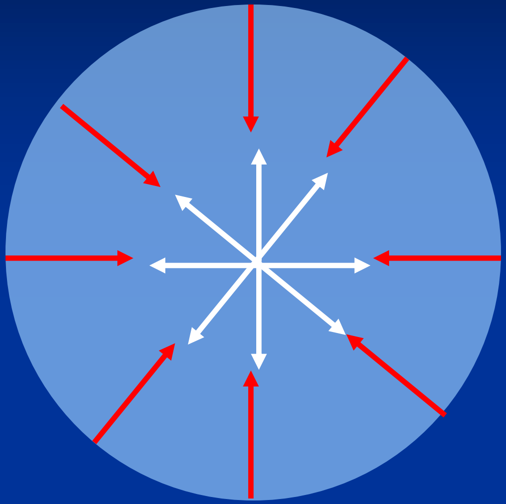
- Light transports energy (& information)
- But it also has momentum, $p=E/c$
- Usually neglected, because c is very high
- But becomes significant for very bright stars,
- Key question: how big is force vs. gravity??

Light As a Driving Mechanism

- Free Electron (continuum) Scattering
- Bound Electron (line) Scattering
  - Can be much stronger than free electron scattering

details

Driving by free e scattering



Radiative Force

$$g_e = \frac{\kappa_e L}{4\pi r^2 c}$$

Gravitational Force

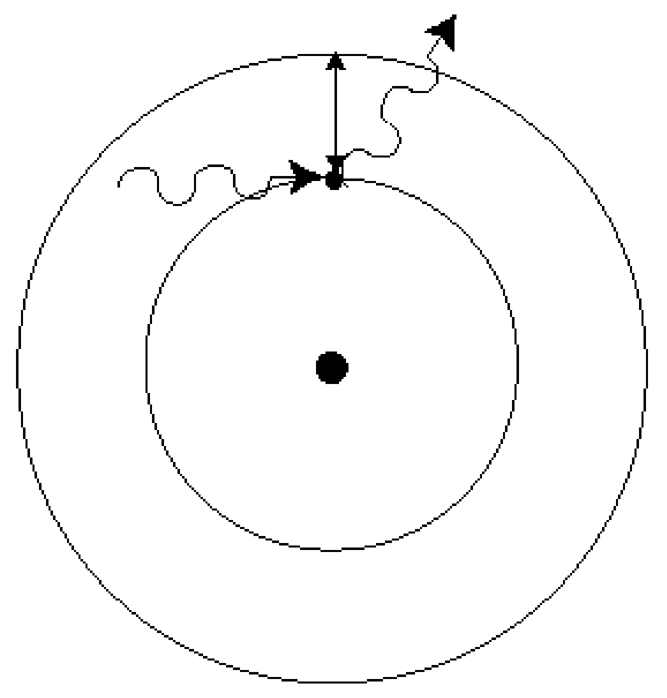
$$\frac{GM}{r^2}$$

$$\Gamma_e \equiv \frac{g_e}{g} = \frac{\kappa_e L / 4\pi r^2 c}{GM / r^2} = \frac{\kappa_e L}{4\pi GMc}$$

$$\Gamma_e \approx 2\times 10^{-5} \frac{L / L_{Sun}}{M / M_{Sun}} \frac{\overline{\kappa_F}}{\kappa_e}$$

for hot stars
Γ~0.5

Driving by Line-Opacity: Thin Lines



for high **Q**uality **L**ine **R**esonance,

cross section >> electron scattering

$Q \sim \nu\,\tau \sim 10^{15}$ Hz * 10^{-8} s $\sim 10^{7}$

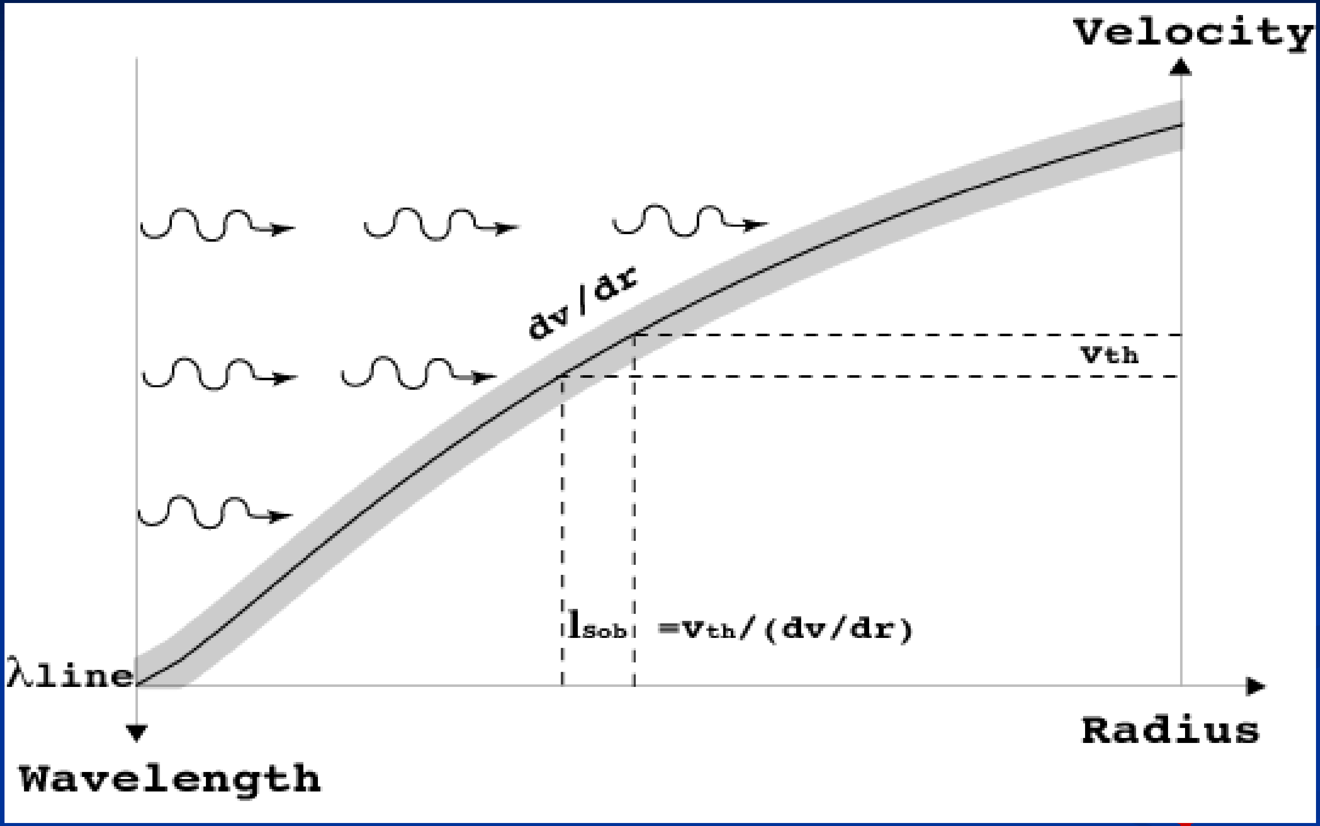
$\bar{Q} \sim Z\,Q \sim 10^{-4}\,10^{7} \sim 10^{3}$

$$\kappa_{lines} \sim \bar{Q} \times \kappa_e$$

$$g_{lines} \sim 10^3 \times g_{el}$$

$$\Gamma_{thin} \sim Q\Gamma_e \sim 1000\Gamma_e$$

The Other Extreme: Optically Thick Line-Absorption in an Accelerating Stellar Wind



$$L_{sob} << R_*$$

For strong, **optically thick** lines:

$$g_{thick} \sim \frac{g_{thin}}{\tau} \sim \boxed{\frac{1}{\rho}\frac{dv}{dr}}$$

$$\tau \equiv \kappa\rho \boxed{\frac{v_{th}}{dv/dr}} \sim \frac{v_{th}}{v_\infty} R_*$$

Line Force From an Ensemble of Lines in CAK theory

If we take into account all available thick and thin lines, the line force is:

$$g_{lines} \approx \overline{Q}\,\frac{\kappa_e L}{4\pi r^2 c}\left(\frac{dv/dr}{\rho c\overline{Q}\,\kappa_e}\right)^{\alpha}$$

Free e^- scattering

α , the fraction of thick lines compared to thin lines

details

CAK Line-Driven Wind for OB stars

combination of
thin and thick lines

$$\Gamma_{lines} \sim \frac{Q\Gamma}{\tau^{\alpha}} \sim \frac{Q\Gamma^{1-\alpha}}{\dot{M}^{\alpha}} \sim 1$$

$$\dot{M} \sim \frac{L}{c^2}\left[\frac{Q\Gamma}{1-\Gamma}\right]^{(1-\alpha)/\alpha} \sim 10^{-6}\,\frac{M_{Sun}}{yr}$$

$$V(r) \approx V_{\infty}(1-R_{*}/r)^{\beta} \qquad V_{\infty} \sim V_{esc} \sim \sqrt{g}$$

Hydrodynamic Equations

$$\frac{D\rho}{Dt}+\rho\nabla\bullet v=0$$

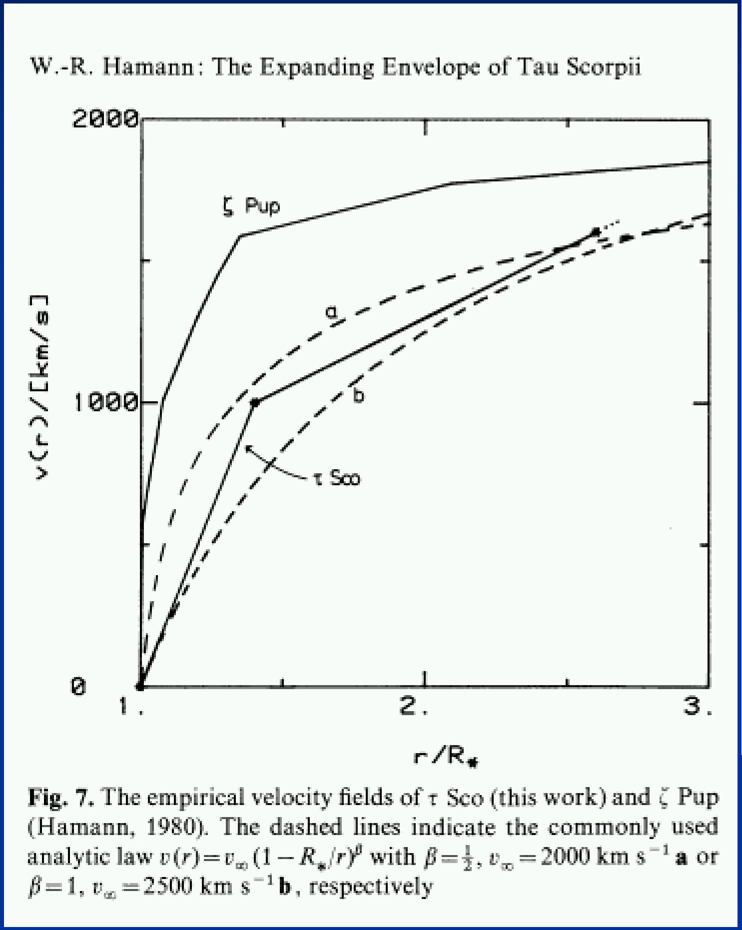
Mass conservation

$$\frac{Dv}{Dt}=-\frac{1}{\rho}\nabla p+g_{grav}+g_{lines}$$

Momentum

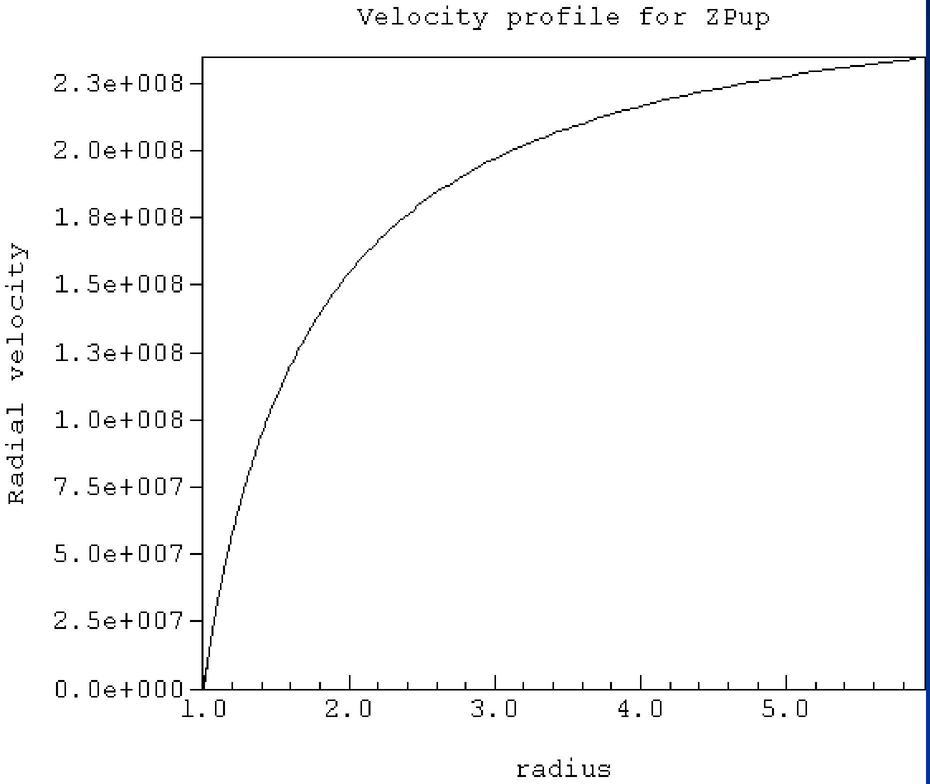
Assume ***isothermal wind***: energy equation redundant
Add: Finite Disk Correction factor

1D CAK Solutions vs Observations



W.-R. Hamann: The Expanding Envelope of Tau Scorpii

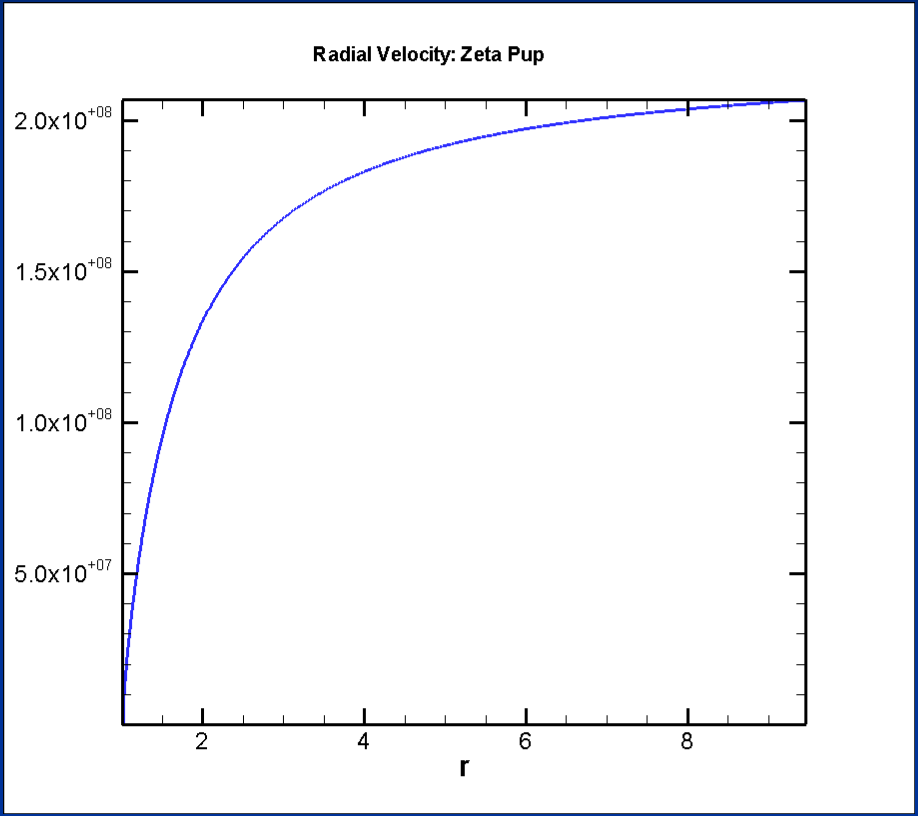
Fig. 7. The empirical velocity fields of τ Sco (this work) and ζ Pup (Hamann, 1980). The dashed lines indicate the commonly used analytic law $v(r)=v_\infty(1-R_*/r)^\beta$ with $\beta=\frac{1}{2}$, $v_\infty=2000$ km s^{-1} **a** or $\beta=1$, $v_\infty=2500$ km s^{-1} **b**, respectively



Simulations: Steady & spherically symmetric

1D CAK Model of ZPup

Using **AMRVAC**

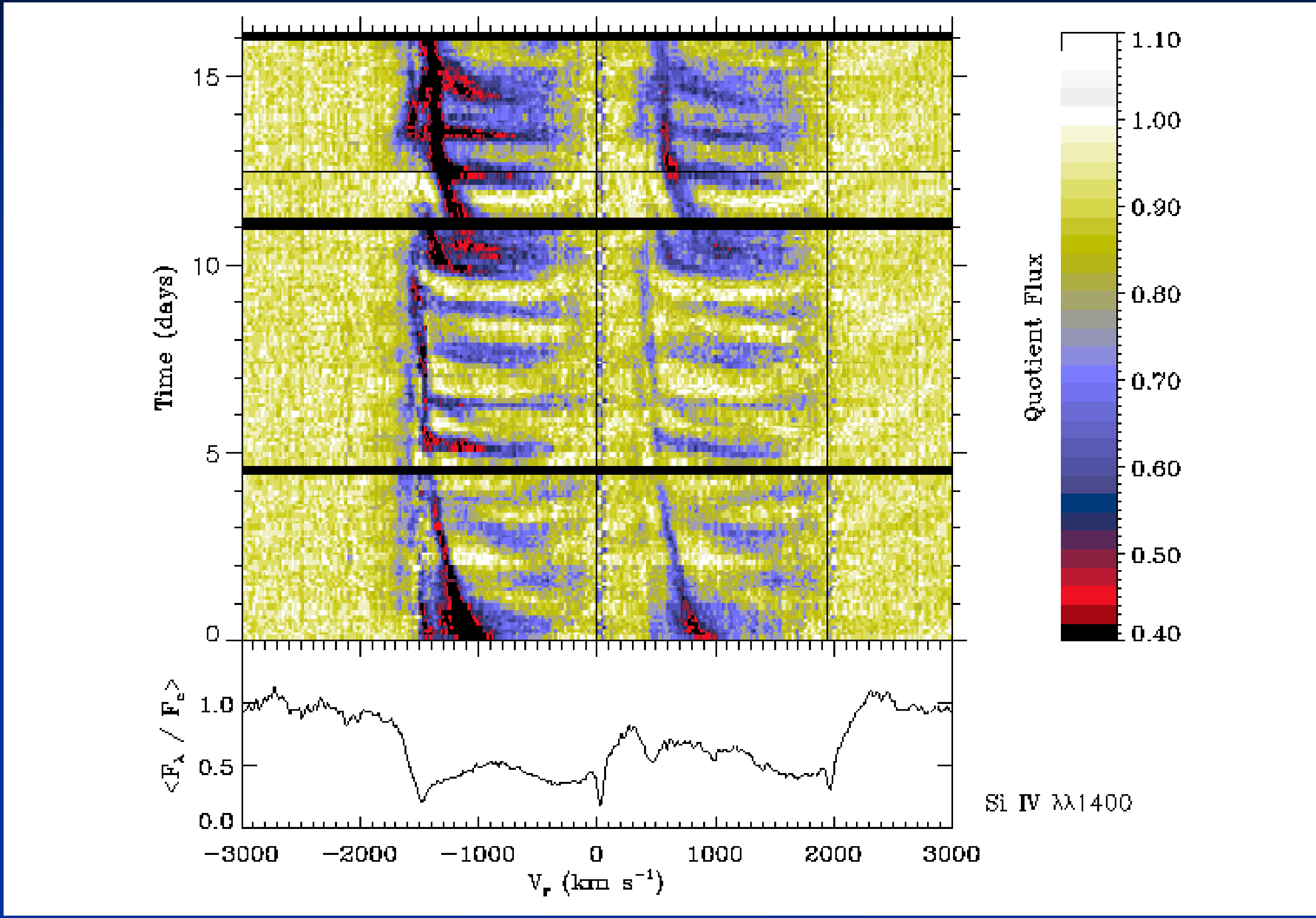


well characterized by

$$v(r) = v_{\infty}(1 - R_{*}/r)^{\beta}$$

Discrete Absorption Features

HD 64760
(B Star)



CAK Line-Driven Wind for OB stars

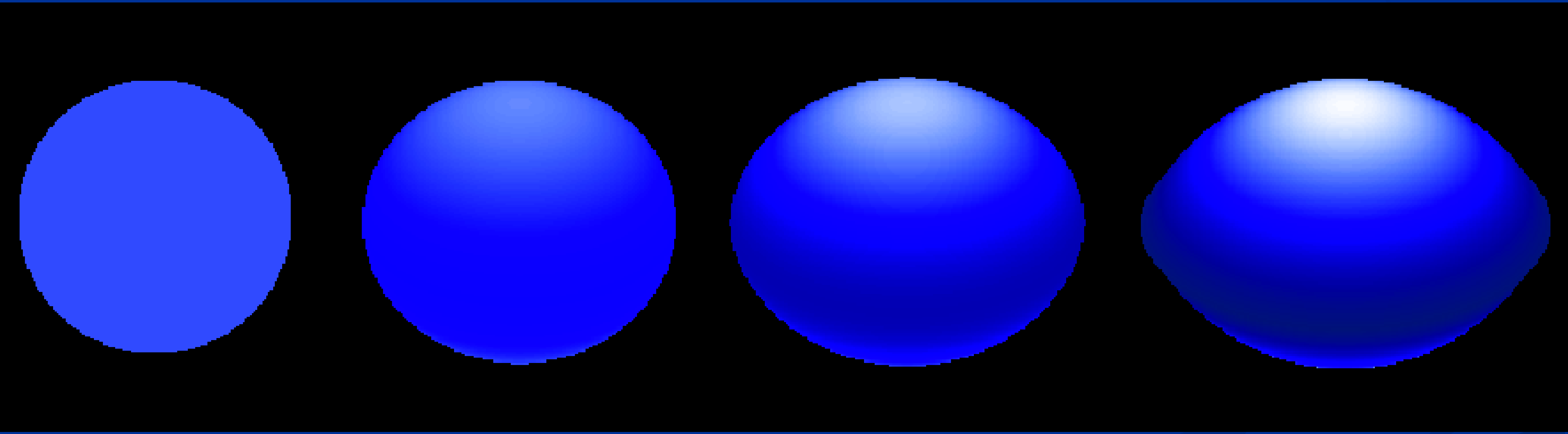
Mass flux can be latitude dependent

$$\dot{m}(\theta) \sim \frac{F(\theta)}{c^2} \left[\frac{Q\Gamma}{\underbrace{1-\Gamma}_{g_{eff}(\theta)}} \right]^{(1-\alpha)/\alpha}$$

$$V_\infty(\theta) \sim V_{esc}(\theta) \sim \sqrt{g(\theta)}$$

Gravity Darkening

increasing stellar rotation →



the gravity and the flux the highest at the poles

Effect of gravity darkening on line-driven mass flux

Recall:

$$\dot{m}(\theta) \sim \frac{F(\theta)^{1/\alpha}}{g_{eff}(\theta)^{1/\alpha-1}} \sim \frac{F^2(\theta)}{g_{eff}(\theta)}$$

e.g., for

$$\alpha = 1/2$$

w/o gravity darkening,
if F(θ)=const.

$$\dot{m}(\theta) \sim \frac{1}{g_{eff}(\theta)}$$

highest at equator

w/ gravity darkening,
if F(θ)~g_{eff}(θ)

$$\dot{m}(\theta) \sim F(\theta)$$

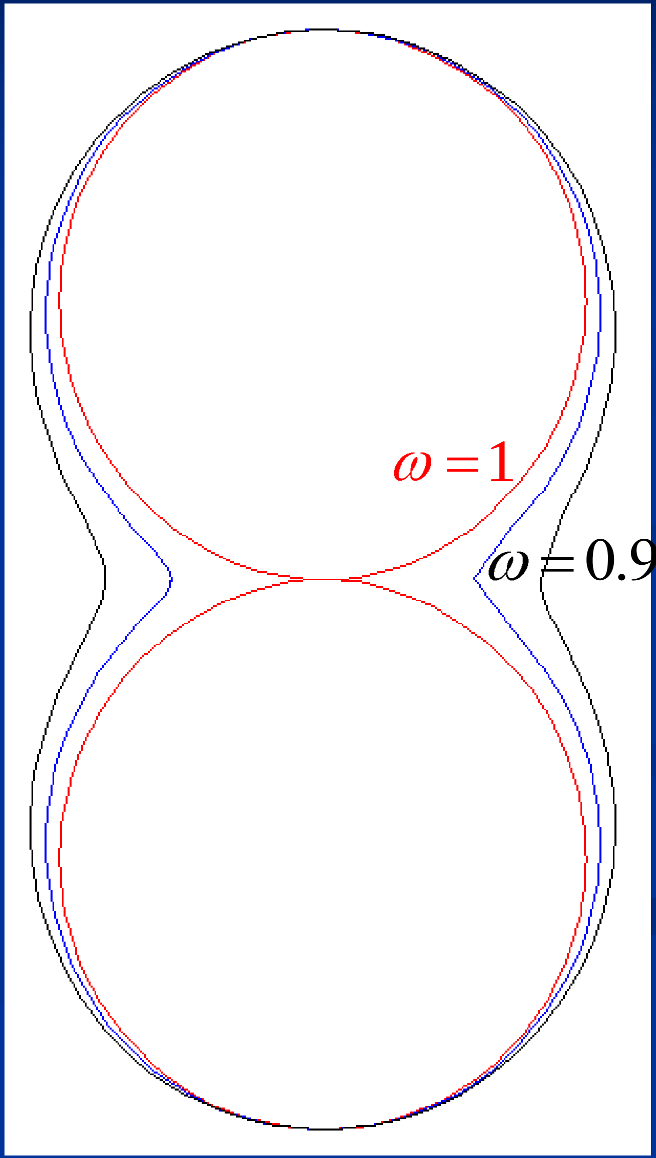
highest at pole

Effect of rotation on flow speed

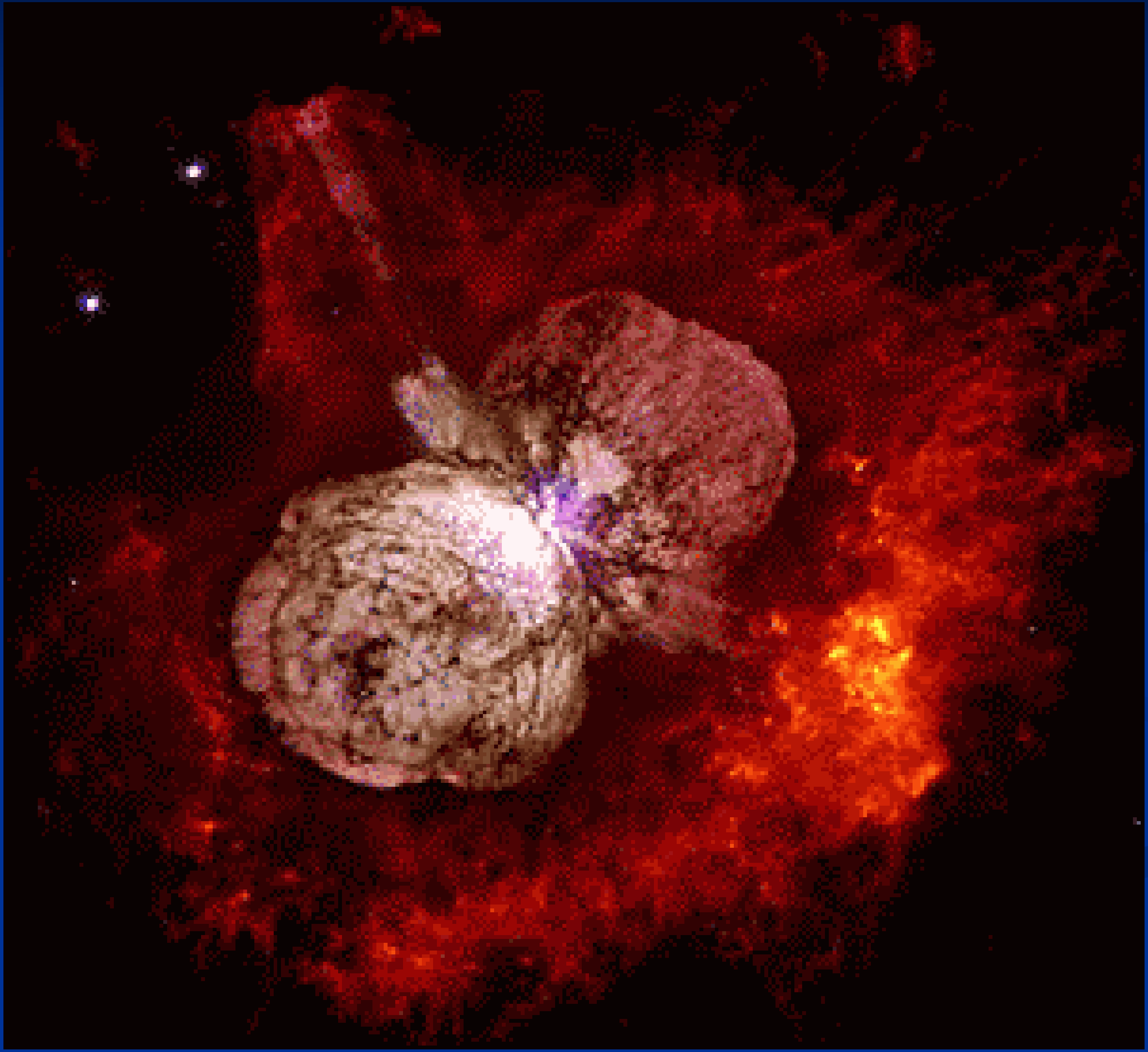
$$V_\infty(\theta) \sim V_{eff}(\theta) \sim \sqrt{g_{eff}(\theta)}$$

$$g_{eff}(\theta) \sim 1-\omega^2\, Sin^2\theta$$

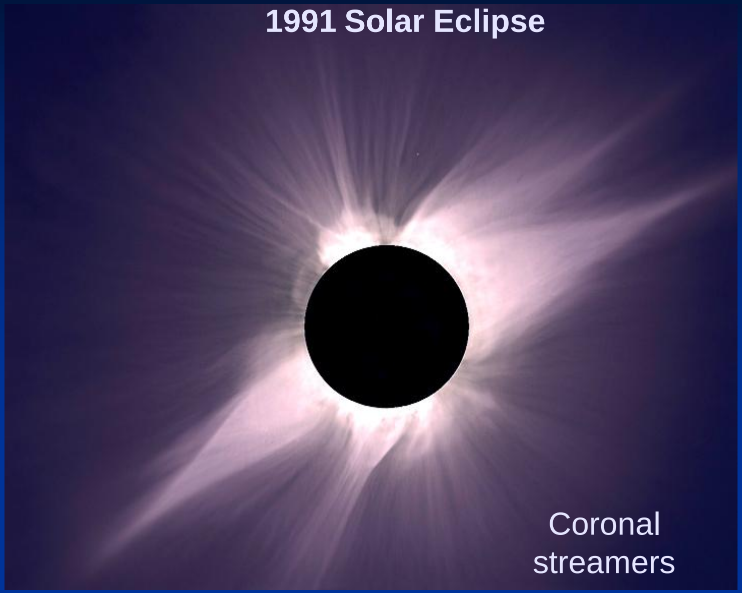
$$\omega \equiv \Omega / \Omega_{crit}$$



Eta Carinae

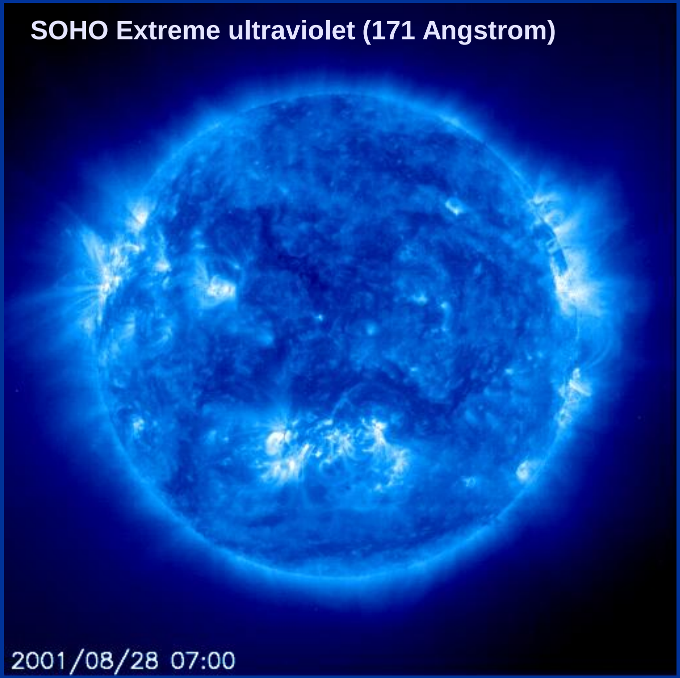


Magnetic Effects on Solar Coronal Expansion



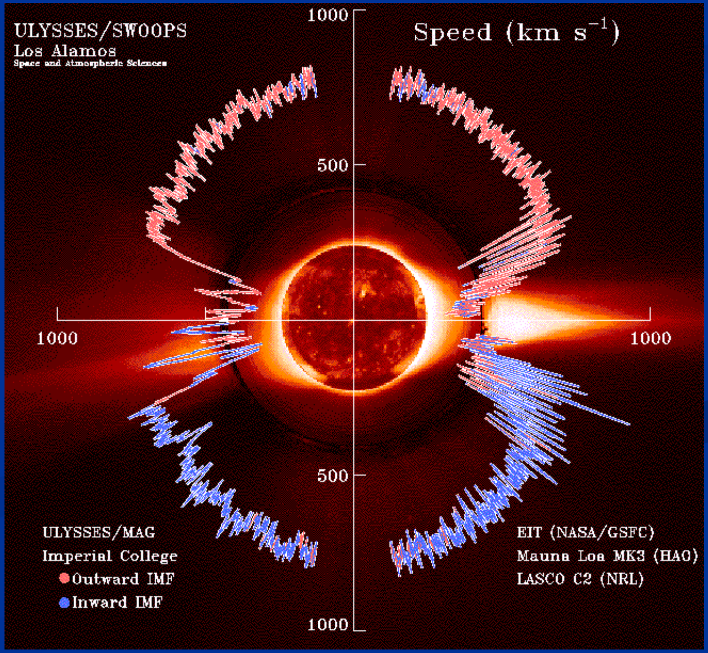
1991 Solar Eclipse

Coronal streamers



SOHO Extreme ultraviolet (171 Angstrom)

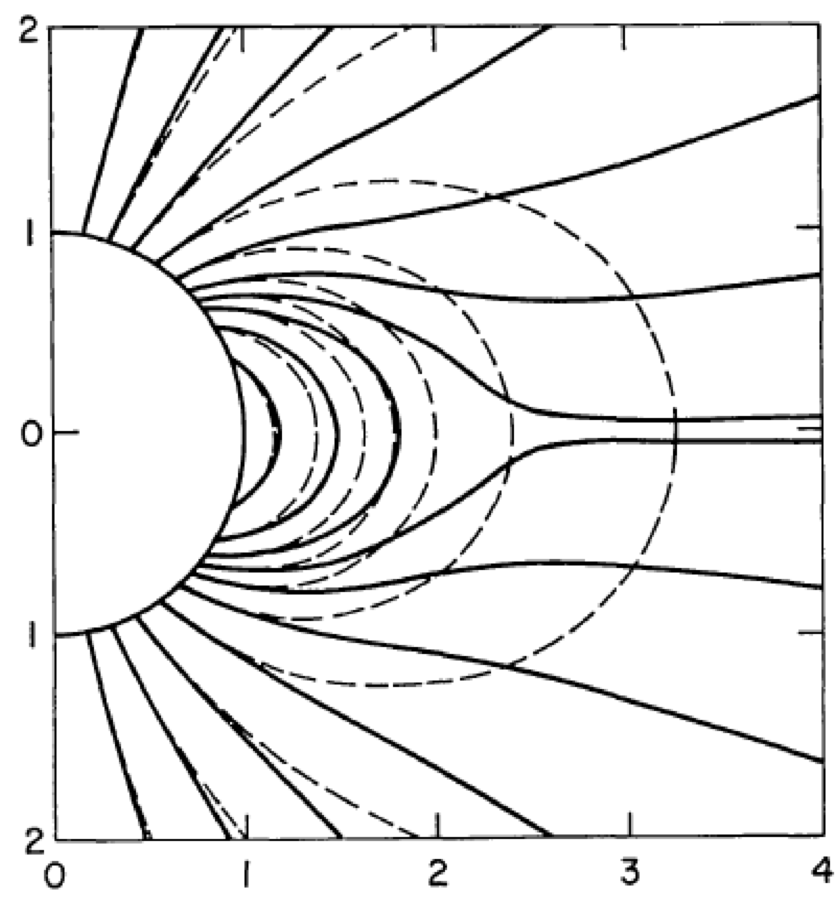
2001/08/28 07:00



Pneuman and Kopp (1971)

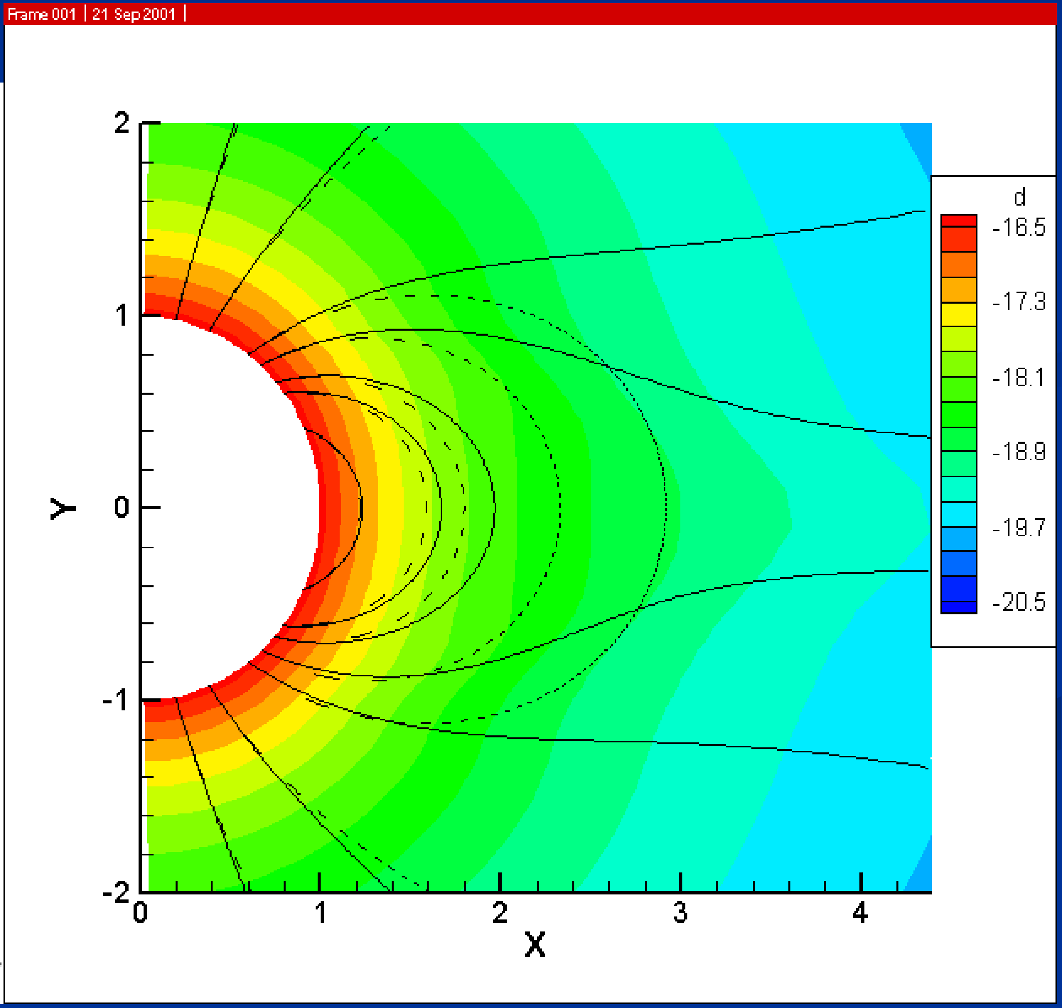
Iterative scheme

MHD model for base dipole with B_o=1 G



Fully dynamic, time dependent

Our Simulation



Maxwell's equations

$$\nabla \bullet E = 4\pi\rho_e$$

Coulomb

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

Induction

$$\nabla \bullet B = 0$$

no mag. monopoles

$$\nabla \times B = \frac{4\pi}{c}J + \frac{1}{c}\xcancel{\frac{\partial E}{\partial t}}$$

Ampere

$$O\left(\frac{v}{c}\right)^2 << 1$$

Frozen Flux theorem

Ideal MHD induction eqn.: $$\frac{\partial B}{\partial t} = \nabla \times \ \ v \times B$$

implies **flux** F through any **material surface** σ,

$$F \equiv \int_{\sigma} B \bullet dA$$

does not change in time, i.e. is “frozen”:

$$\frac{dF}{dt} = 0$$

Magnetohydrodynamic (MHD) Equations

$$\frac{D\rho}{Dt} + \rho\nabla \bullet v = 0$$

Mass

$$\rho\frac{Dv}{Dt} = -\nabla p + \frac{1}{4\pi} \; \nabla \times B \;\; \times B + \rho(\boxed{g_{lines}} - g_{grav})$$

Momentum

$$T = const.$$

Energy

$$P = \rho a^2$$

Ideal Gas E.O.S.

$$\frac{\partial B}{\partial t} = \nabla \times \;\; v \times B$$

mag Induction

$$\nabla \bullet B = 0$$

Divergence free B

Magnetohydrodynamic (MHD) Equations

$$\frac{D\rho}{Dt} + \rho\nabla \bullet v = 0$$

Mass

$$\rho\frac{Dv}{Dt} = -\nabla p + \frac{1}{4\pi}\;\nabla\times B\;\;\times B + \rho(g_{lines} - g_{grav})$$

Momentum

$$\frac{\partial e}{\partial t} + \nabla\cdot e\mathbf{v} = -P\nabla\cdot\mathbf{v} + \boxed{H_M - n^2\Lambda(T)}$$

Energy

$$P = \rho a^2 = (\gamma - 1)e$$

Ideal Gas E.O.S.

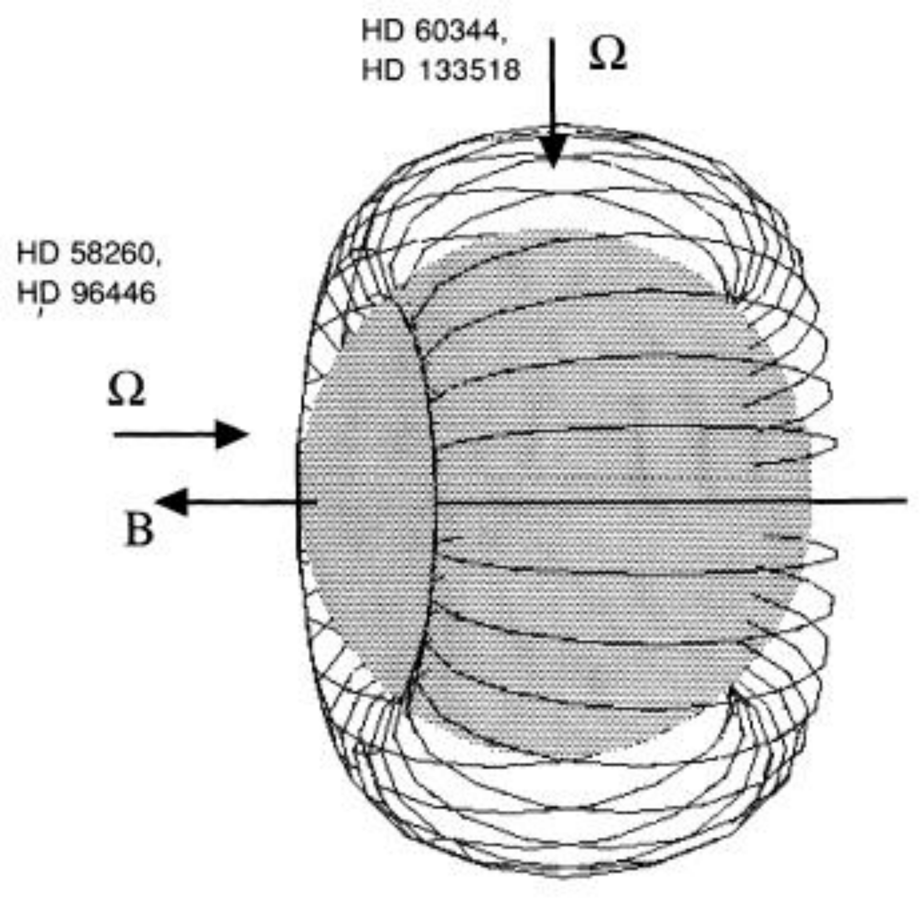
$$\frac{\partial B}{\partial t} = \nabla\times\;\;v\times B$$

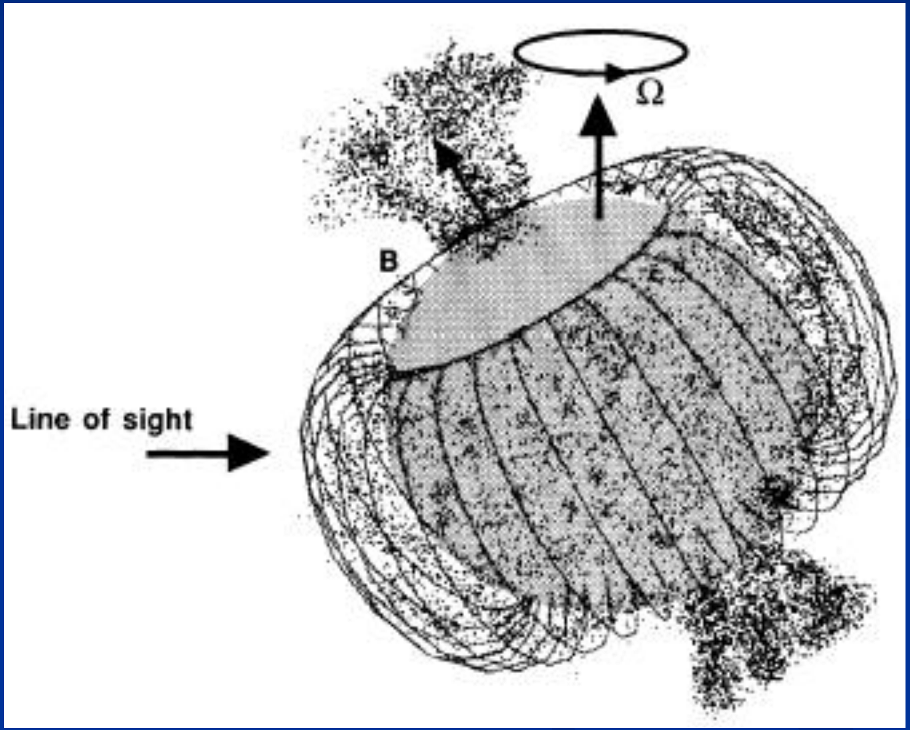
mag Induction

$$\nabla \bullet B = 0$$

Divergence free B

Wind confinement in magnetic B-stars





Shore & Brown 1990

Wind confinement in magnetic B-stars

Shore & Brown 1990

Magnetically Confined Wind-Shocks

Babel & Montmerle 1997

Magnetic A_p-B_p stars

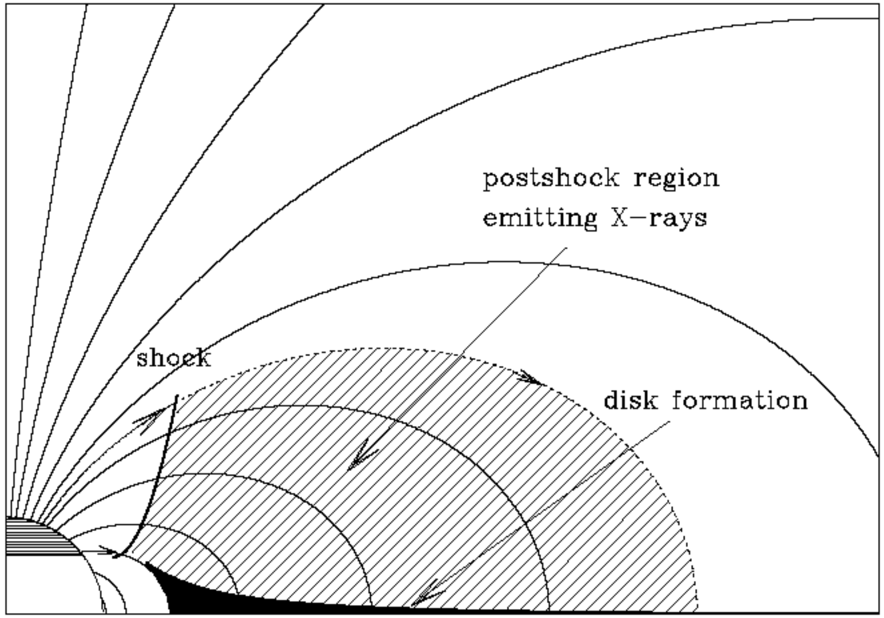


Fig. 7. Schematic view of the model proposed for the X-ray emission from IQ Aur (see text).

Wind Magnetic Confinement

$$\frac{\text{magnetic energy density}}{\text{kinetic energy density}} \longrightarrow \eta(r) \equiv \frac{B^2/8\pi}{\rho v^2/2}$$

$$\eta(r) = \frac{B^2 r^2}{\dot{M} v} = \underbrace{\left[\frac{B_0^2 R_*^2}{\dot{M} v_\infty}\right]}_{=\eta_*}\left[\frac{(r/R_*)^{-2n}}{(1-(R_*/r))^{\beta}}\right]$$

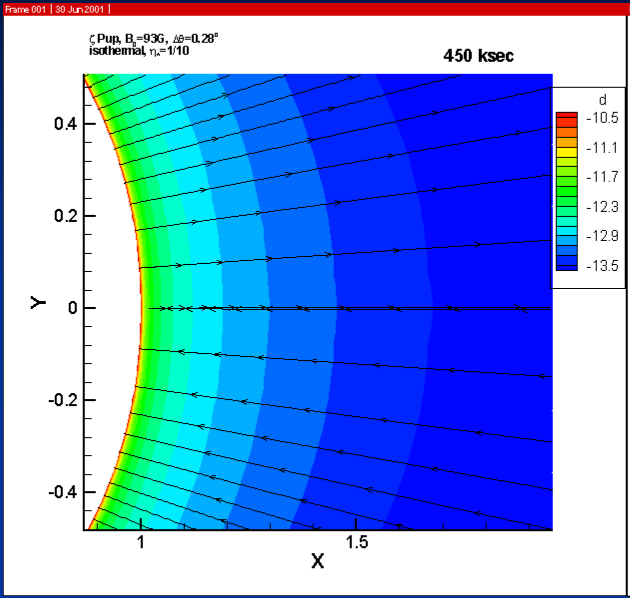
$$\boxed{\eta_* \equiv \frac{B_0^2 R_*^2}{\dot{M} v_\infty} = 0.4 \times \frac{B_{100}^2 R_{12}^2}{\dot{M}_{-6} v_8}}$$

for solar wind, $\eta_* \sim 40$ but for O-stars, to get $\eta_* \sim 1$, need:

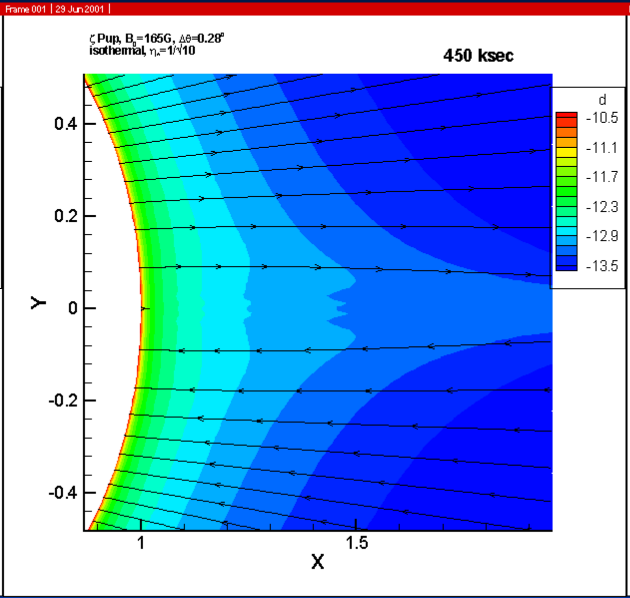
$B_* \sim 300$ G for ζ Pup

Final state of ζ Pup isothermal models

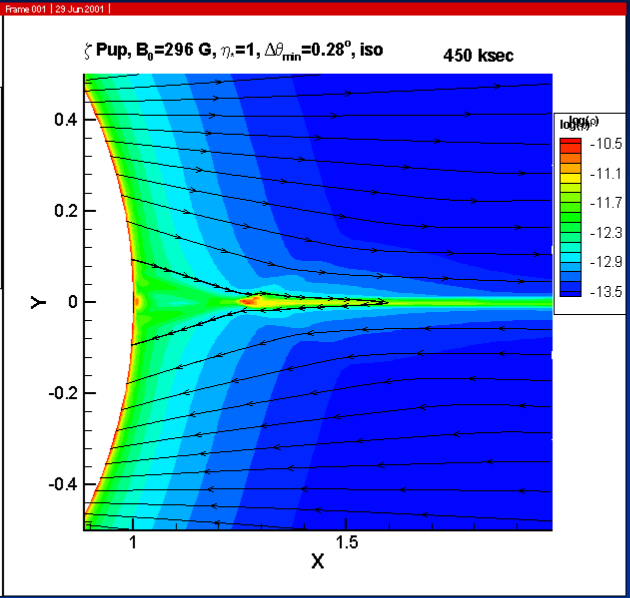
93 G ; $\eta_* = 0.1$



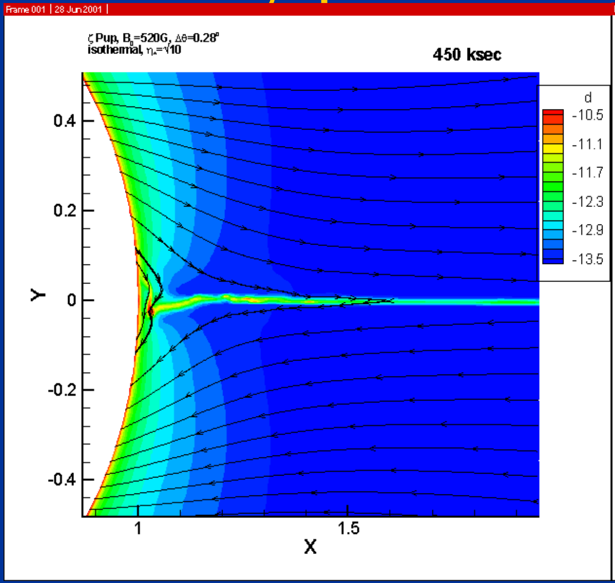
165 G ; $\eta_* = 0.32$



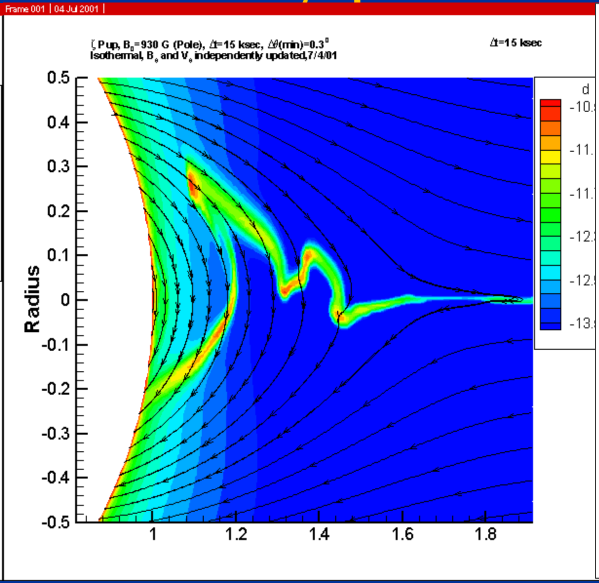
295 G ; $\eta_* = 1$



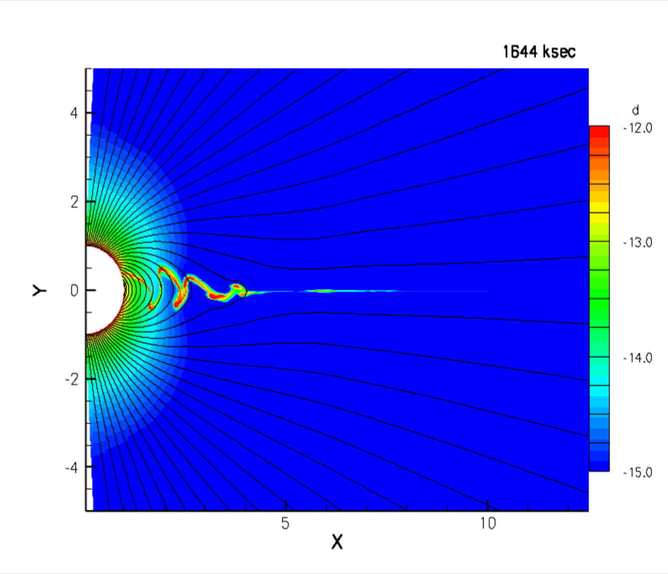
520 G ; $\eta_* = 3.2$



930 G ; $\eta_* = 10$



2950 G ; $\eta_* = 100$



Basic Results

Alfven Radius
determines wind modulation

$\eta(R_A)=1$
$R_A=\eta_*^{1/4}R_*$

(For dipole)

Wind Magnetic Confinement Parameter

- $\eta_* << 1$: radial outflow and field.
- $\eta_* >> 1$: strong confinement, infall
- $\eta_* \sim 1$: at equator, high density, low speed
- Can explain X-ray from θ^1 Ori C

Magnetic confinement vs. Wind + Rotation

Wind mag. confinement

$$\eta_* \equiv \frac{B_*^2 R_*^2}{\dot{M} V_\infty}$$

Rotation vs. critical

$$W \equiv \frac{V_{rot}}{\sqrt{GM / R_*}}$$

Alfven radius

$$R_A = \eta_*^{1/4} R_*$$

Kepler radius

$$R_K = W^{-2/3} R_*$$

Alfven vs. Kepler Radius

Kepler co-rotation Radius, R_K:

$$GM/\, R_K^2 = V_\phi^2/\boxed{R_K} = V_{rot}^2 R_K/R_*^2$$

$$\boxed{R_K = w^{-2/3}\, R_*} \qquad w = V_{rot}/V_{crit} \qquad V_{crit}^2 = GM/R_*$$

Alfven radius: $\eta(R_A)=1$

e.g, for dipole field,
$\eta \sim 1/r^4$

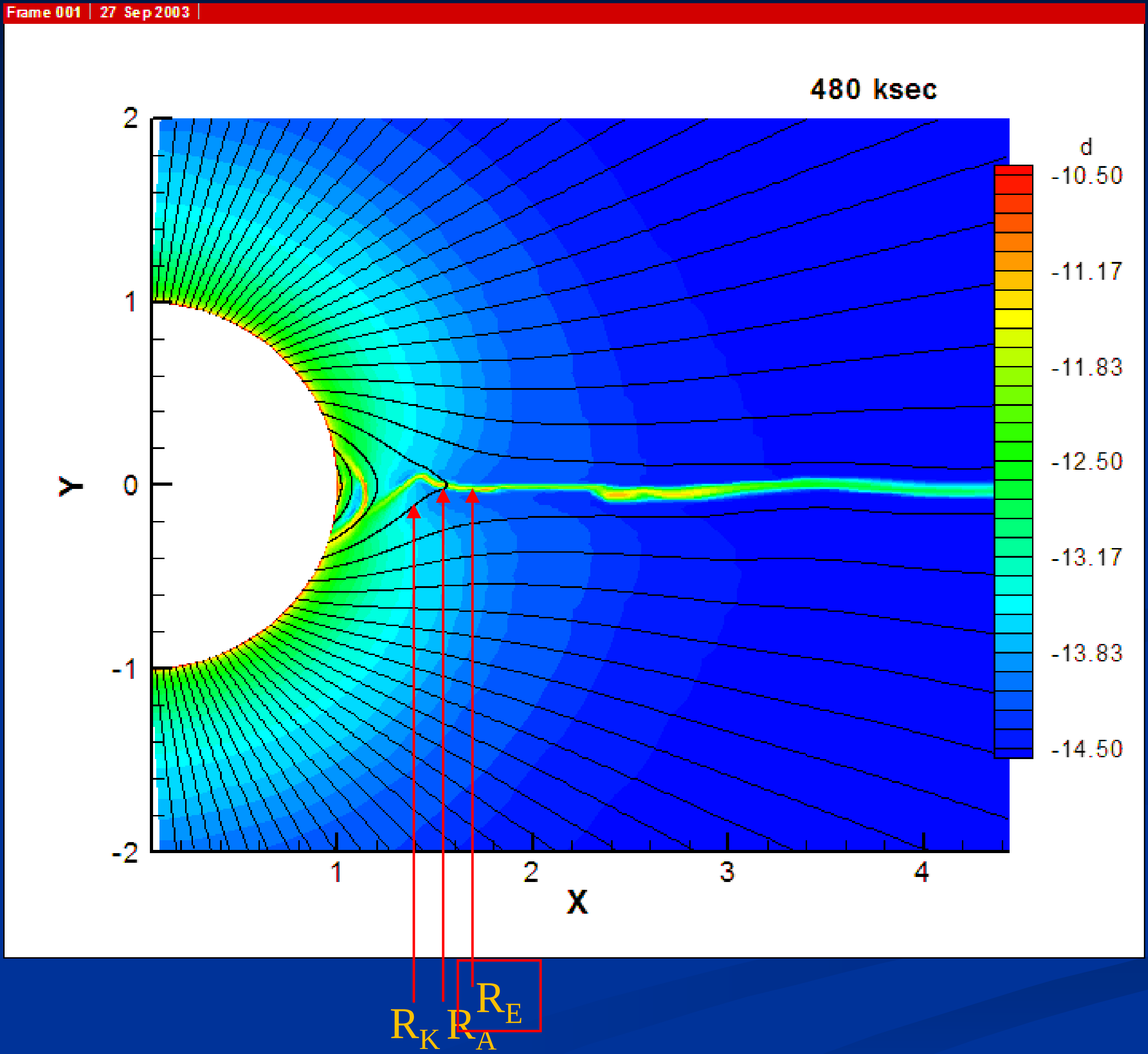
$$R_A = \eta_*^{1/4}\, R_*$$

when $R_A > R_K$:
Magnetic spin-up => centrifugal support & ejection

A Sample Simulation

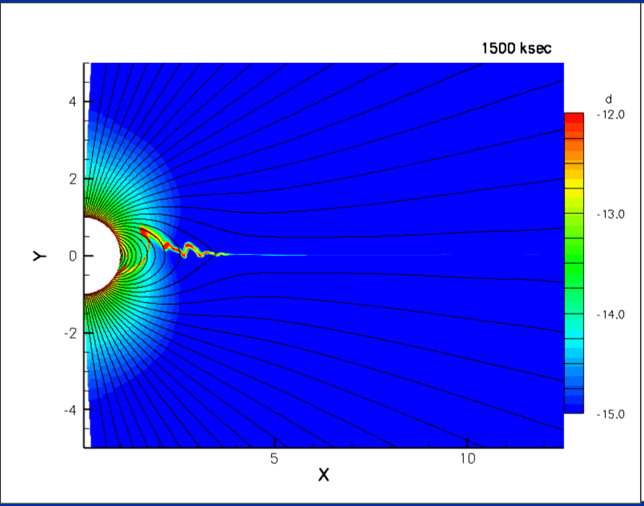
$W=1/2$

$\eta_{*}=10$

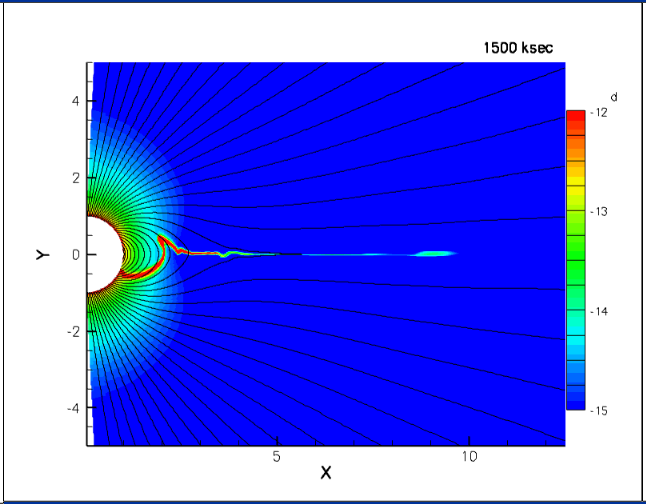


Rotating Wind with High η_*

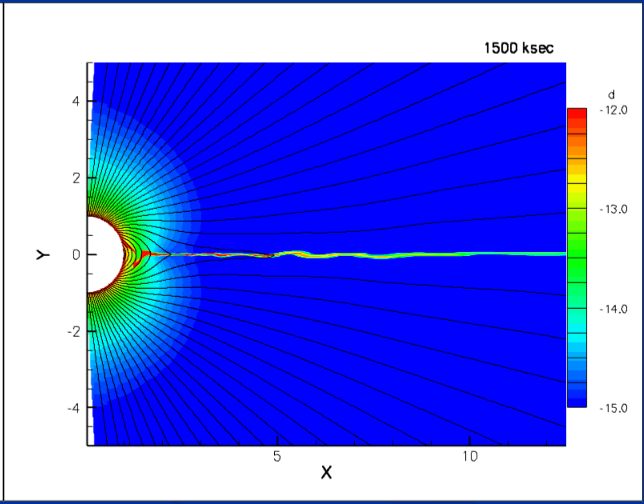
η_*=100
W=0



W=1/4



W=1/2

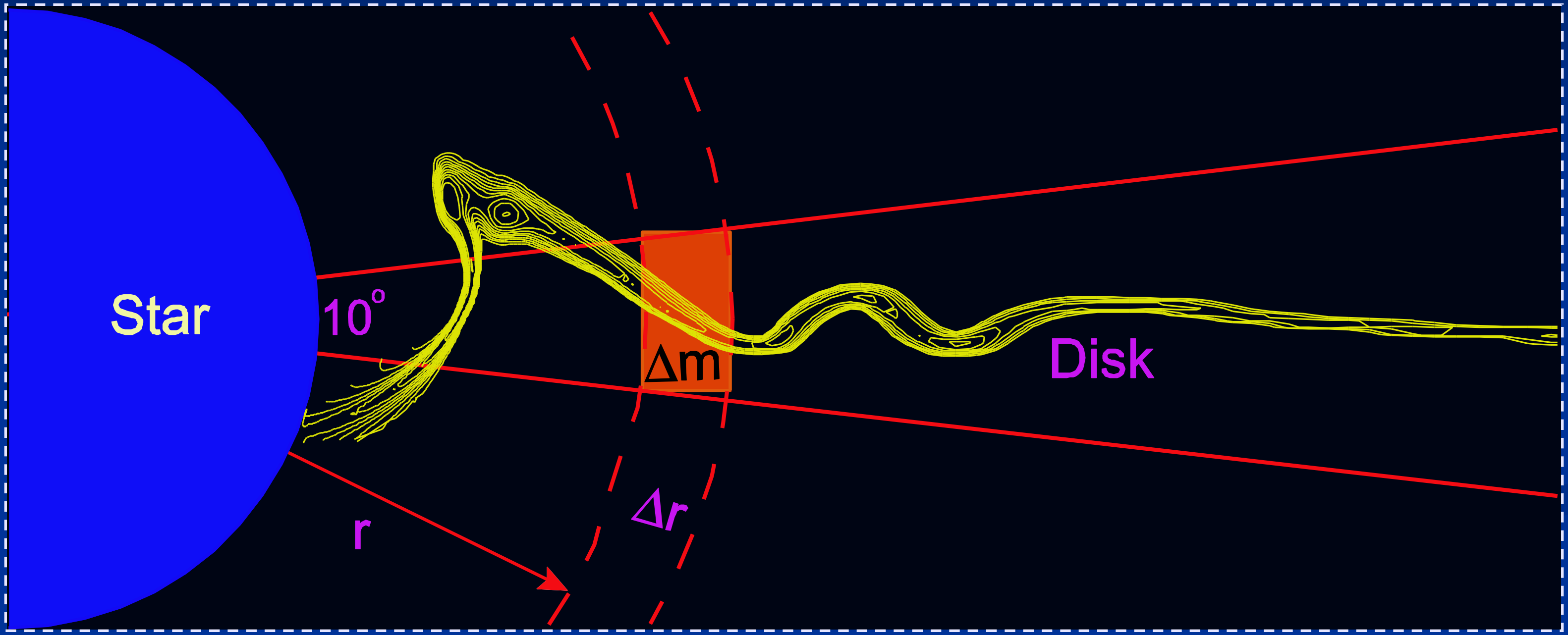


zoom in

Can these form disks?

- Mais bien sûr …
- But NOT Keplerian
- Instead Rigidly Rotating Disks are formed
- Subject to episodic ejections/flares

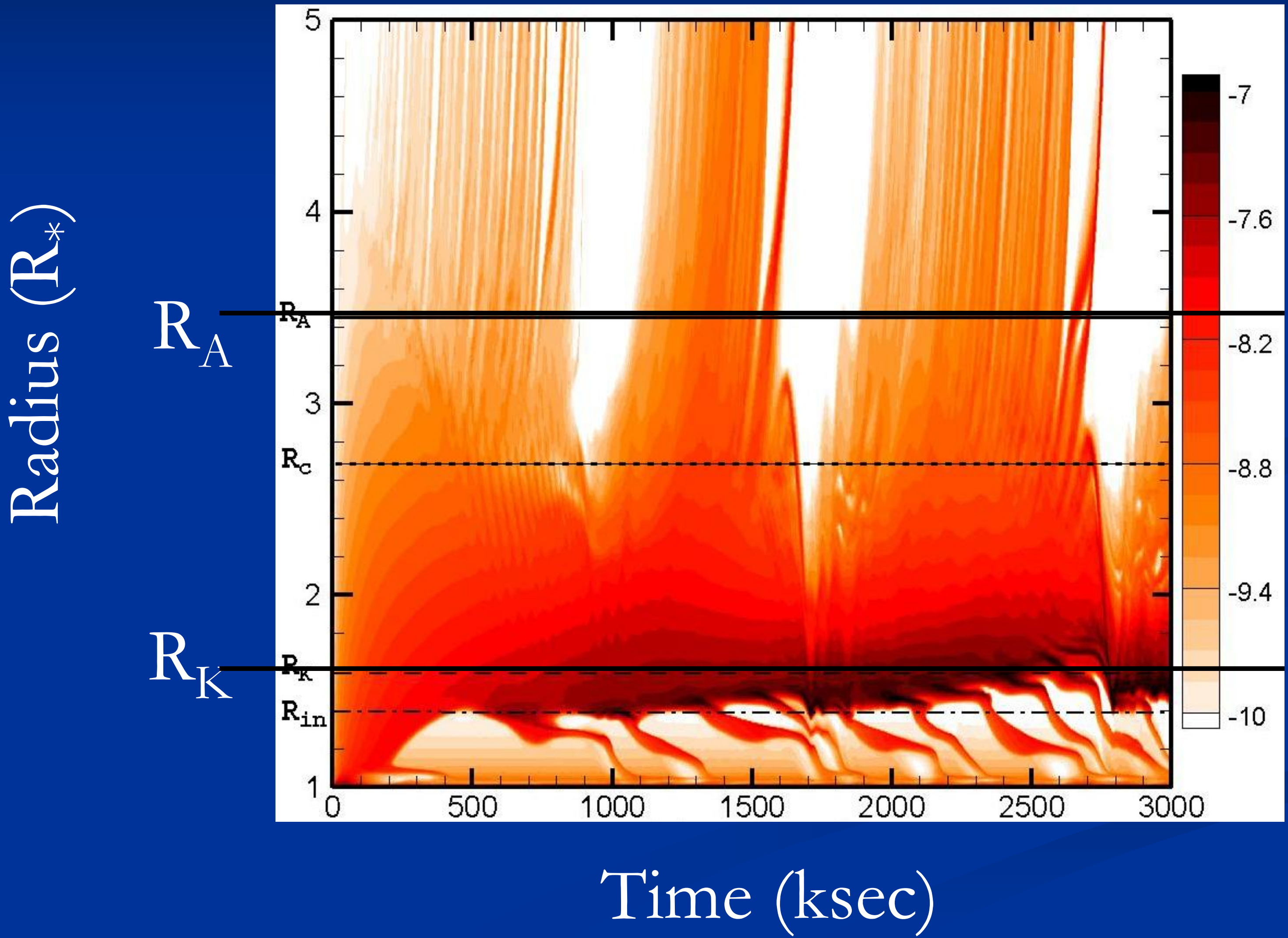
Radial Mass Distribution



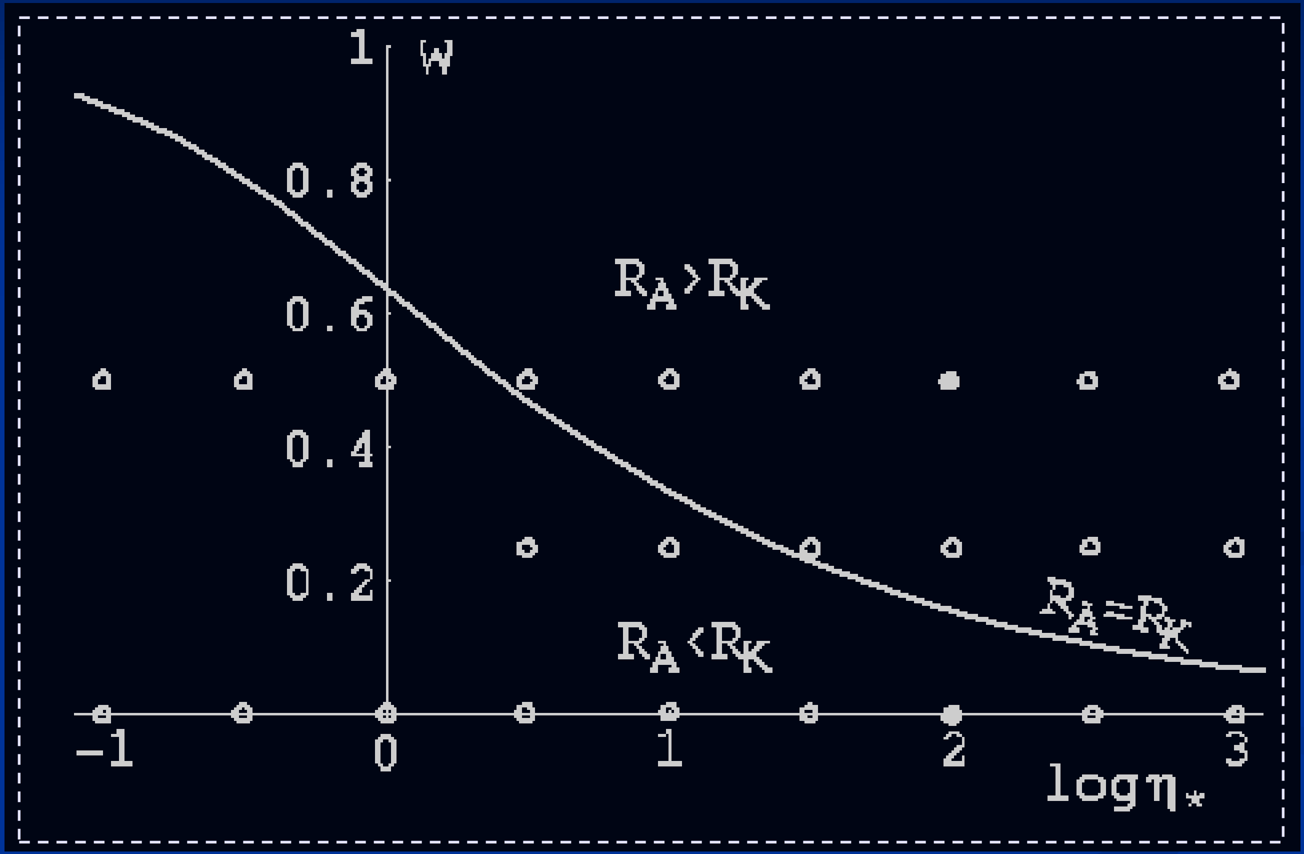
$$\frac{dm_e(r,t)}{dr} \equiv 2\pi r^2 \int_{\pi/2-\Delta\theta/2}^{\pi/2+\Delta\theta/2} \rho(r,\theta,t)\,\sin\theta\,d\theta$$

Time evolution of Radial distribution of equatorial disk mass

$\eta_* = 100$ & Vrot/Vcrit =1/2



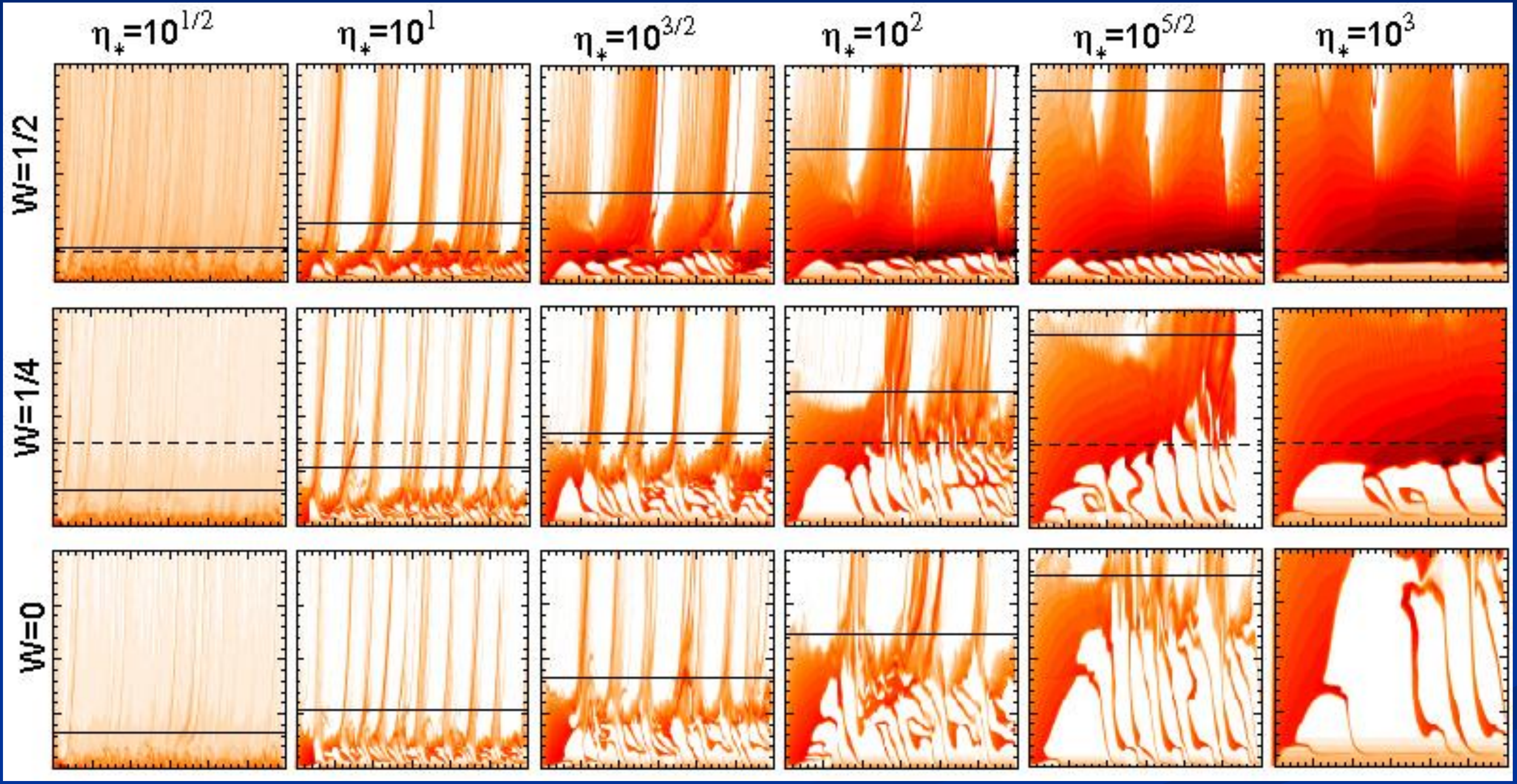
Two Parameter Study



More Rapid Rotation --->

Stronger Magnetic Confinement --->

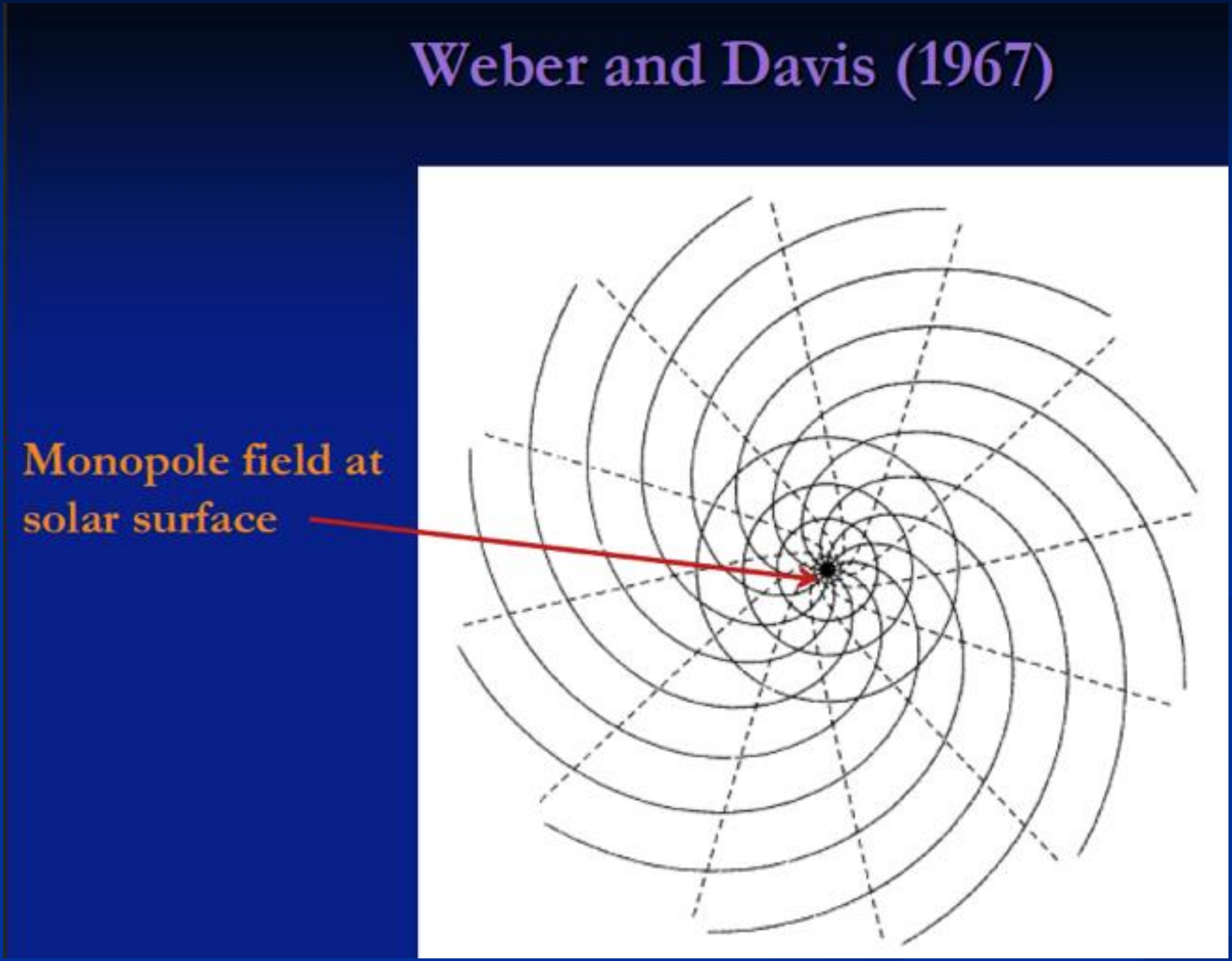
Radial Distribution of equatorial disk mass



More Rapid Rotation --->

Stronger Magnetic Confinement --->

Angular Momentum Loss



Weber and Davis (1967)

Monopole field at solar surface

$$\dot{J} \approx \frac{2}{3}\dot{M}\Omega R_A^2$$

Angular Momentum Loss

Contribution from the field

$$\dot{J}_B = \int -\frac{B_r B_\phi(r,\theta)}{4\pi} r\sin\theta dA$$

Contribution from gas

$$\dot{J}_{gas} = \int \rho v_r v_\phi r\sin\theta dA$$

Need to compute

Total loss $$\dot{J}_{tot} = \dot{J}_B + \dot{J}_{gas}$$

Weber and Davis $$\dot{J}_{tot} \approx \frac{2}{3}\dot{M}\Omega R_A^2$$

As if gas co-rotates to R_A

Computing Alfven Radius

$$\eta(r) = \eta_* \left[\frac{(r/R_*)^{-2n}}{(1-(R_*/r))^{\beta}} \right]$$

For hot stars, β~1

Alfven radius

$$\eta(R_A) \equiv 1$$

For a monopole n=1

$$\frac{R_A}{R_*} \approx \eta_*^{1/2}$$

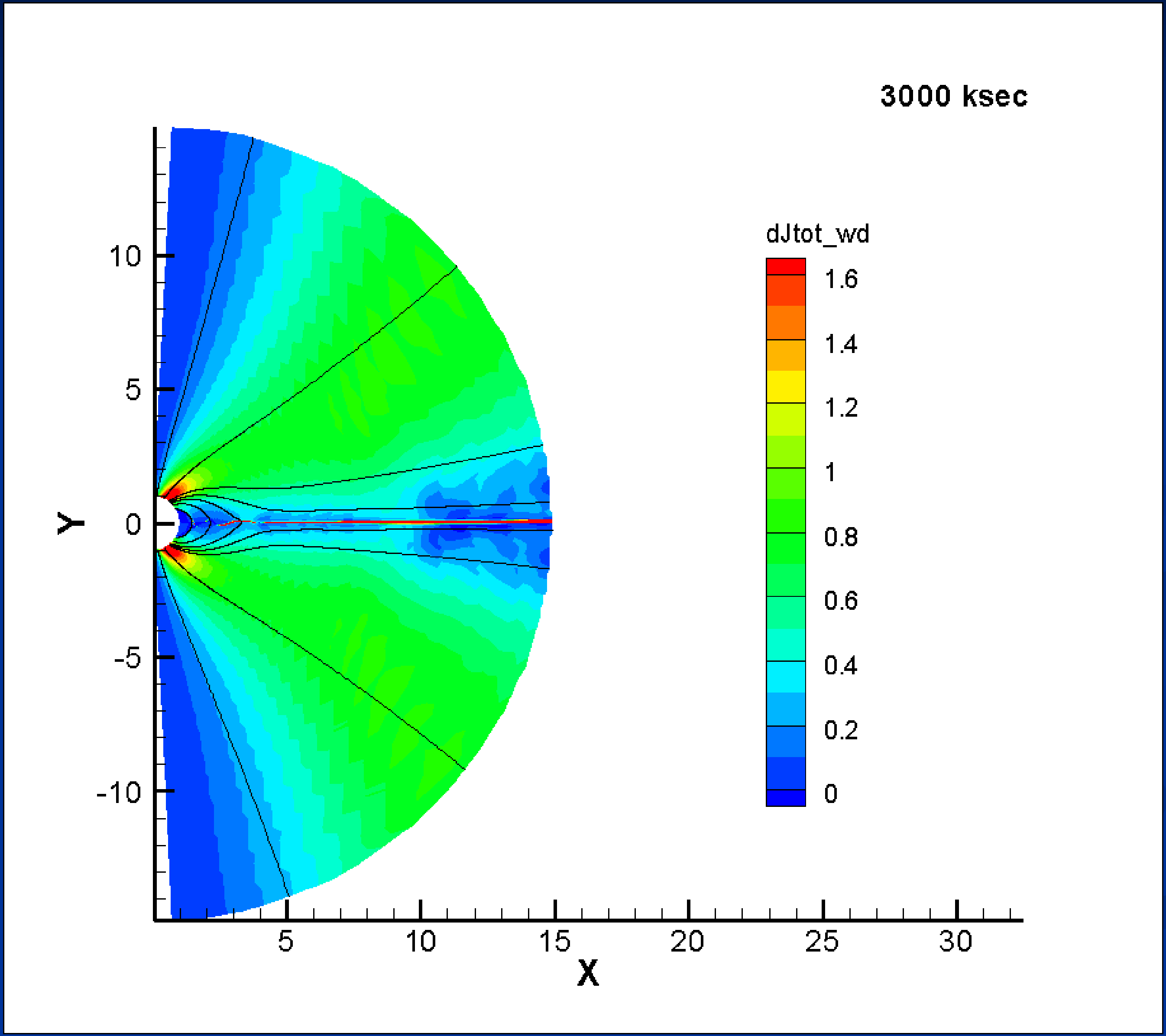
$$\dot{J} \approx \tfrac{2}{3} \dot{M} \Omega R_*^2 \eta_*$$

For a dipole n=2

$$\frac{R_A}{R_*} \approx \eta_*^{1/4}$$

$$\dot{J} \approx \tfrac{2}{3} \dot{M} \Omega R_*^2 \sqrt{\eta_*}$$

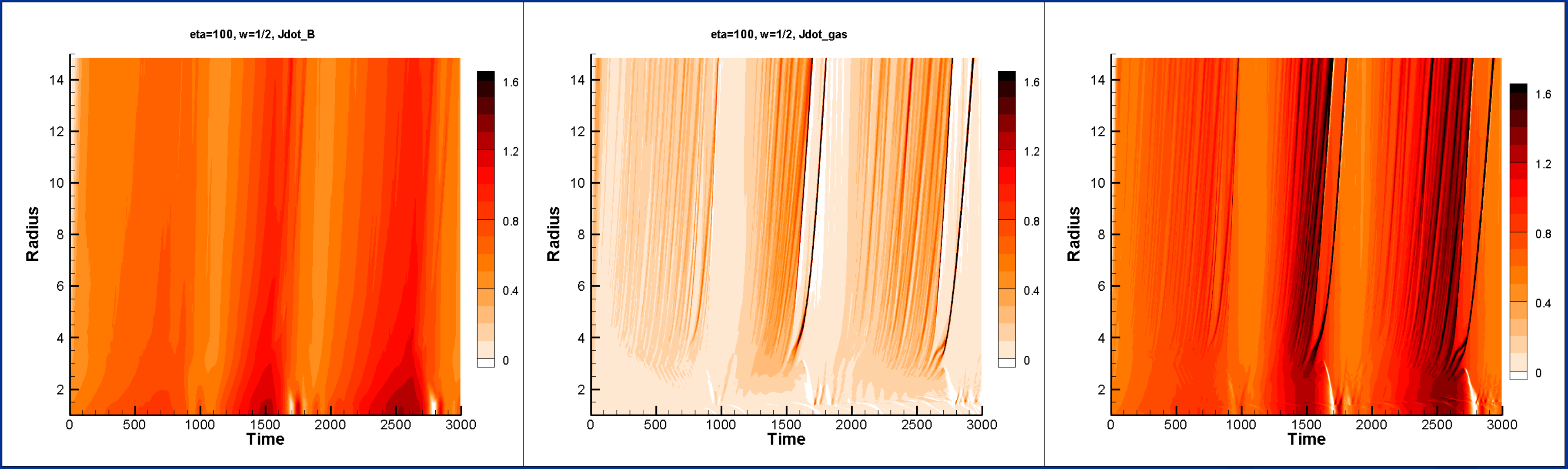
Angular Momentum Loss: Sims



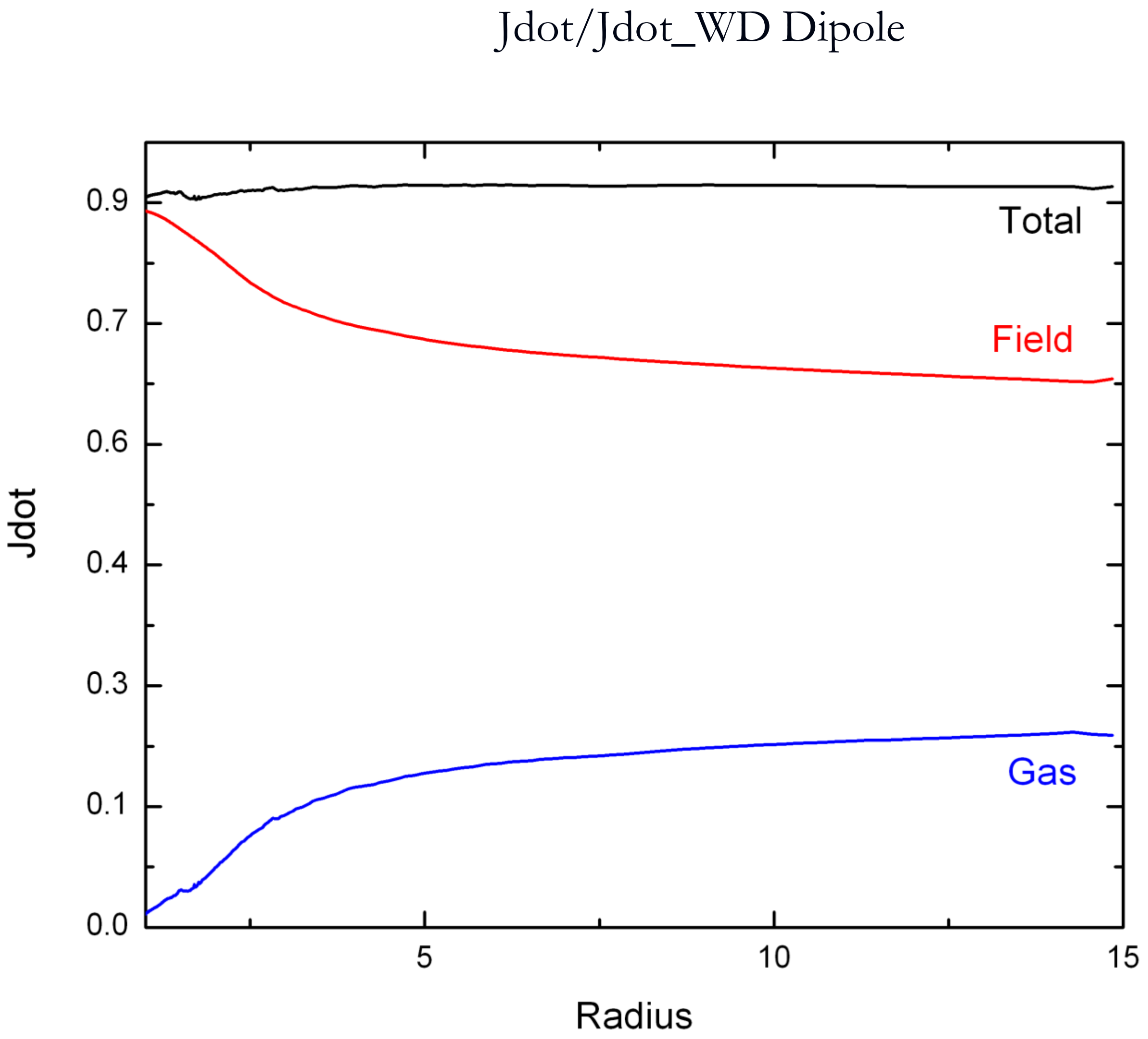
Dead Zone Lives!

Angular Momentum Loss

Magnetic Field + Gas = Total



Time-Averaged Angular Momentum Loss



Angular Momentum Loss
Weber and Davis

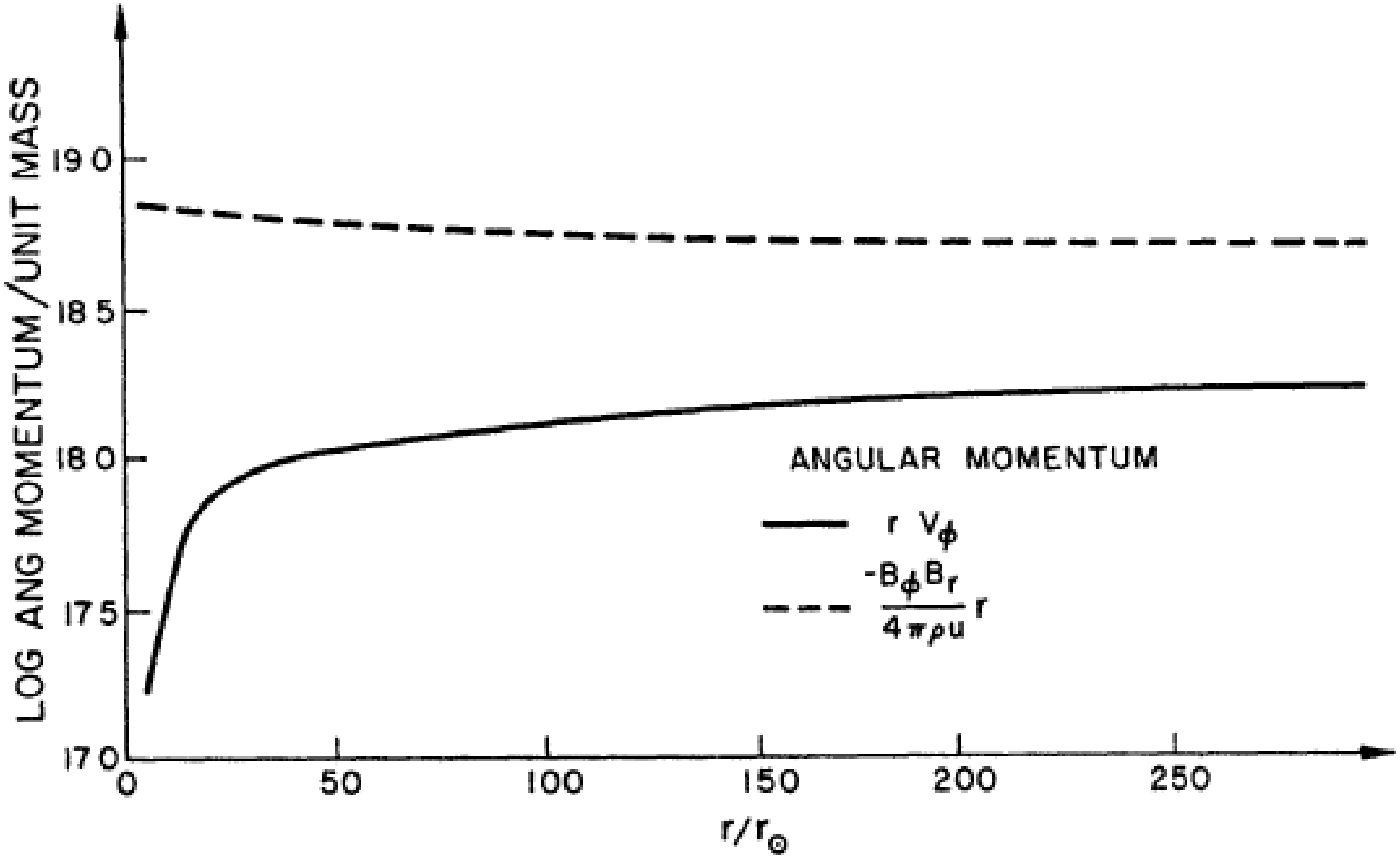


FIG. 5.—Angular momentum and magnetic torque in the solar wind

Spindown

$$\dot{J} \approx \frac{2}{3} \dot{M} \Omega R_A^2$$

contribution from both field and gas

For Dipole

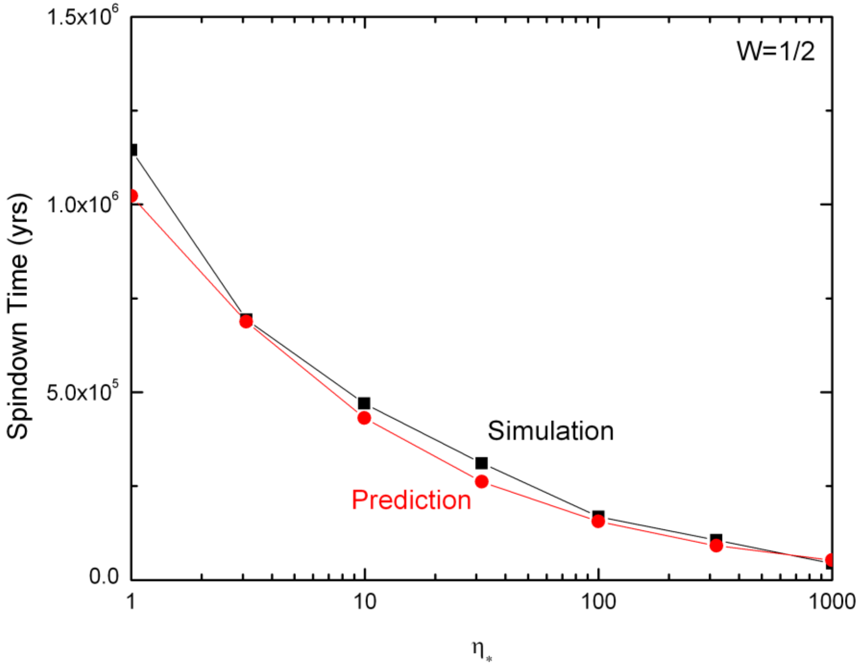
$$\tau_{spin} \equiv \frac{J}{\dot{J}} \approx \frac{\frac{3}{2} I}{MR^2} \frac{M}{\dot{M}} \frac{1}{\sqrt{\eta_*}} = \tau_{mass} \frac{\frac{3}{2} k}{\sqrt{\eta_*}}$$

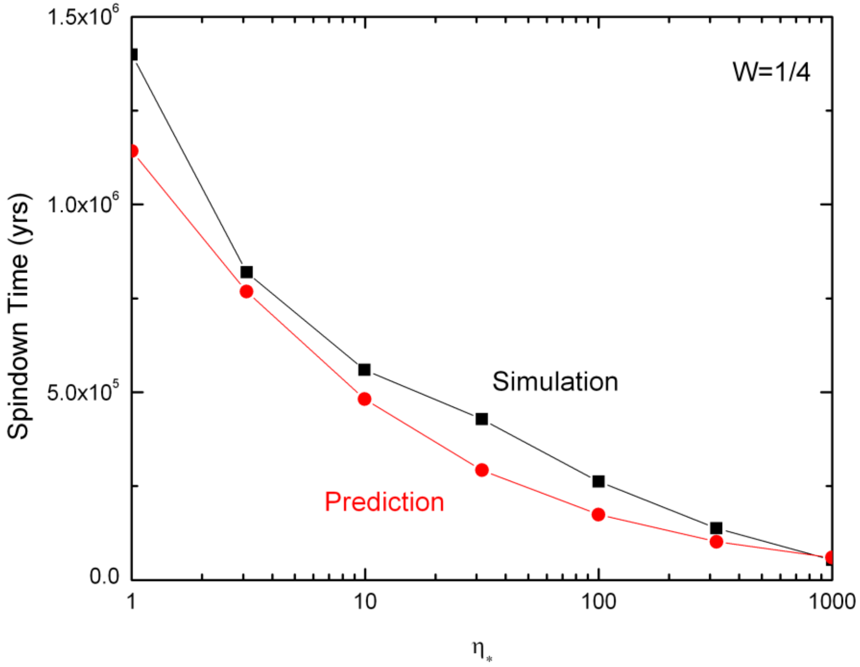
$$\frac{\tau_{spin}}{\tau_{mass}} \approx \frac{0.15}{\sqrt{\eta_*}}$$

Spindown Time

W=1/2

W=1/4





Spindown Time

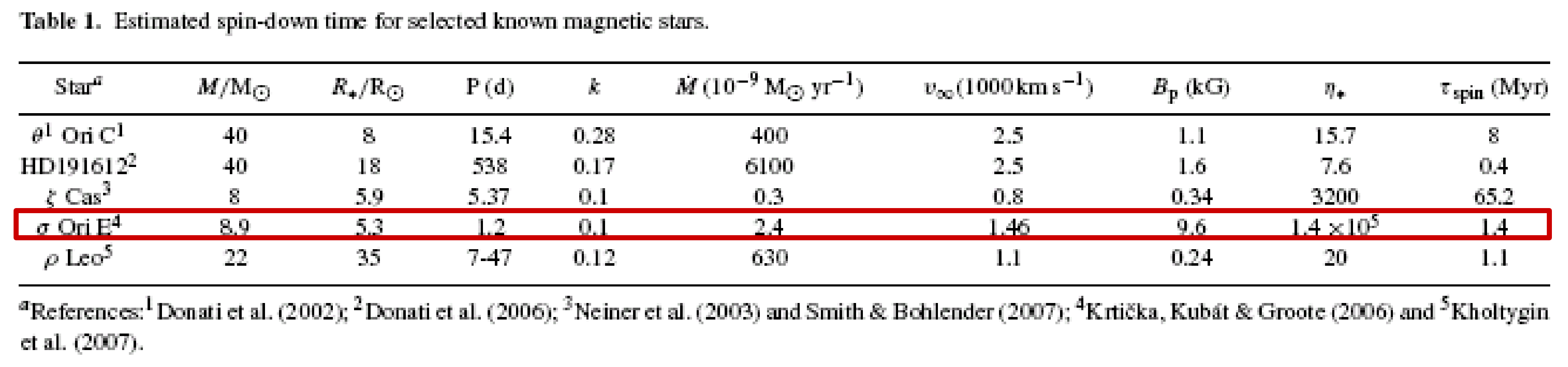


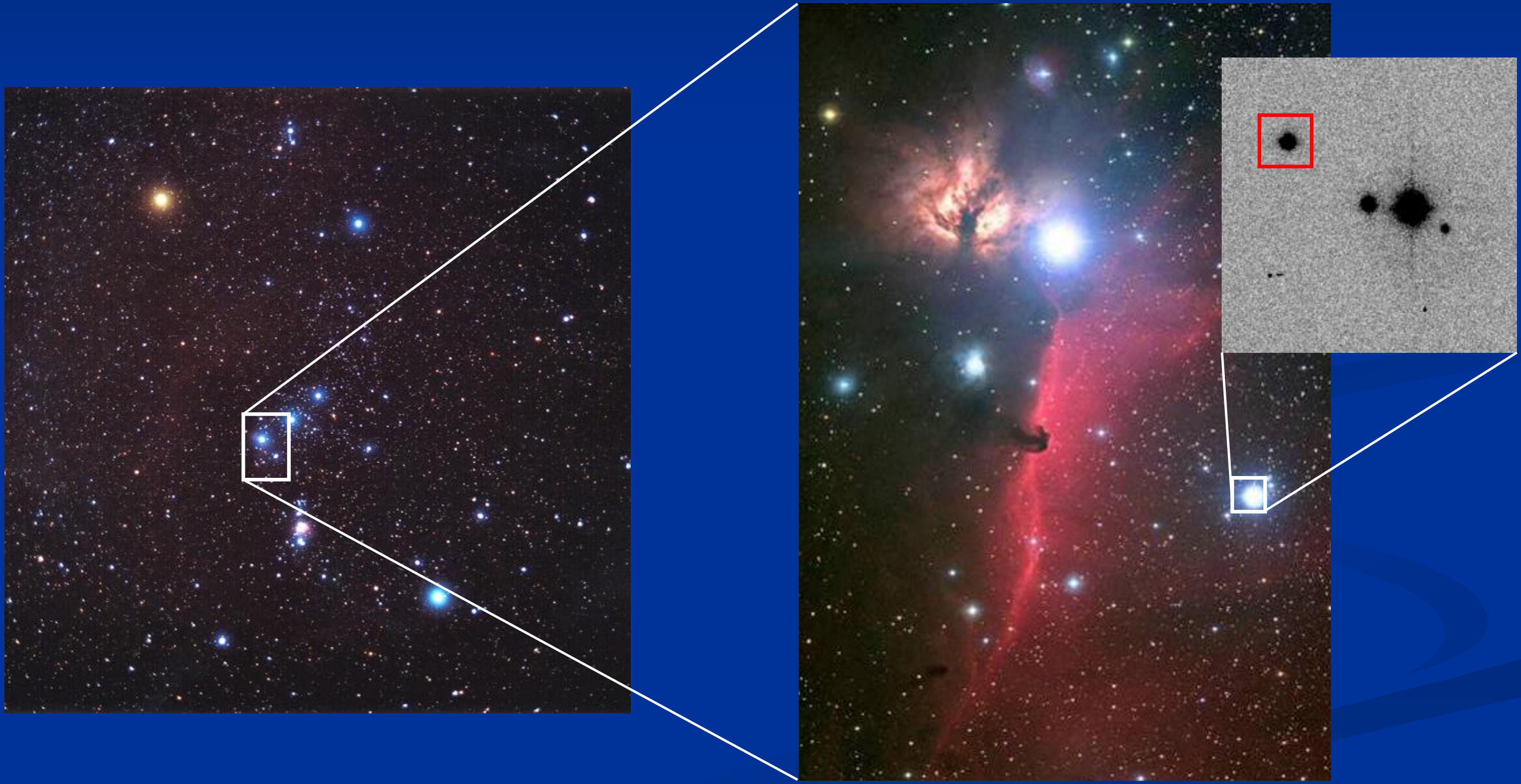
Table 1. Estimated spin-down time for selected known magnetic stars.

Star[a]	$M/\mathrm{M}_\odot$	$R_*/\mathrm{R}_\odot$	P (d)	k	$\dot{M}$ ($10^{-9}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$)	v_∞ ($1000\,\mathrm{km\,s}^{-1}$)	B_p (kG)	η_*	τ_spin (Myr)
θ^1 Ori C[1]	40	8	15.4	0.28	400	2.5	1.1	15.7	8
HD191612[2]	40	18	538	0.17	6100	2.5	1.6	7.6	0.4
ζ Cas[3]	8	5.9	5.37	0.1	0.3	0.8	0.34	3200	65.2
σ Ori E[4]	8.9	5.3	1.2	0.1	2.4	1.46	9.6	1.4×10^5	1.4
ρ Leo[5]	22	35	7-47	0.12	630	1.1	0.24	20	1.1

[a] References: [1] Donati et al. (2002); [2] Donati et al. (2006); [3] Neiner et al. (2003) and Smith & Bohlender (2007); [4] Krtička, Kubát & Groote (2006) and [5] Kholtygin et al. (2007).

ud-Doula et al. 2009

σ Ori E



Rotational Braking of σ Ori E

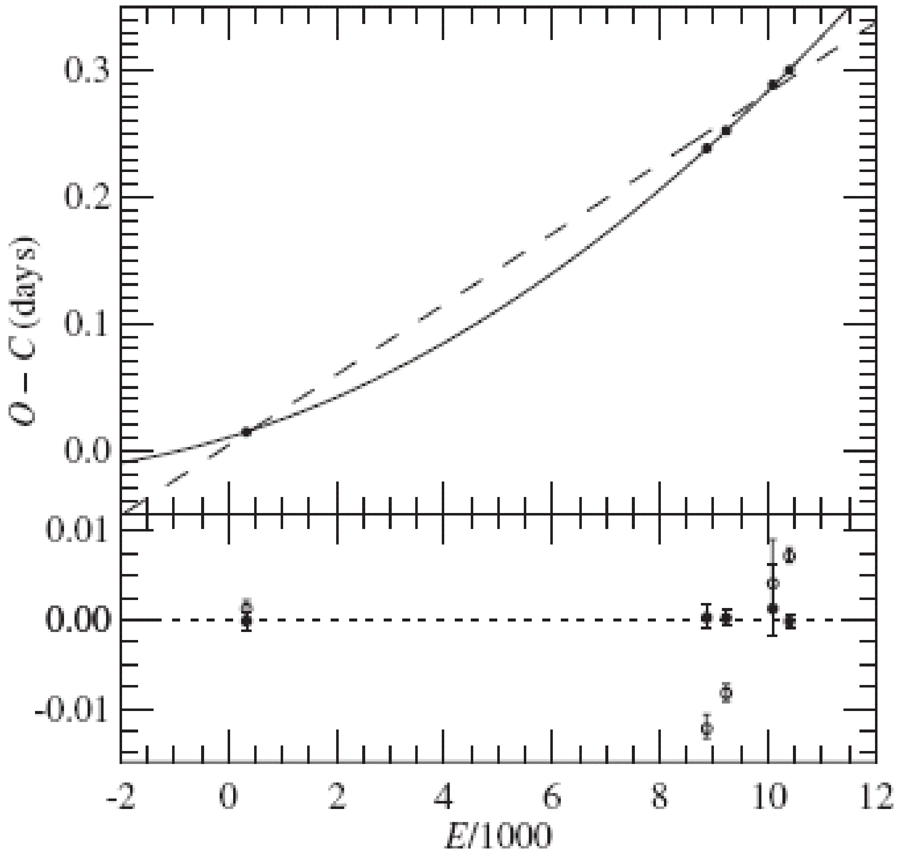


Figure 3. Observed-minus-corrected diagram for the primary minimum measurements. The solid (dashed) lines indicate the best-fit quadratic (linear) models; the residuals relative to these models are shown below the $O - C$ diagram as filled (open) symbols.

Townsend et al. 2010

Predicted

τ_{spin}=1.40 Myr

Measured

τ_{spin}=1.34 Myr

Quick Summary of J loss

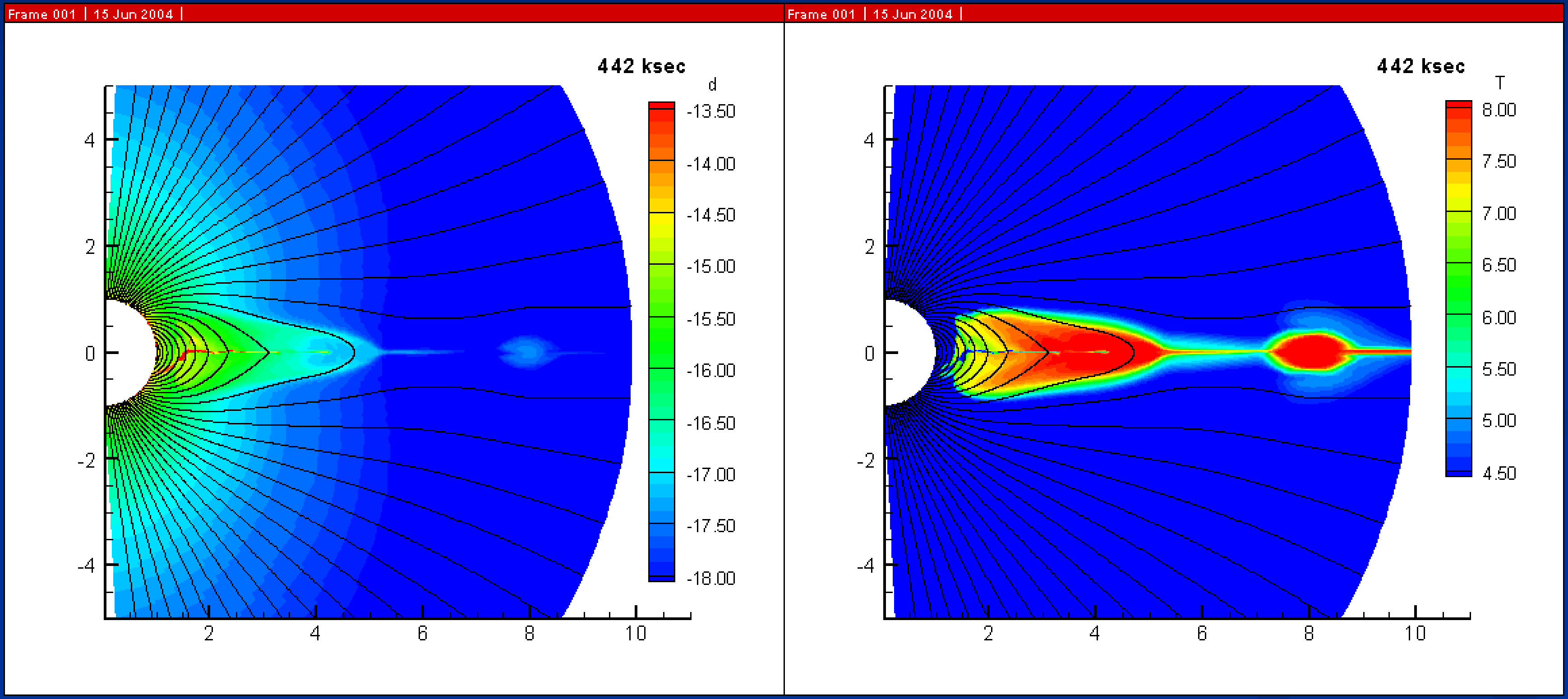
- Angular momentum loss $\dot{J} \propto \Omega R_A{}^2$
- Spindown time ~ Mass time / $\sqrt{\eta_*}$
- $\dot{J}_B > \dot{J}_{gas}$
- breakout events: J stored and released

Model with Full Energy Equation

log ρ

$\eta_* \sim 600, V_{rot}=V_{crit}/2$

log T



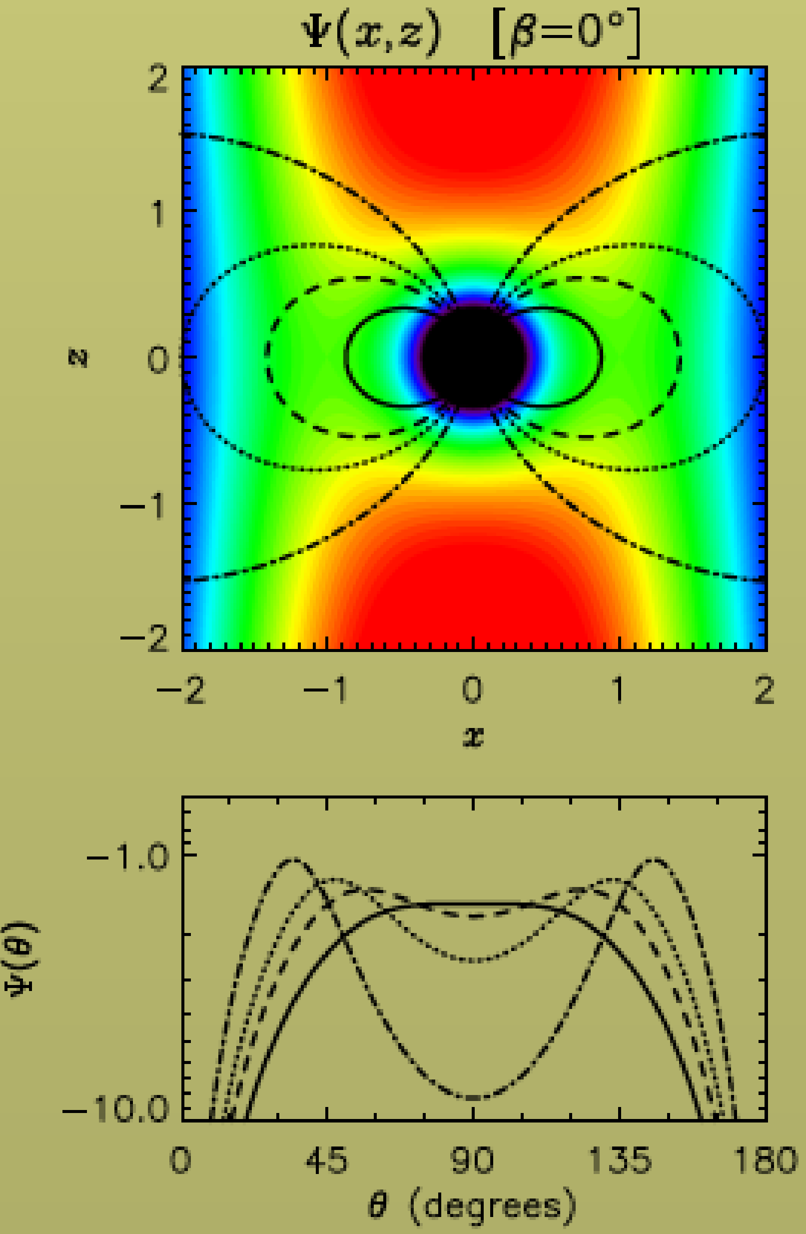
~ 4-5 kev X-ray flares, as seen in σ Ori E: reconnection?

(ud-Doula et. al 2006)

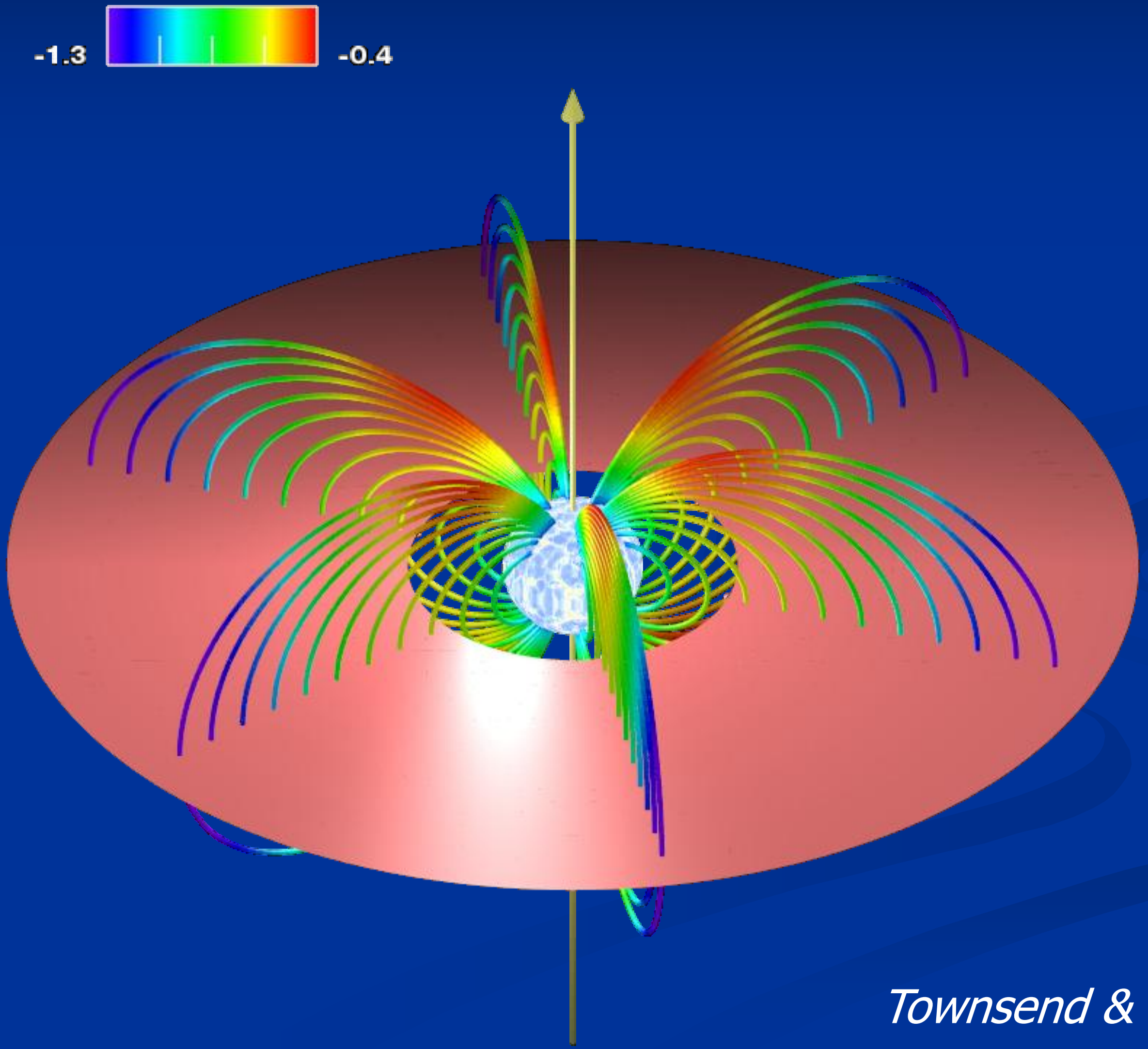
Magnetic Bp Stars

- σ Ori E (B2p V)
  - P_{rot} = 1.2 days => $v_{rot}/v_{crit} \sim 1/2$
  - $B_{obs} \sim 10^4$ G => $\eta_* \sim 10^7$!
  - => V_{Alfven} very large => Courant time very small
  - => Direct MHD impractical
- Instead treat fields lines as Rigid guides
  - **Torque up** wind outflow
  - **Hold down** disk material vs. centrifugal force

Eff. Grav.+Centrifugal **Potential**

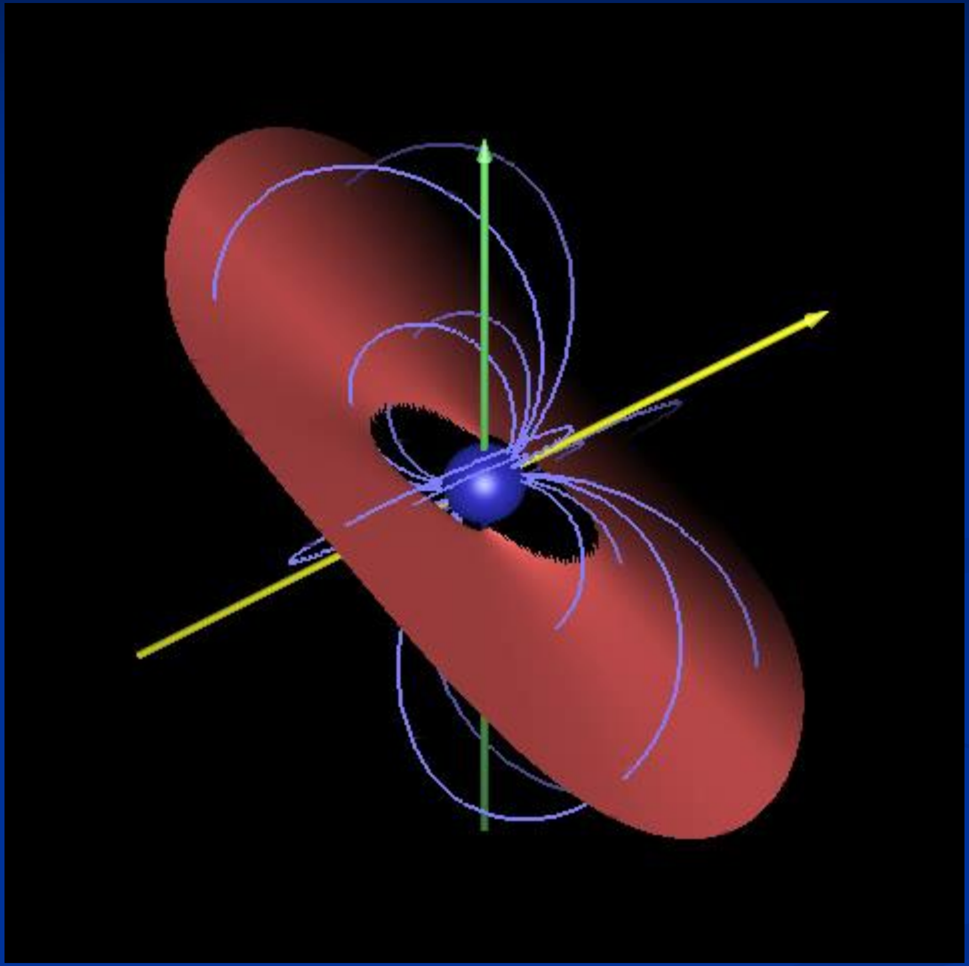


Rigidly Rotating Magnetosphere



Townsend & Owocki (2005)

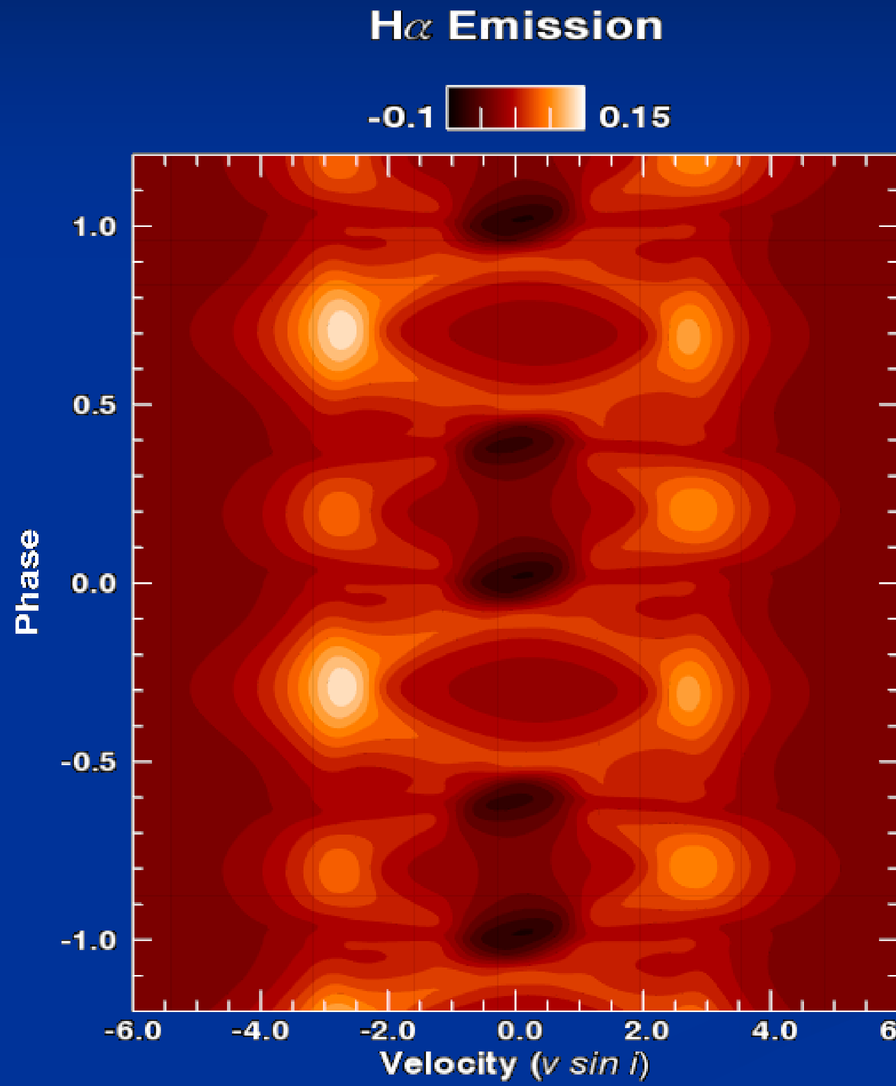
Oblique-Dipole RRM



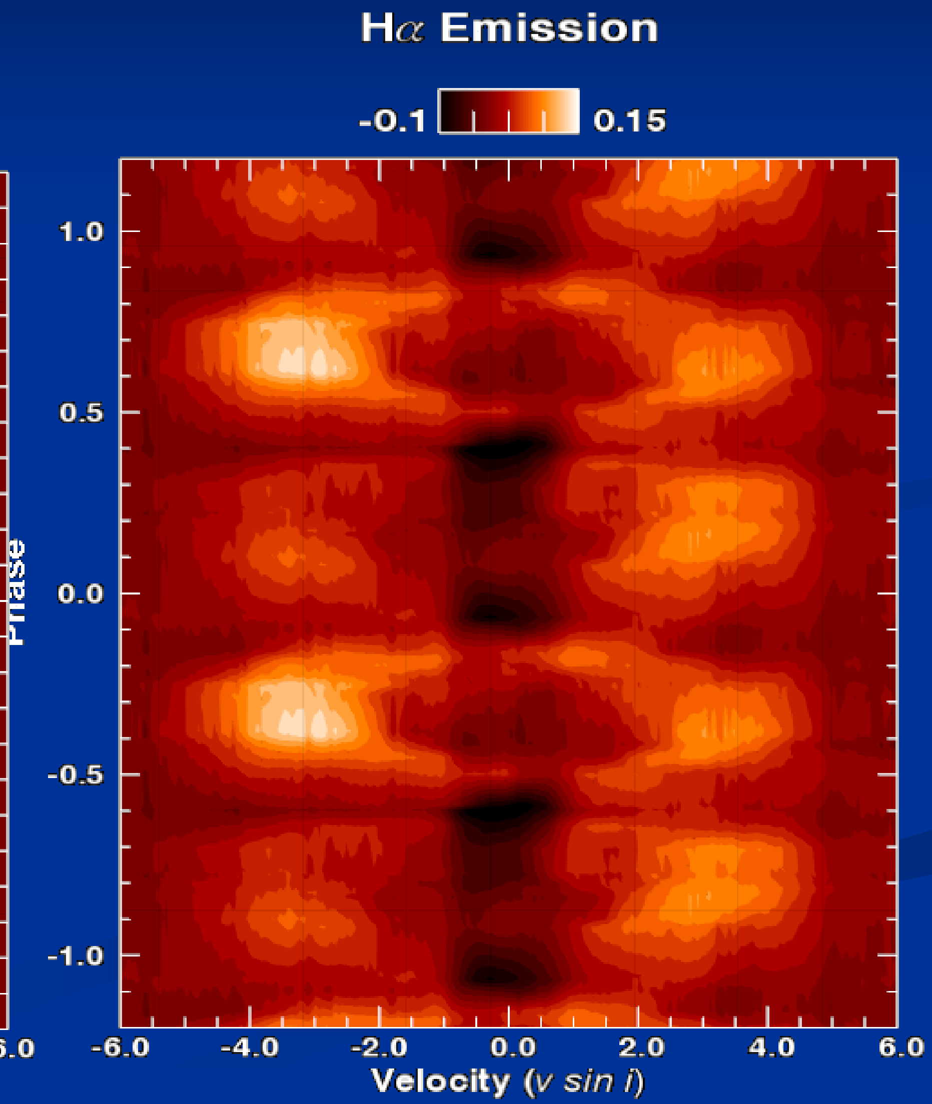
Not a cartoon!!
Analytic result.

σ Ori E

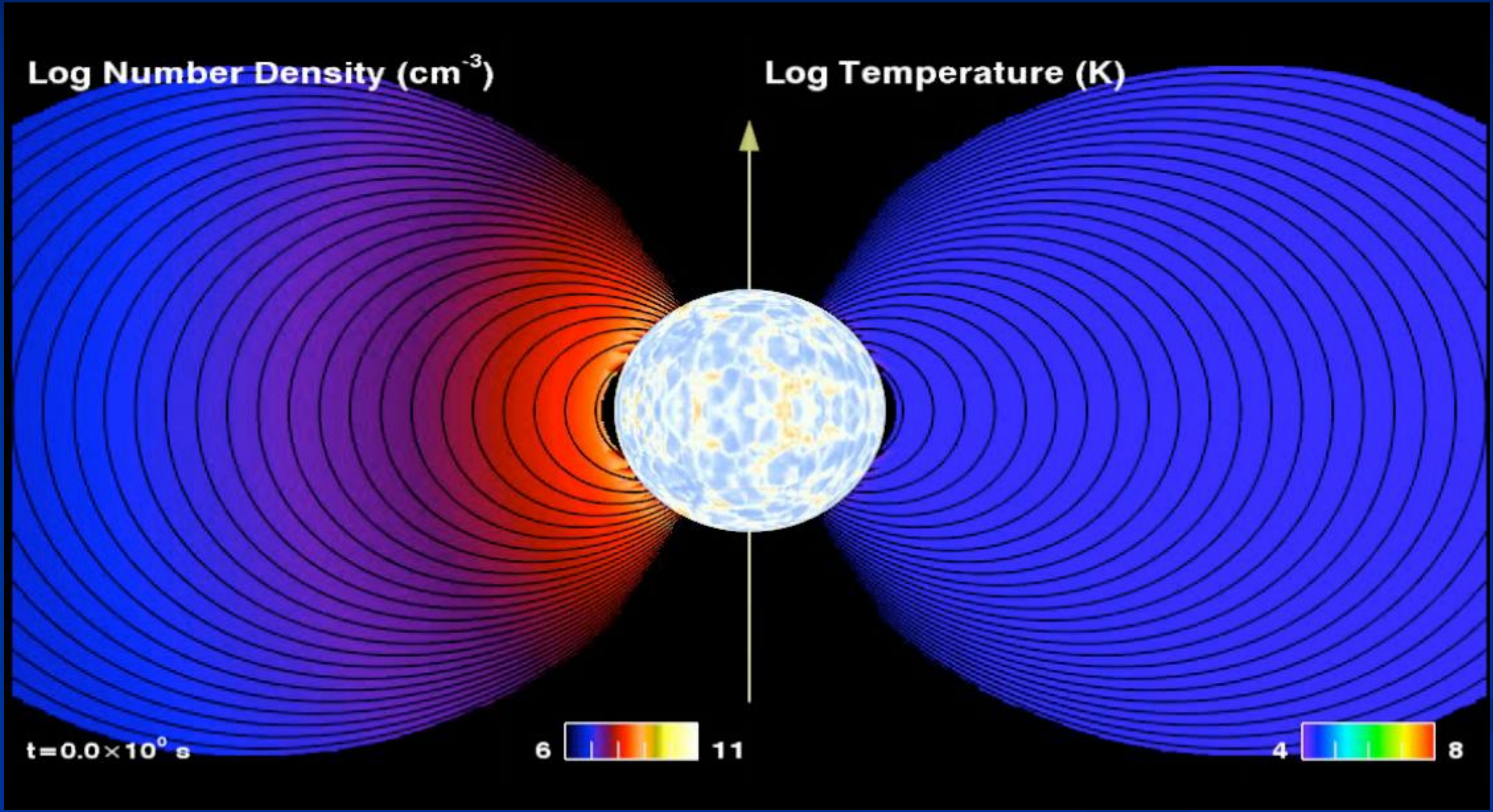
RRM Model



Hα Observations

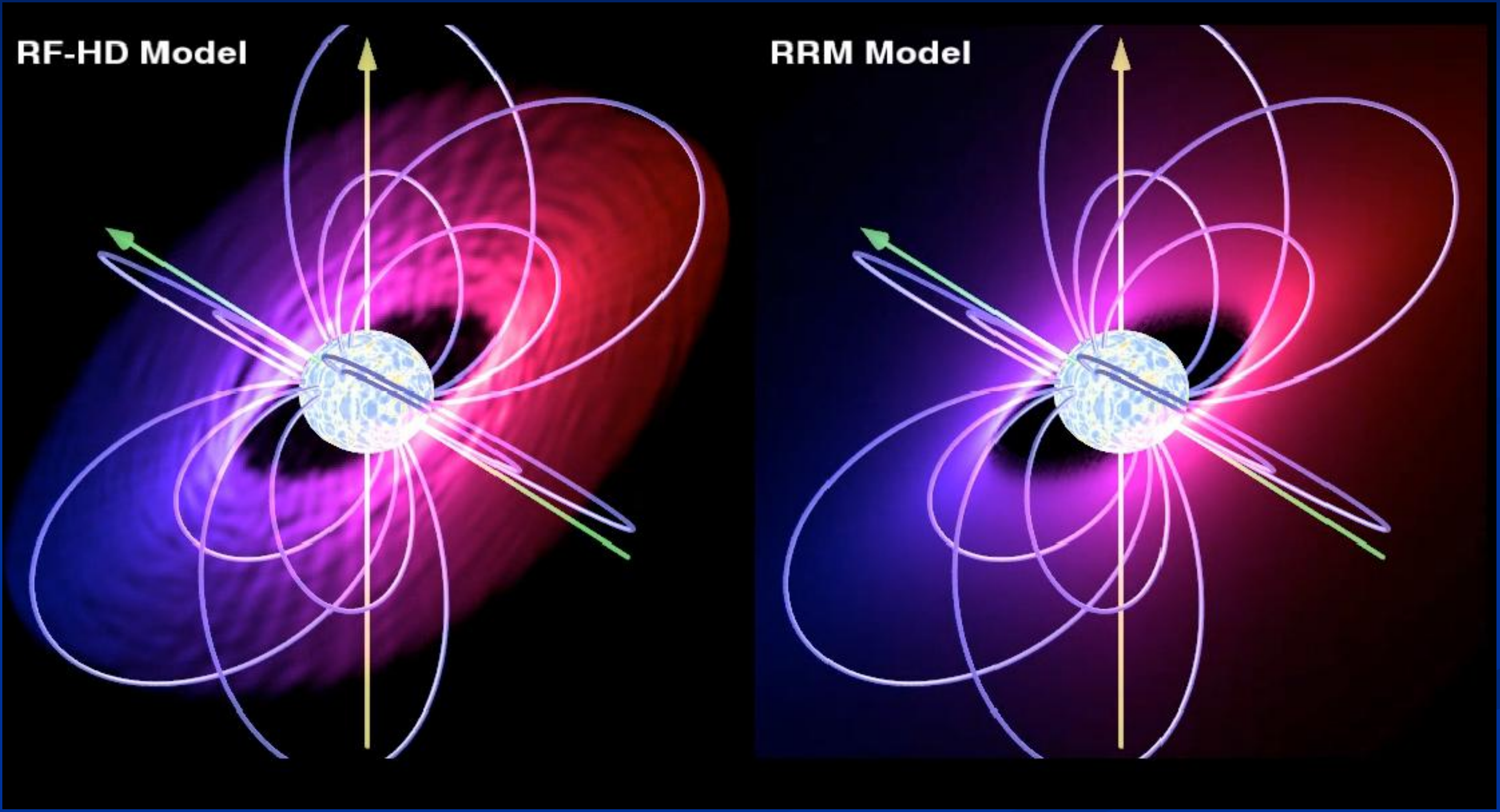


Rigid Field – Hydro Model



Townsend , Owocki and ud-Doula (2007)

Compare (3D)



Summary

- Magnetic field + hot star wind:
  - $\eta_* > 10$: **channel** wind into shock (MCWS)
    - explains X-rays from Θ^1 Ori C
  - **spin up** wind past Kepler co-rotation radius
  - **confine** material against centrifugal
    - Rigidly Rotating Magnetosphere (RRM)
    - explains H-α vs. rot. phase in σ Ori E
  - mass build up => **centrifugal breakout**
    - reconnection => T >~ 10^8 K
    - can explain hard X-ray **flares** in σ Ori E
    - "centrifugally driven reconnection"

Current & Future Work

- 3D MHD of MCWS
  - lateral structure scale, tilted field case
- Comparison with observations
  - Chandra & XMM
    - X-rays from shocks and flares
  - Optical photometry, spectroscopy, polarimetry
- Application to other types of magnetospheres?
- RRM shock as site for Fermi acceleration??
  - Could produce up to TeV Gamma Rays!