Seismic diagnostics for rotating ***massive main*** ***sec*** … ***rs***

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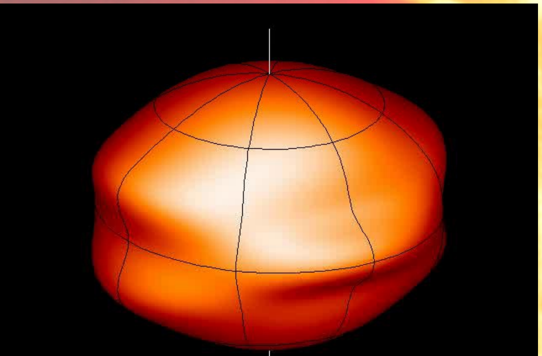
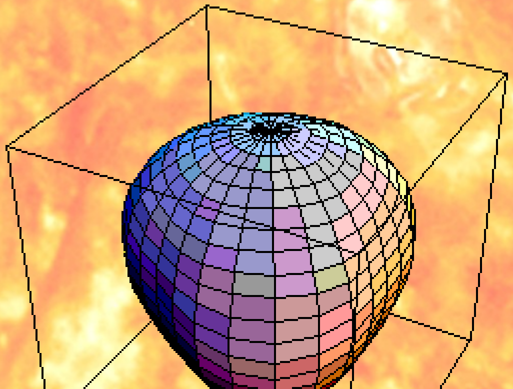
What information can we obtain- about rotation - from the oscillations of stars:

here massive, MS stars → defines the type detected modes





http://sohowww.nascom.nasa.gov/gallery/images/large/cropplume.jpg

OUTLINE

- I Introduction

- II. Theoretical framework: brief reminding

No rotation, notation

- III. Seismic analyses of the 4 β Cep *

III.1 Rotational splittings and overshoot
III.2 Splitting asymmetries : distorsion
III.3 Axisymmetric modes : mixing

* Continued after Anne’s talk

Introduction

O- B stars are characterized by a convective core and an envelope
which is essentially radiative apart in the region of the Z-opacity bump

Important incertainties regarding the structure and futur evolution of these stars are:

-the extent of chemical element mixing beyond the core instable
layers as defined by the Schwarzschild criterium

-Transport of angular momentum because the rotation also plays some role in element mixing

1) Convective instability

Rising eddy: $\rho < \rho_{env}$ after travelling distance l

buoyancy : $f = - g\, \delta\rho = m\, \gamma$

Eddy stops when $\gamma = 0$

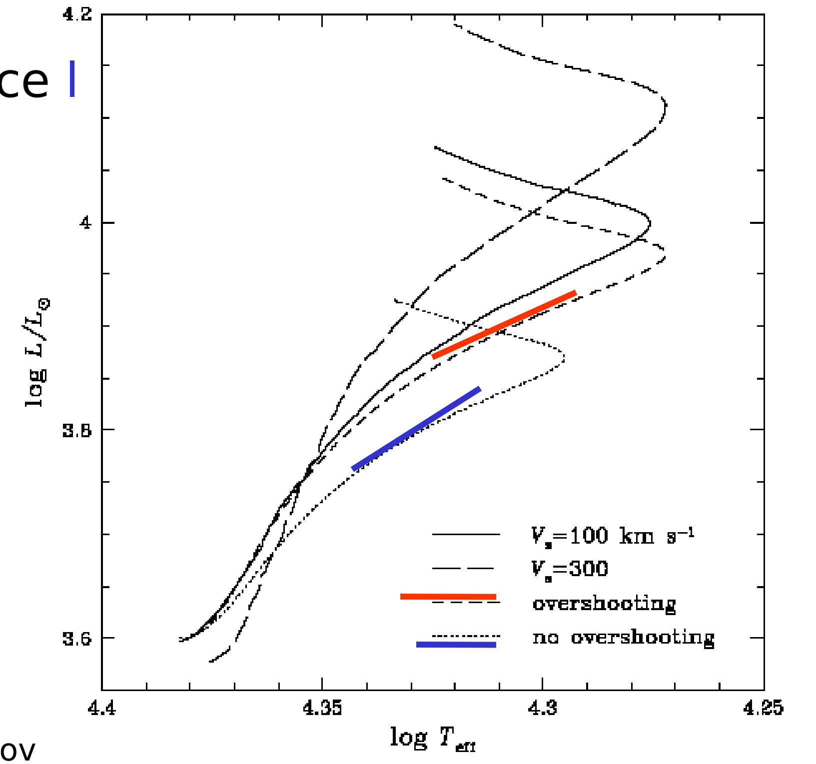
→ $\nabla_{ad} < \nabla_{rad}$

→ Convective core at Schwarzschild radius r_{zc}

but $v \neq 0$

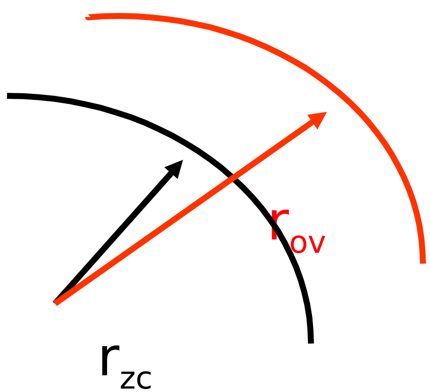
Core overshoot:

Due to inertia, eddies move beyond the Schwarzschild radius up to v=0 : r_{ov}



Evolutionary tracks for a $9M_\odot$ models

(*Talon et al., 1997)*



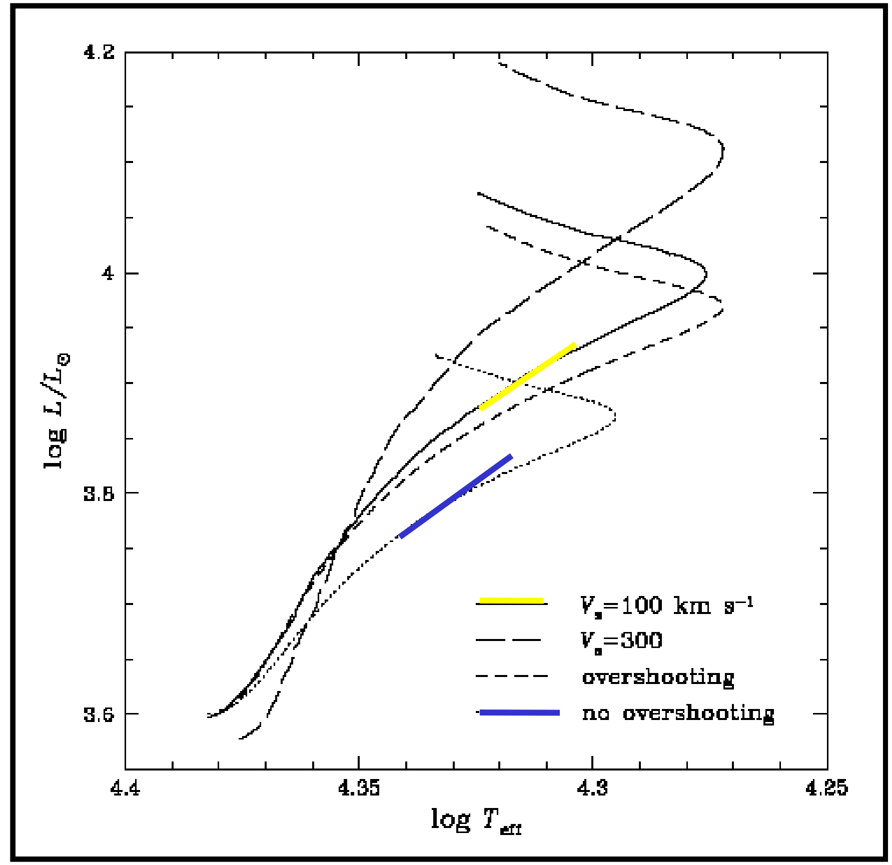
The overshooting distance in a simplist description in 1D stellar evolutionary models: $d_{ov} = \alpha_{ov} \min(r_{zc}, H_P)$ with H_P= local pressure scale height

2) Rotationally induced mixing

Meridional cirdculation +
(rotationally induced) turbulence

→Diffusion of chemical elements

→ mixing



Evolutionary tracks for a $9M_{\odot}$ models
(*Talon et al., 1997)*

We want : to identify

regions of uniform rotation and regions of differential rotation

(depth, latitude dependence)

inside the star ($\Omega_{core}/\Omega_{surf}$) → constraint on transport of angular momentum

Another goal is

to disentangle effects of overshooting and rotation on mixed central regions and extension of convective core

Seismology of O-B stars can bring some light about these processes:

Fitting axisymmetric modes m=0 : overshoot distance
Non axisymmetric modes : rotation profile

βCephei stars are good candidates for this purpose

Advantage over delta Scuti stars: no near surface convective layers
→ mode identification is trustworthy

β Cephei stars

Pulsating stars with masses roughly > 5 Msol

A few modes around the fundamental radial mode:

low radial order, low degree p/g modes with periods

around 3-8 h:

sofar observed and identified p1, p2, g1 modes

often of *mixed* p ang g nature

Reviews: Kurtz 2006, Handler 2006, Stankov, Handler 2005, Pigulski 2007, Aerts 2008 g modes p propagative when $\omega^2 > N^2$ and $\omega^2 > S_l^2$

Brunt-Vaissala (buoyancy) frequency

Lamb frequency

(sound speed)

$$N^2 = \frac{g}{r}\frac{\mathrm{d}\ln P}{\mathrm{d}\ln r}\left[\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P (\nabla_{\mathrm{ad}} - \nabla) - \nabla \mu\right]$$

$$S_l^2 = (k_h c_s)^2 = l(l+1)\, c_s^2/r^2$$

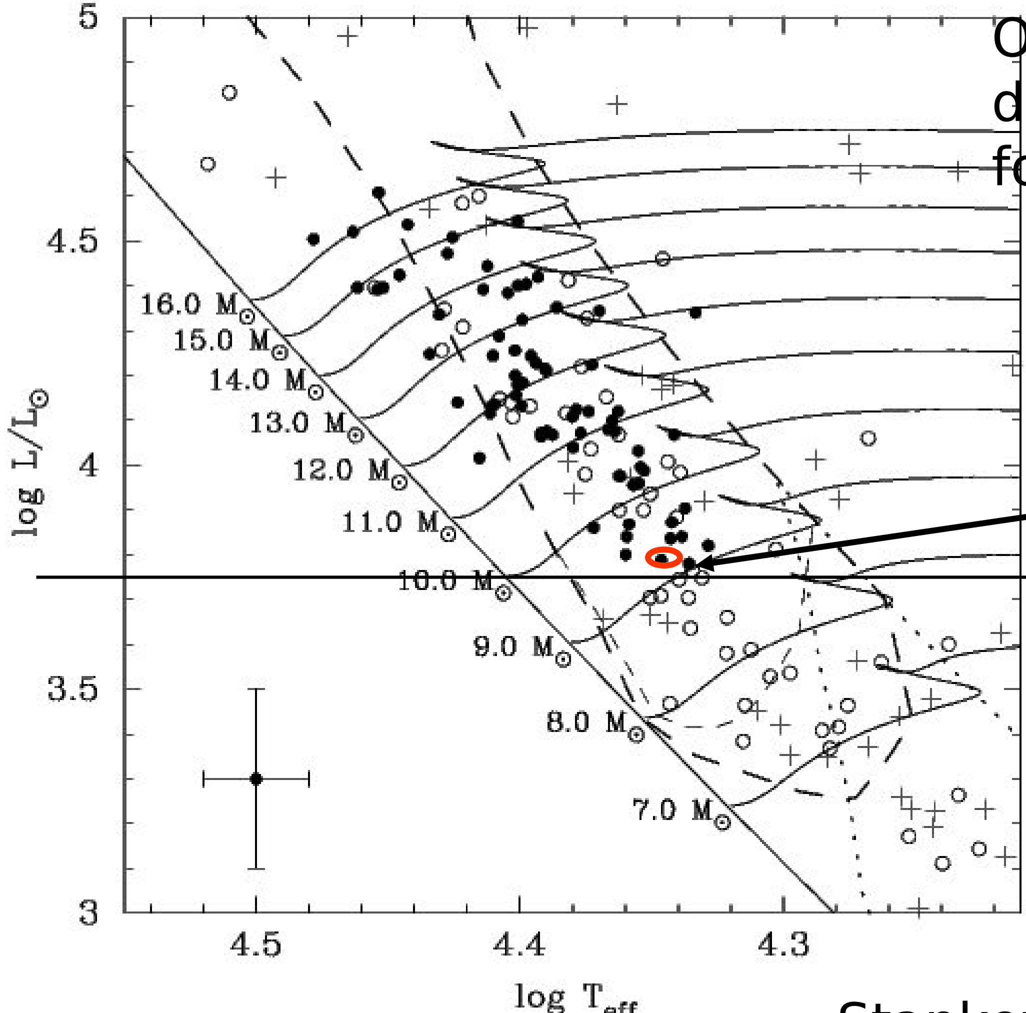
Modes g propagative when $\omega^2 < N^2$ and $\omega^2 < Sl^2$

Mixed modes : g mode in the inner part and p mode in the outer part

β Cephei stars

HR Diagram and instability strip

Full dots: confirmed beta Cep
Open : candidates
dashed lines delimitate the IS
for the fundamental radial mo



A typical model_

Stankov, Handler 2005

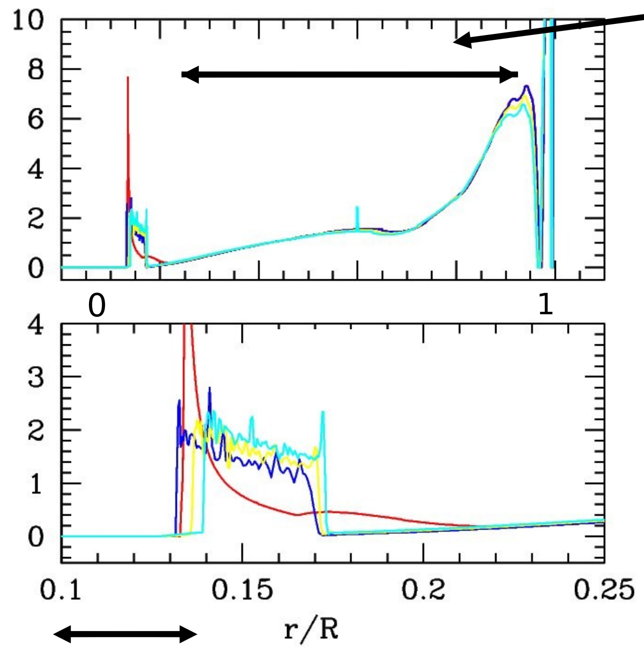
Brunt-Vaissala for a typical model N^2

$$N^2 /g = (1/\Gamma_1) (d\ln p /dr - d\ln\rho/dr)$$

$8.5\ M_{sol}$, T_{eff} = 22 230K

Radiative envelope

$N^2 r/g$

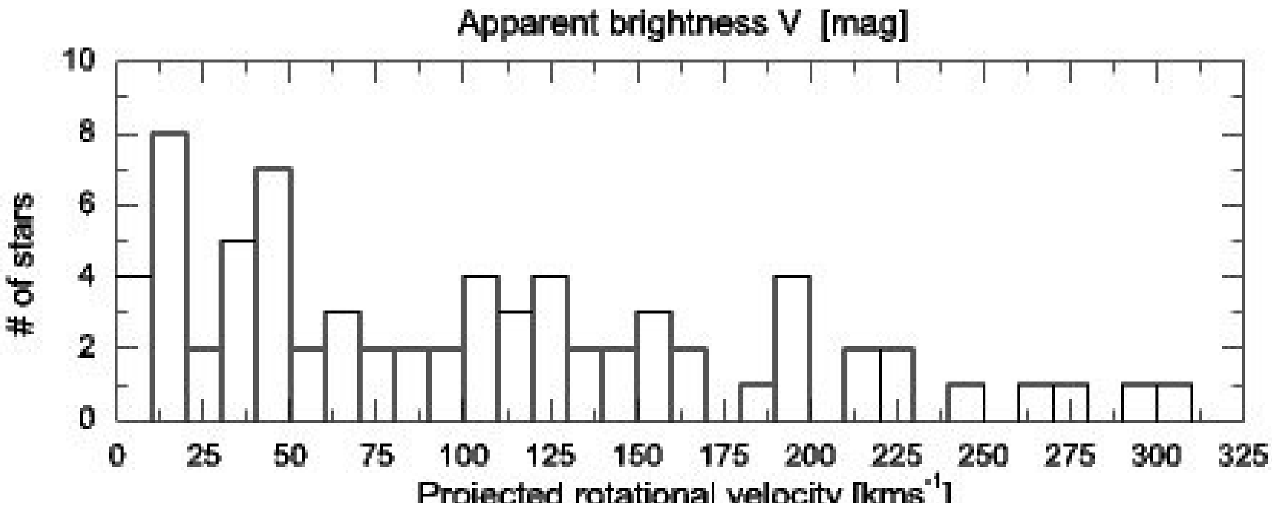


Modes with frequencies around that of the fundamental radial (σ=2-3) can be mixed modes

Convective core_

$\nabla\mu$ region

β Cephei stars



Stankov, Handler 2005

slow rotators <50 km/s up to rapid > 250 km/s

Ω/Ω_K ~0.003 up to 0.015

$\Omega_K = (GM/R^3)^{1/2}$ break up angular velocity

Rapid rotation for these stars ~100 km/s C. Lovekin, R. Deupree

Ω/Ω_K ~0.06

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II

Theoretical background : a brief reminder

Tools for a theoretical interpretation

Study pulsation → linearized equations about a static equilibrium stellar model

P_0, ρ_0, $\Gamma 1$, ϕ_0

Rotation → centrifugal and Coriolis accelerations come into play : $\Omega(r,\vartheta)$

Centrifugal force affects the structure of the star : oblatness, distorsion

→ meridional circulation, chemical mixing

Coriolis force enters the equation of motion → affects the motion of waves and frequencies of normal modes

Equation of motion is perturbed, resonant cavity is modified

→ Linearized equation of motion is modified

→ static equilibrium stellar model is modified

No rotation

adiabatic oscillations:
linearized equations of momentum, continuity and adiabatic relation + boundary conditions leads to an eigenvalue problem:

$$\mathcal{L}_0 \boldsymbol{\xi} - \rho_0 \hat{\omega}^2 \boldsymbol{\xi} = 0$$

with

$$\mathcal{L}_0 \xi = \nabla p' - \frac{\rho'}{\rho} \nabla p_0 + \rho_0 \nabla \phi'$$

+ B(oundary) C(onditions)

→ Eigenvalue problem: ω is the eigenvalue

ξ is the eigenfunction for the displacement

No rotation: axisymmetric modes: m=0

$$\int d3r\, \xi^* \left(\mathcal{L}_0 \boldsymbol{\xi} - \rho_0 \hat{\omega}^2 \boldsymbol{\xi} \right) = 0$$

$$\omega_0^2 = \frac{1}{I} < \vec{\xi}^* | \mathcal{L}_0 \, \vec{\xi} > \quad ; \quad I = < \vec{\xi}^* | \vec{\xi} >$$

Eigenmode: displacement
At zeroth order, it is written with a single harmonics

l: degre
m:nombre de nœuds
le long de l'equateur

$$\vec{\xi}\,(\vec{r}) = \xi_r(r) Y_{\ell,m} \vec{e}_r + \xi_h(r) \vec{\nabla}_h Y_{\ell,m}$$

Composante horizontal

Composante radiale
du deplacement

Add rotation

Results discussed here are obtained with perturbation methods

Rotation: Seismic Diagnostics:

1- Splitting

2 Splitting Asymmetries

3- Centroid modes ν (m=0)

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III

Four β Cepheid stars

with seismic analyses

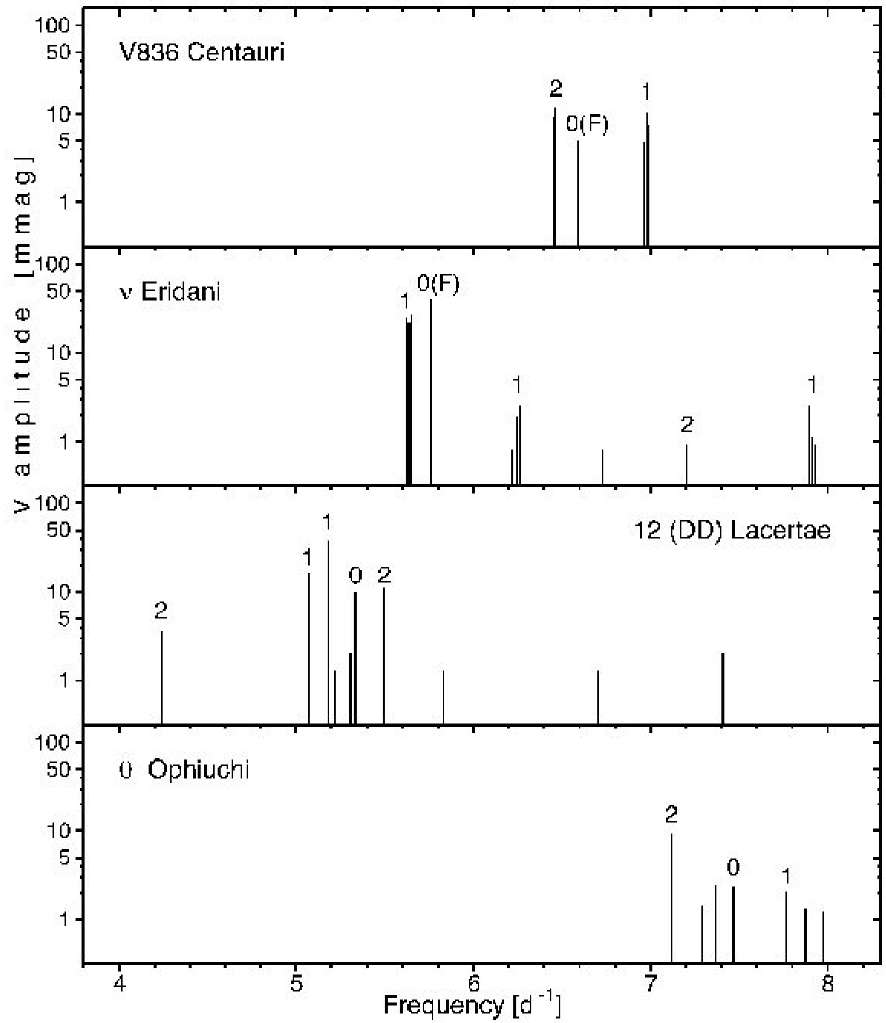
β Cephei stars

4 such stars have been the subject of seismic analyses:

V836 Cen (HD 129929) *Aerts 2003, Dupret et al 2004*
ν Eri *Pamyatnykh et al 2004, Ausseloos et al 2004, Dziembowski et al 2007 Dziembowski, Pamyatnykh 2008*
12 Lac *Dziembowski et al 2003, Dziembowski, Pamyatnykh 200*
θ Ophiuchi *Briquet et al 2005, 2007*

For which information about *rotation* And *core overshoot* has been inferred

Some others are under investigation



Pigulski 2007

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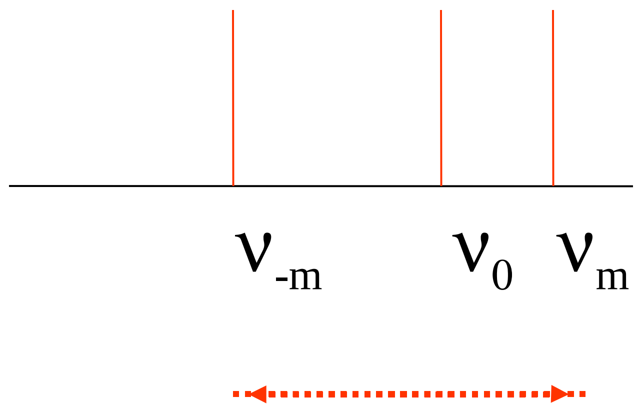
III.1 Rotational splitting

ν_{0nlm} = frequency for a given oscillation mode: n, l , m

Rotation breaks the azimuthal symmetry , lifts the degeneracy: 2l +1 modes (given n,l):

schematique l=1 triplet



→ S = $(\nu_m - \nu_{-m})/2m$

also $\delta\nu = \nu_m - \nu_0$ or $\delta\nu = \nu_{(m+1)} - \nu_m$

when only a few successive components are available

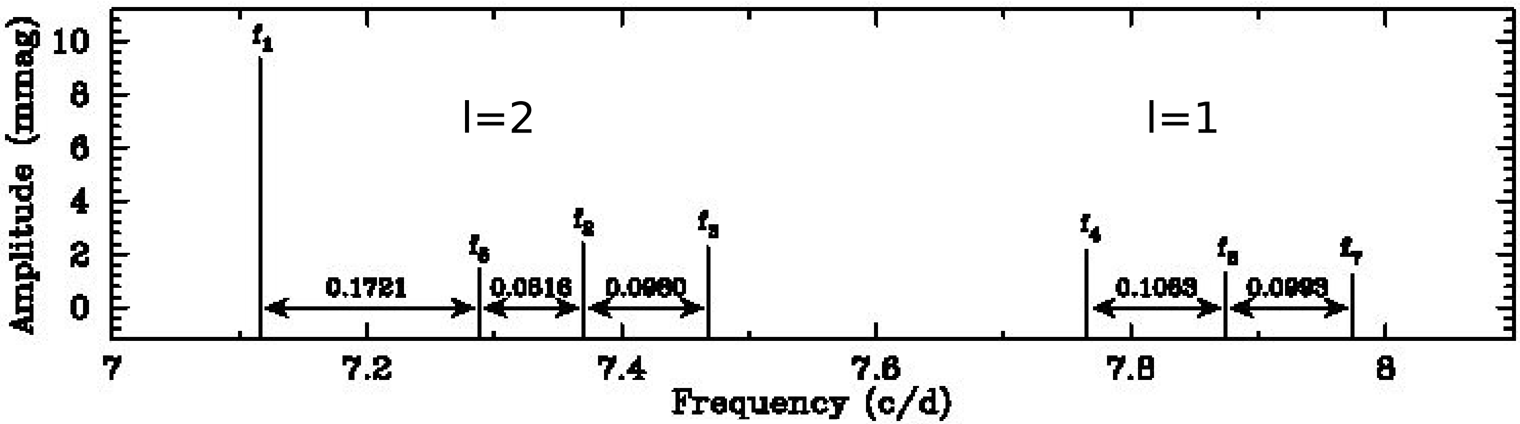
Ophiuchi: a β Cephei with a mass ¬ 9 Msol and an effecitve temperature ¬22 900 K

identified frequencies:

he radial fundamental l=0 (p1)

one triplet l=1 (p1)

3 components (m=-1,1,2) d'un quintuplet l=2 (g1)



3 multisite_campains (Handler et al 2005)

f1 = 82.3611 μHz

f4= 89.8831 μHz

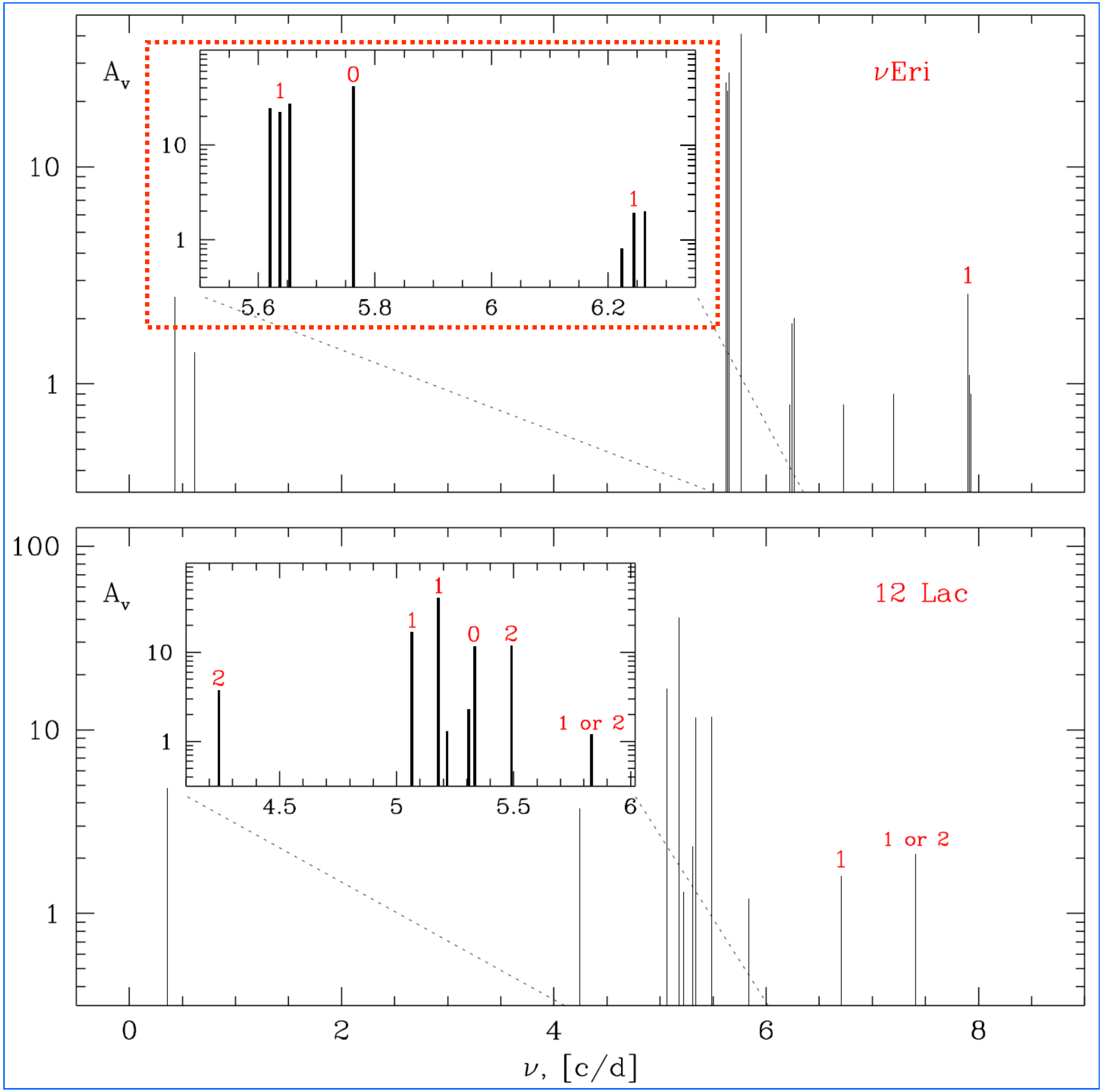
f5= 91.1366

f6 =92.2847

Seismic analysis Briquet et al 2007

Oscillation spectra of ν Eri and 12 Lac

Two rotationally splitted triplets of *l* = 1 modes (g1 and p1)



Dziembowski & Pamyatnykh 2008

III.1a) Theoretical expression

Linearized equation of motion

From Coriolis acc. $2\Omega\times v$

First order

$$\mathcal{L}_0\boldsymbol{\xi} - \rho_0\hat{\omega}^2\boldsymbol{\xi} - 2\rho_0\hat{\omega}\Omega\boldsymbol{K}\boldsymbol{\xi} + \qquad = 0, \quad (1)$$

$\hat{\omega} = \omega + m\,\Omega$ ω is the eigenfrequency

ξ is the displacement eigenvector (ξr radial component

ξh : horizontal component)

$\int d^3r\,(\xi^* . (1)) \rightarrow$

no rotation Coriolis

from which one derives $\omega_{nlm} = \omega_{nl0} + m\,S_{nl}$

Assuming a shellular rotation Ω(r)

Splitting (m azimuthal order)

Rotation angular velocity

$$S = \int_0^R dr K_{n,\ell}(r)\Omega(r)$$

avec

$\Lambda^2 = l(l+1)$

$$K_{n\ell}(r) = \frac{\rho_{00}\, r^2}{I}\left(\xi_r^2 + \Lambda^2\xi_h^2 - 2\xi_r\xi_h + \xi_h^2\right)$$

rotational kernel

avec

$$I = \int_0^R \rho_{00} r^2\, (\xi_r^2 + \Lambda^2\xi_h^2) dr$$

Mode inertia_

Equivalent splitting definition at first order

$$S \equiv \frac{\sigma_m - \sigma_0}{m} = \frac{\sigma_m - \sigma_{-m}}{2m} = \sigma_m - \sigma_{m-1}$$

depending which componenbts of the multiplets qre available

inferred

For a uniform rotation

III.1.b) HD 129929 is a beta Cephei d'environ 9 Msol, MS

Aerts et al 2003, Aerts et al 2004, Dupret et al 2004

1 For a uniform rotation

$$S = \Omega \beta$$

with β known from model , measured splittings S gives :

l=1 p=1 triplet → vrot = 3.61 km/s

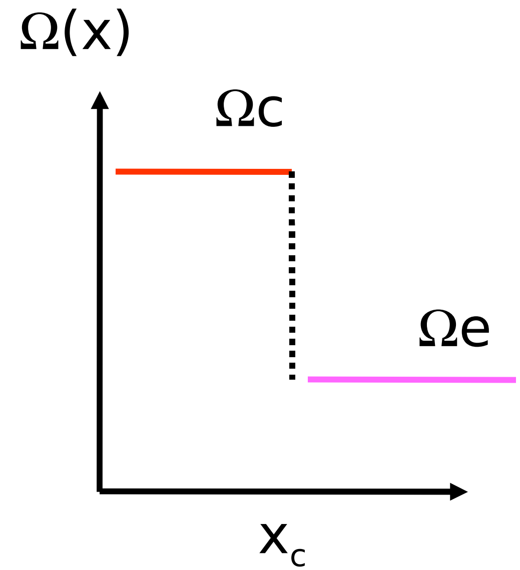
l=2 g1 2 successive components yields vrot = 4.21

→ Non uniform rotation

2 Rotation of the convective core ?

Assume a uniform rotation for the convective core with the angular velocity $\Omega = \Omega_c$ and a uniform rotation for the envelope $\Omega = \Omega_e$. Both values now are the unknowns

Inserting into Eq(1), the splitting becomes



$$\frac{\sigma_m - \sigma_0}{m} = \frac{\sigma_m - \sigma_{-m}}{2m} = \Omega_c \beta_c + \Omega_e \beta_e$$

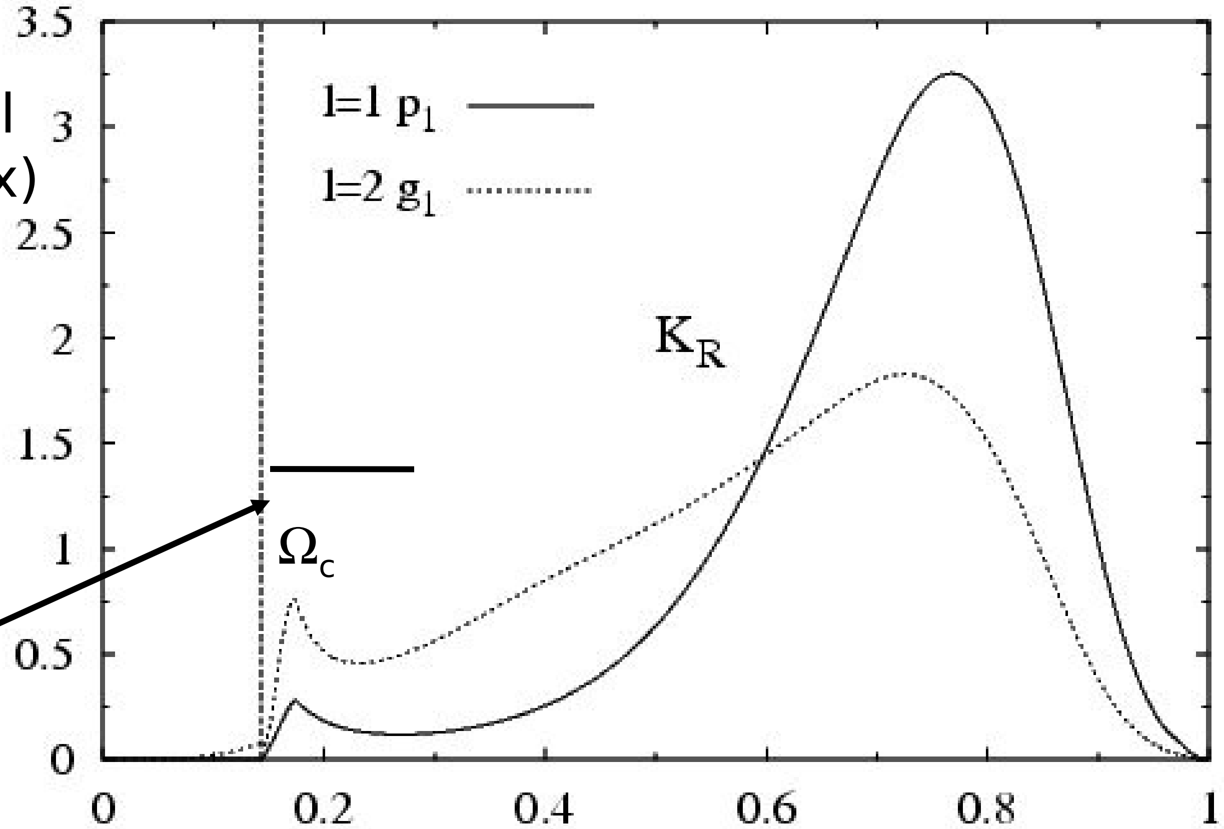
$$\beta_c \equiv \int_0^{x_c} K(x)\, dx$$

$$\beta_e \equiv \int_{x_c}^{1} K(x)\, dx$$

$$S = \int_0^1 K(x)\, \Omega(x) dx$$

βc < βe , these modes do not efficiently probe convective core

Rotational
Kernel K(x)



r/R

Ωc

Dupret et al 2004

→ Non uniform rotation with Ωc/Ωe=3.6

3- Depth variation of the rotation in the envelope

Assuming a linear depth variation of the angular velocity

in the
envelope

$$\Omega(x) = \Omega_0 + (x - x0)\ \Omega'$$

→ The splittings must obey $S = \Omega_0\ \beta_0 + \Omega_1\ \beta_1$

$\beta 0$ and $\beta 1$ are known from the models; the knowledge of S1 and S2 yields $\Omega 0$ and Ω'

$$\beta_0 = \int_0^{x_c} K\ (x)\ dx \qquad \beta_1 = \int_{x_c}^{x_e} (x - x_c)\ K\ (x)\ dx$$

→ non uniform rotation with avec $\Omega c/\Omega s=3.6$

Simple but efficient !

th the splittings of the l=1 triplet and et the components of the l=2 ultiplet, it is found that in the envelope, the rotation gradient is small and ta are compatible with a solid rotation

Conclusions:

l=1 p1 triplet and l=2 2 successive frequencies do not probe rotation of the core for this star. A small rotation gradient in the envelope

Core extension:

d mixing = 0.2 Hp is rejected

d mixing = 0.1 Hp is better than 0

.c) θ Ophiuchi: a β Cephei with a mass ¬ 9 Msol and an effective temperature ¬22 90

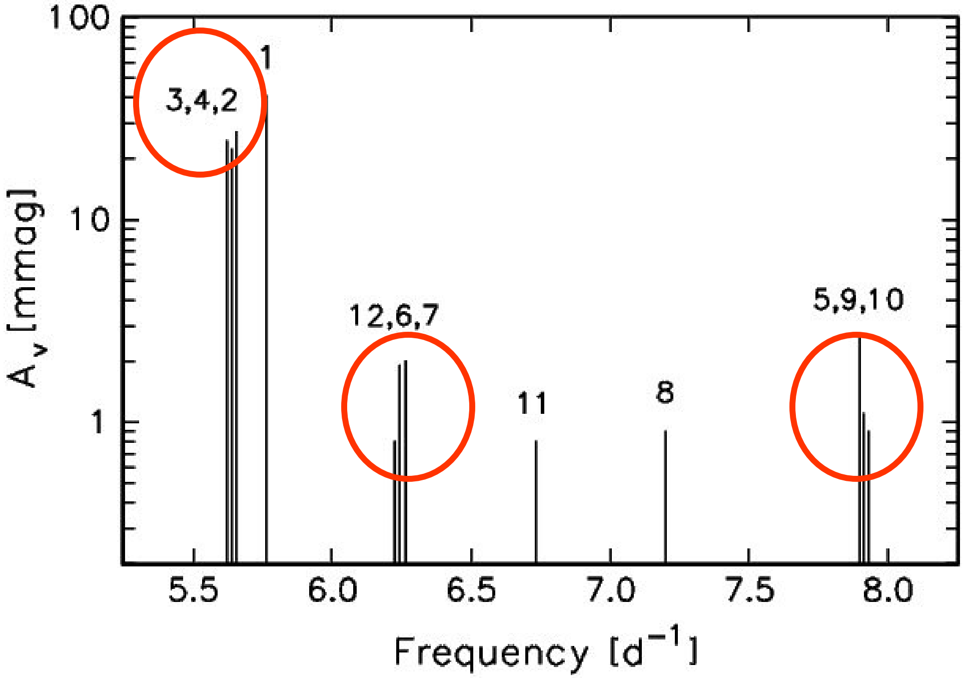
ultisite_campains (Handler et al 2005) → **7 identified frequencies:**
radial fundamental l=0 (p1) - one triplet l=1 (p1)
components (m=-1,1,2) d'un quintuplet l=2 (g1)

eismic analysis *Briquet et al 2007*
he situation is very similar to the previous case

he modes do not provide strong constraint about the convective core
however
rom its edge to the surface, a uniform or slightly varying rotation

Convective core extension : $\alpha_{mix} = 0.44 \pm 0.07$

III.1d) ν Eri



3 triplets l=1
(g1,p1,p2)
One radial mode p1
One l=2 component

Jerzykiewicz et al 2005

smic studies : *Pamyatnykh et al 2004, Aussellos et al 2004, Suarez et al*
embowski, Pamyatnykh 2008

Linear depth variation of Ω in the mu gradient zone
Ω =Ωc in the convective core
Ω=Ωe in the envelope above the grad mu region

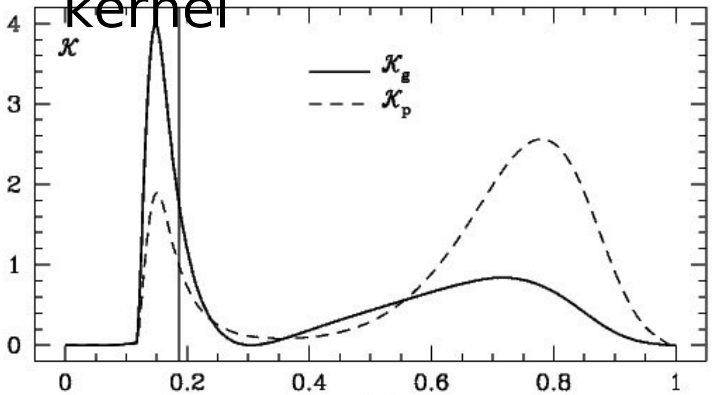
$$\Omega(x) = \Omega_c \quad \text{for } x_c > x$$

$$\Omega(x) = \Omega_c + (x - x_c)\ \Omega' \quad \text{for} \quad x_c < x < x$$

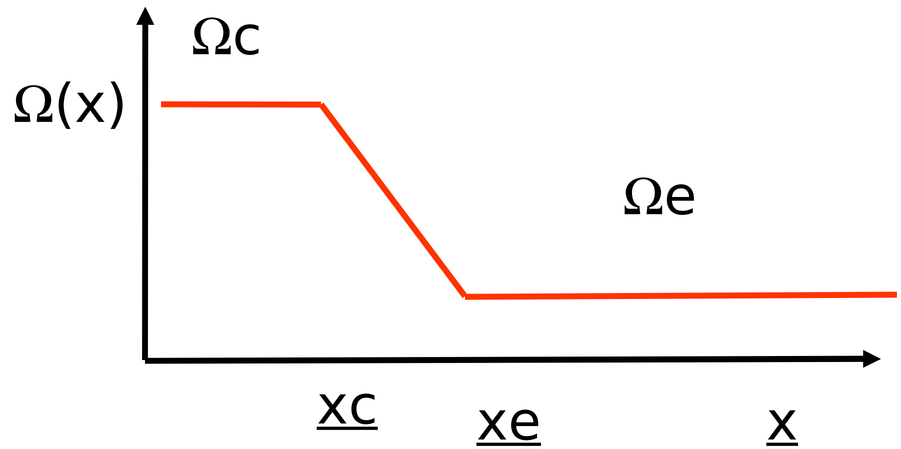
$$\Omega' = \frac{\Omega_e - \Omega_c}{x_e - x_c}$$

$$\Omega(x) = \Omega_e \quad \text{for } x_c > x$$

Rotational kernel



PD04



l=1 triplets g1 and p1 yield Ωc/Ωe = 5.3-5.8

DP08

Model fitting yields mixed core extension 0.1-0.2

III.1e)Summary

A few modes are enough to get some important information about internal rotation and core overshoot

→ If they are identified

→ If enough precise measurements

→ If age of the star such that ex
→ cited modes have mixed g, p modes

–

Desantangling overshoot/rotation effect on core element mixing ?

$d_{ov} = \alpha_{ov}$ HP   represents extension of mixed central layers

The question is

In the seismically measured d_{ov}, what part comes from eddies overshooting the r_{zc}

and

what part comes from other transport processes , f.i. rotation $\Omega c/\Omega e, \Omega e$?

Summary of III.1

Overshoot versus rotation

	Veq (km/s)	α_{ov}	$\Omega_{inner}/\Omega_{env}$	Z
D 129929	~2	0.1 ± 0.05	$\Omega_{(0.2)}/\Omega_{surf}$ ~3.1	0.019 ± 0.00
Ophiuchi	29 ± 7	0.44* ± 0.07	env. unif. rotation	0.012 ± 0.00
Eric	~6	0.15 ± 0.05	Ω_c/Ω_{env} ~ 5.5-5.8	0.0172 ± 0.001
Lac	~47	-	Ω_c/Ω_{env} ~ 4.65	0.015

2 others only α_{ov} ~ 0.2

*= Asplund mixture

It requires

faster rotators

other indicators

but one must take into account a Z-α_{ov} anticorrelation : Dupret et al, Thoul et al, Briquet et al

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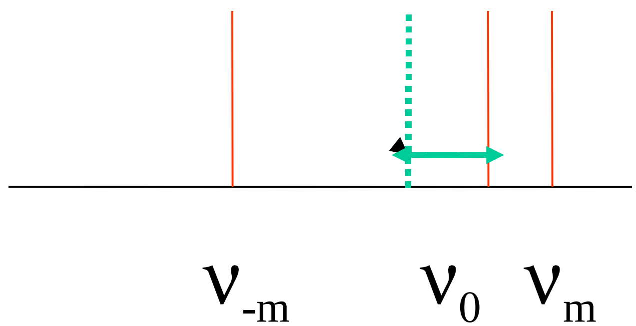
III.2 Splitting asymmetries

schematic l=1 triplet

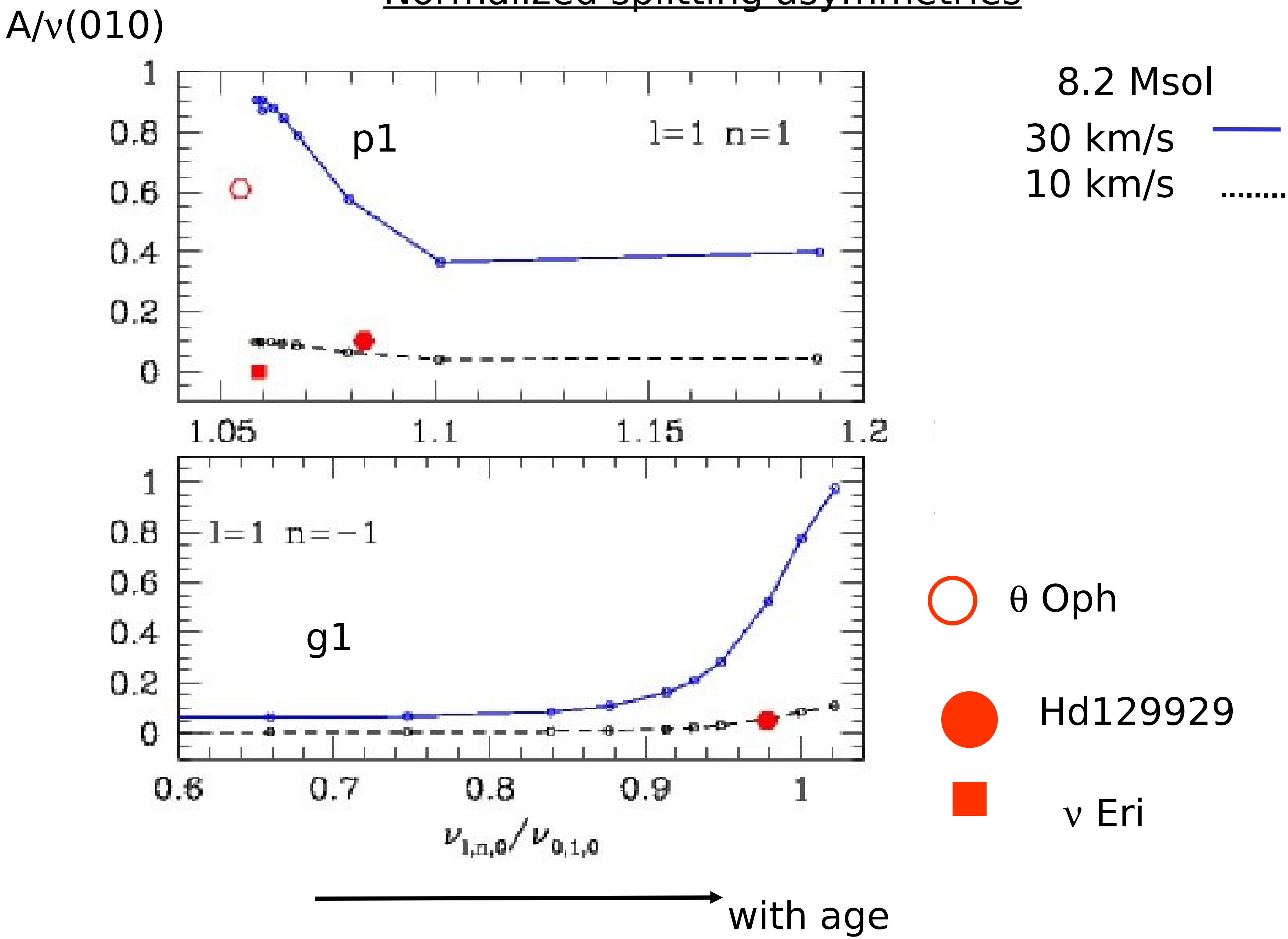


→ A_m splitting asymmetries due to 2nd order effect

$$A_m = \sigma_0 - \frac{1}{2}\left[\sigma_m + \sigma_{-m}\right]$$

ν in μHz, c/d ; σ normalized frequency

Normalized splitting asymmetries



Observed asymmetries deduced from l=2 seem to disagree:

θ Ophiuchi Briquet et al 2007

ν Eri splitting too small for l=1 p2 triplets (Dziembowski, Jerzykiewicz 2003)

Disagreement real ? Asymmetry values marginally above observed uncertainties

Not components of same multiplet ? (Dziembowski, Pamyathnykh, 2008 for nu Eri)

worth to infer

Splitting asymmetries= different probes as kernels are different

For given (n, ℓ, m) mode:

$$\nu = \nu_0 + \frac{\Omega}{2\pi}C + (\frac{\Omega}{2\pi})^2(D_0 + m^2 D_1)....$$

Splitting: slow rotators:Coriolis effect

asymmetry
Centrifugal distorsion dominates but
for low radial modes
Coriolis contribution remains significant

→ $A_m = \nu_0 - (1/2)(\nu_m + \nu_{-m}) = (\Omega/2\pi)^2 D_1$

Where does it come from ?

Ordre 2

(Centrifugal)

Linearized equation of motion:

$$\mathcal{L}_0\boldsymbol{\xi} - \rho_0\hat{\omega}^2\boldsymbol{\xi} - 2\rho_0\hat{\omega}\Omega\boldsymbol{K}\boldsymbol{\xi} + \left(\mathcal{L}_2 - \rho_2\hat{\omega}^2\right)\boldsymbol{\xi} = 0,$$

$$\mathcal{L}_0\xi = \nabla p' - \frac{\rho'}{\rho}\nabla p_{00} + \rho_{00}\nabla\phi'$$

spherical distorsion

et

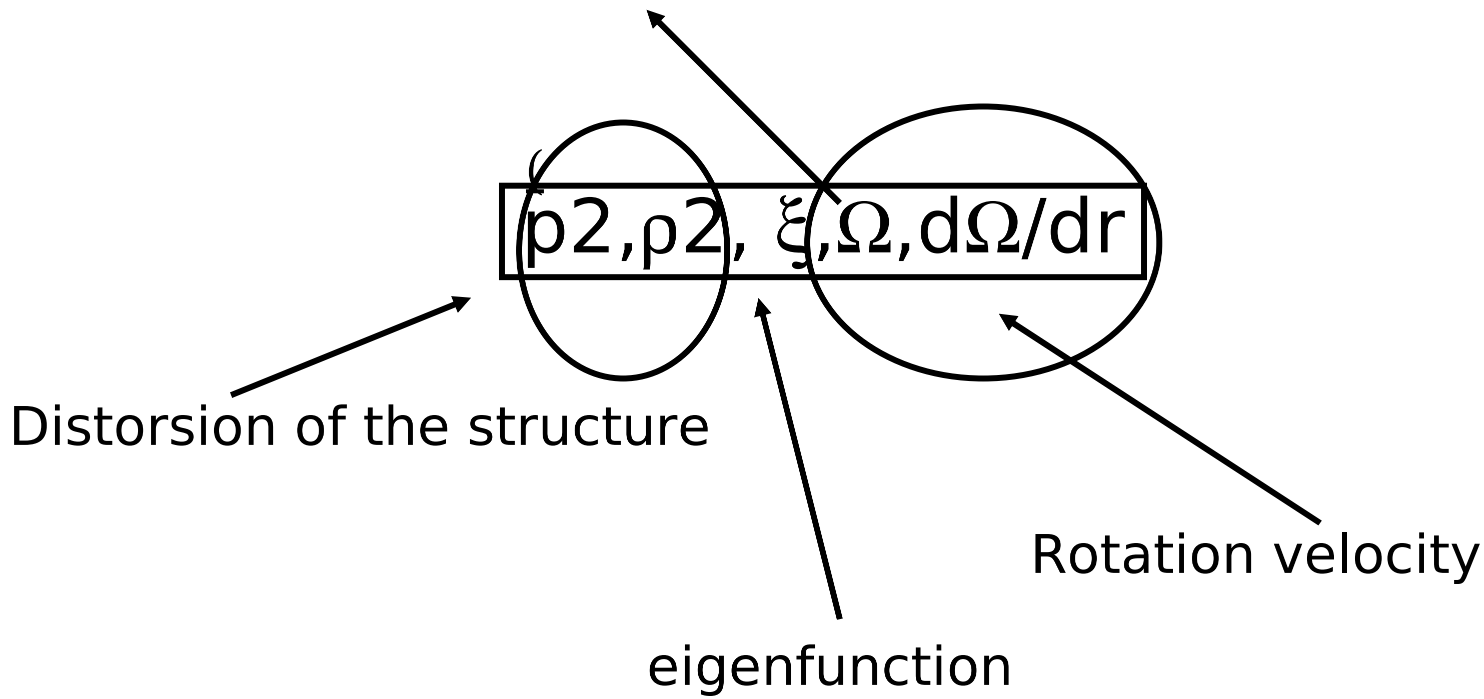
$$L_2\boldsymbol{\xi} = \frac{\rho'}{\rho}\left[\frac{\rho_2}{\rho}\nabla p - \nabla p_2\right] + \rho_2\nabla\phi' + \rho\,\boldsymbol{e_s} r\sin\theta\,\nabla\Omega^2.\boldsymbol{\xi}$$

Non spherical distorsion

p2,ρ2,ϕ2

$$A_{--} = \sigma_0 - \frac{1}{2}(\sigma_m + \sigma_{-m})$$

$$A_{--} = \int_0^1 \frac{\Omega(x)^2}{(2\pi)^2} K_2(x)\, dx$$



(normalized) K2(r)

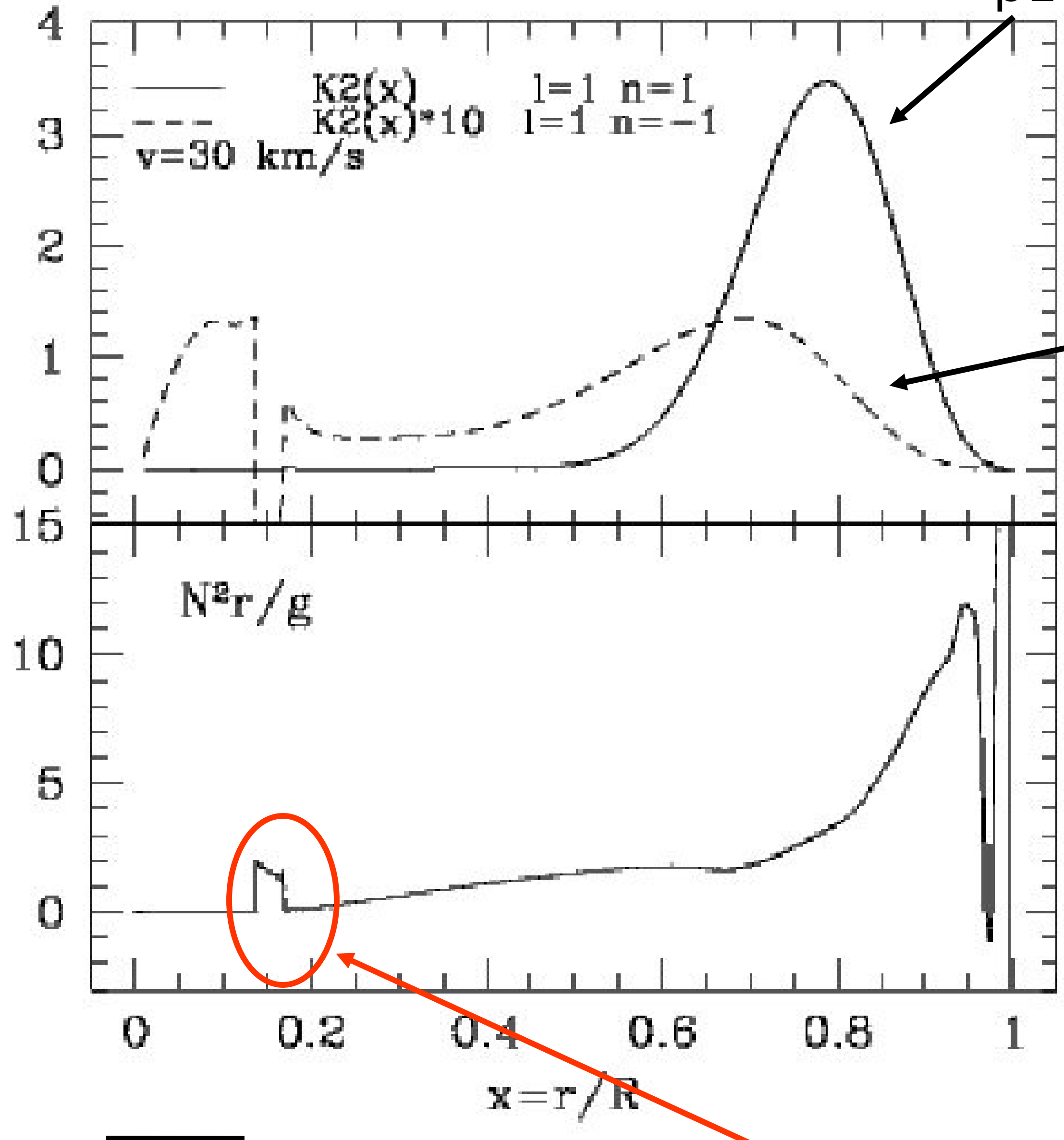
l=1 p1

l=1 g1

8.2 Msol
Xc=0.35

p1

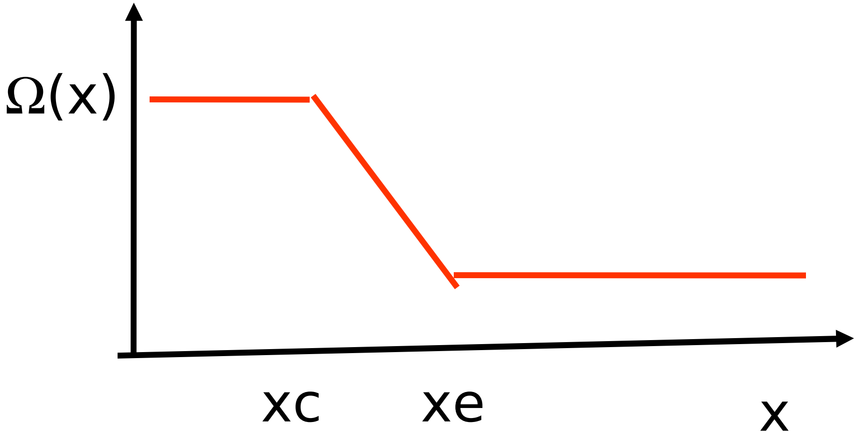
g1



Convective core

grad mu

With a few modes, similar inference than for splitting:



$$\Omega^2(x) = \Omega_c^2 \quad \text{for } x_c > x$$

$$\Omega^2(x) = \Omega_c^2 + 2(x - xc)\ \Omega'\ \Omega_c + (x - xc)^2\ \Omega'^2 \quad \text{for } x_c < x < x_e$$

$$\Omega^2(x) = \Omega_e^2 \text{ for } x_c > x$$

$$\Omega' = \frac{\Omega_e - \Omega_c}{x_e - x_c}$$

$$A_m = \Omega_c^2 \beta_0 + 2\Omega' \Omega_c \beta_1 + \beta_2 \Omega'^2$$

Assumed known from splitting

$$\beta_0 = \int_0^{x_c} K_2(x)\, dx$$

$$\beta_1 = \int_{x_c}^{x_e} (x - x_c)\, K_2(x)\, dx$$

$$\beta_2 = \int_{x_e}^{1} (x - x_c)^2\, K_2(x)\, dx$$

→ Constraint on β's hence on K2(x) ie on distorted structure and/or Ω(r

Promising prospects

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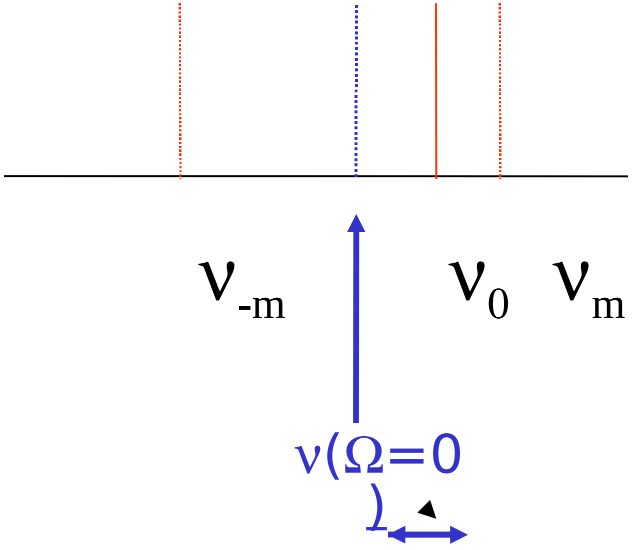
III.3 Axisymmetric modes and mixing

schematique l=1 triplet



Frequency differences $\nu 0$ - $\nu(\Omega=0)$can be efficient diagnostics

but_

Care with defining the $\Omega=0$ stellar model for comparison

Rotationally induces mixing changes the structure, particularly affects N^2 at the border of the convective core hence modifies axisymmetric mode m=0 frequencies

Rotational mixing: *Meridional circulation+ turbulence (Zahn, 1992 and subsequent works) induces some diffusive transport of chemical elements*

Evolution of chemical species:

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right].$$

nuclear transformation   atomic diffusion

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circulation turbulence

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^2 \left(D_{\mathrm{eff}} + D_v\right)\frac{\partial c}{\partial r}\right]$$

nuclear transformation atomic diffusion

‘Extra’ mixing

Chemical and angular momentum must be solved together

Evolution of angular momentum:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right) = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega u_r\right] + \frac{1}{r^4}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right]$$

circulation turbulence

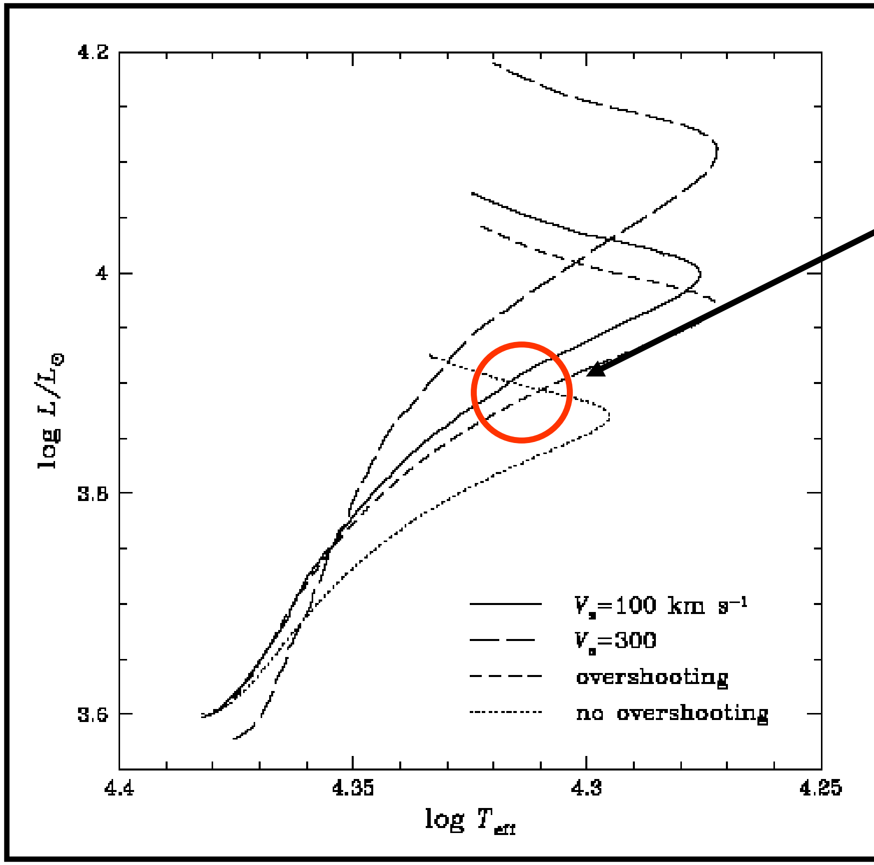
Evolution of chemical species:

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^2 \left(D_{\mathrm{eff}} + D_v\right)\frac{\partial c}{\partial r}\right]$$

nuclear transformation atomic diffusion

‘Extra’ mixing’ Deff,Dv(ur, Ω)

Consequences:

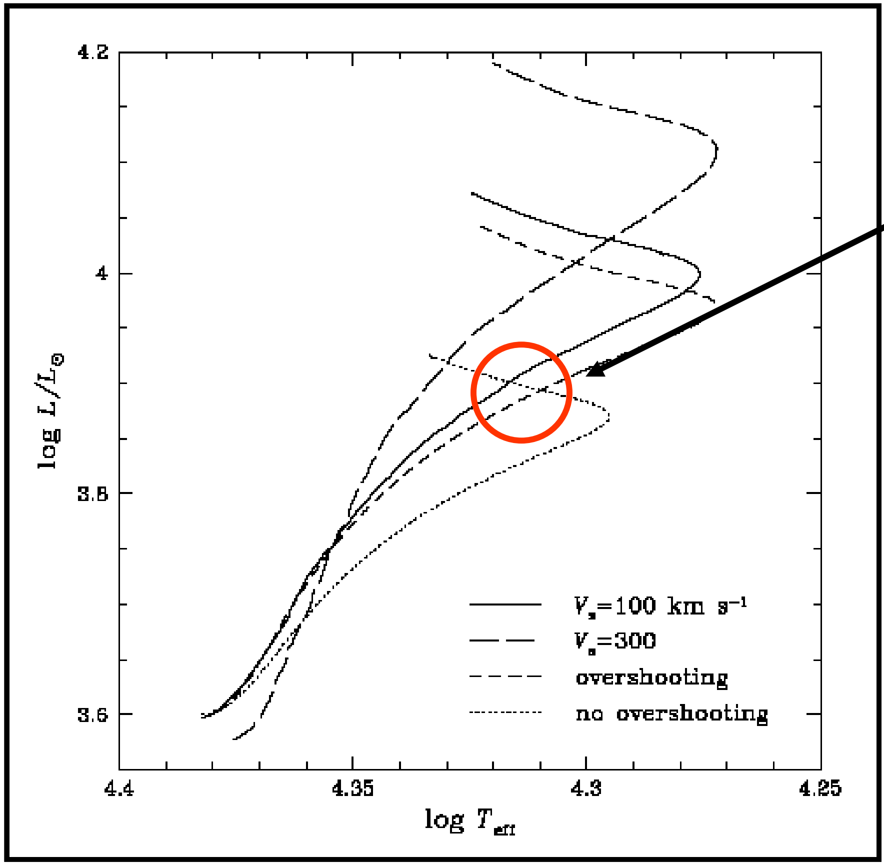


At a given location in a HR diagram, several models with different structures

ie differents axisymmetric modes ?

Evolutionary tracks in a HR diagram
9M models (*Talon et al., 1997*)-

Consequences:



At a given location in a HR diagram, several models with different structures

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Important effect at the edge of the convective core

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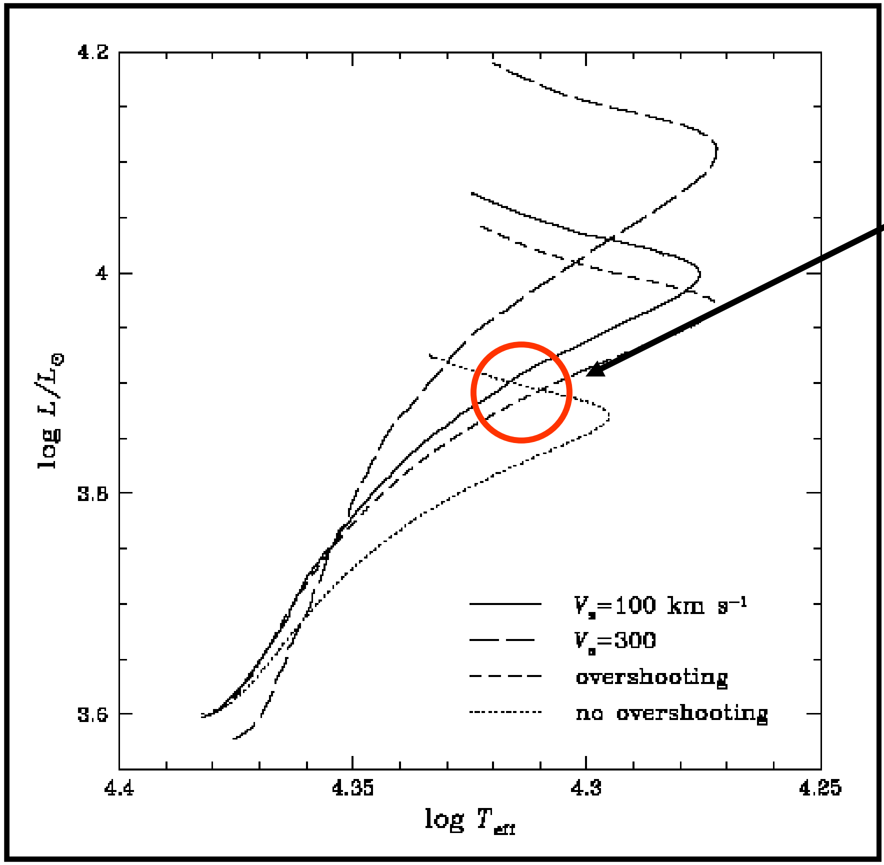

At a given location in a HR diagram, several models with different structures

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Important effect at the edge of the convective core

Evolutionary tracks in a HR diagram 9M models (*Talon et al., 1997)-*

gc mode can efficiently probe such effects as was suggested for overshooting by Dziembowski, Pamyatnykh 1999_

1-

Montalban, Miglio, Eggenberger, Noels, 2008
Miglio, Montalban, Noels, Eggenberger, 2008

nvestigation of effect of turbulent mixing on g mode spectrum and their ability to probe the extent of a convective core

Evolution of chemical species:

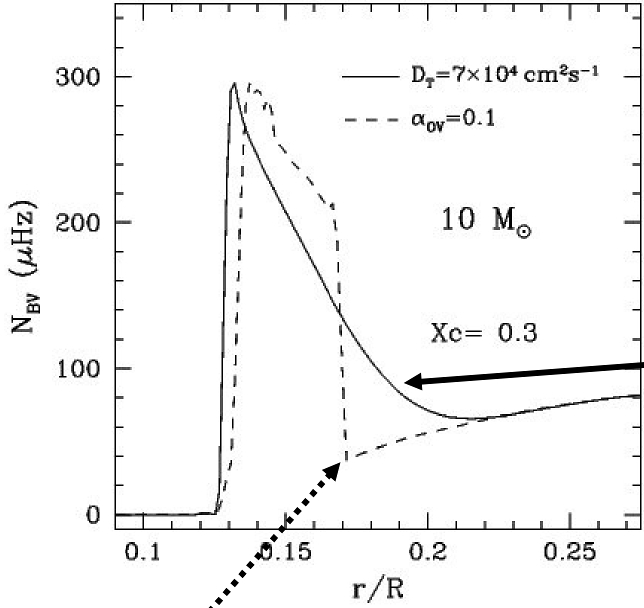
$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho\, c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^2 \left(D_{\mathrm{eff}} + D_v\right)\frac{\partial c}{\partial r}\right]$$

Dt = constant

Dt constant with evolution : for massive stars , roughly OK (J.Montalban)

Dt uniform inside the star, value taken to correspond to the value near
the convective given by Geneva 'rotating' models
OK pour g modes with most amplitude there

Dt chosen such that it corresponds
to a the same value at the edge of
the convective core for a Geneva
model with
10 Msol, Xc=0.3, Vini=50 km/s



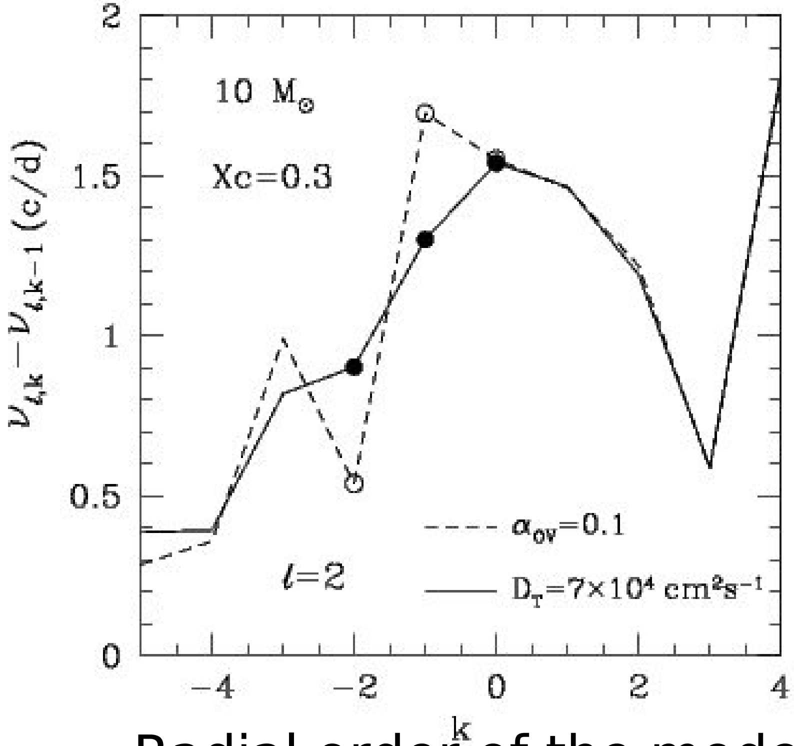
N² profile for a model
with extra element mixing

(Montalban et al 2008)

N² profil for a model with an overshooting distance dov =0.2 Hp

Large difference in N at the edge of the convective core → large effect
on frequencies of g modes and mixed modes

Large frequency separations for O and V models



Radial order of the mode_

O: model with overshoot
V: model with V50 km/s
pseudo rotationally induced
mixing

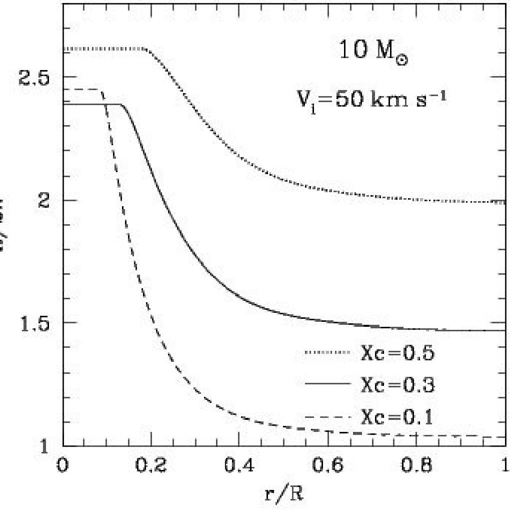
$\Omega c/\Omega e = 1.6$

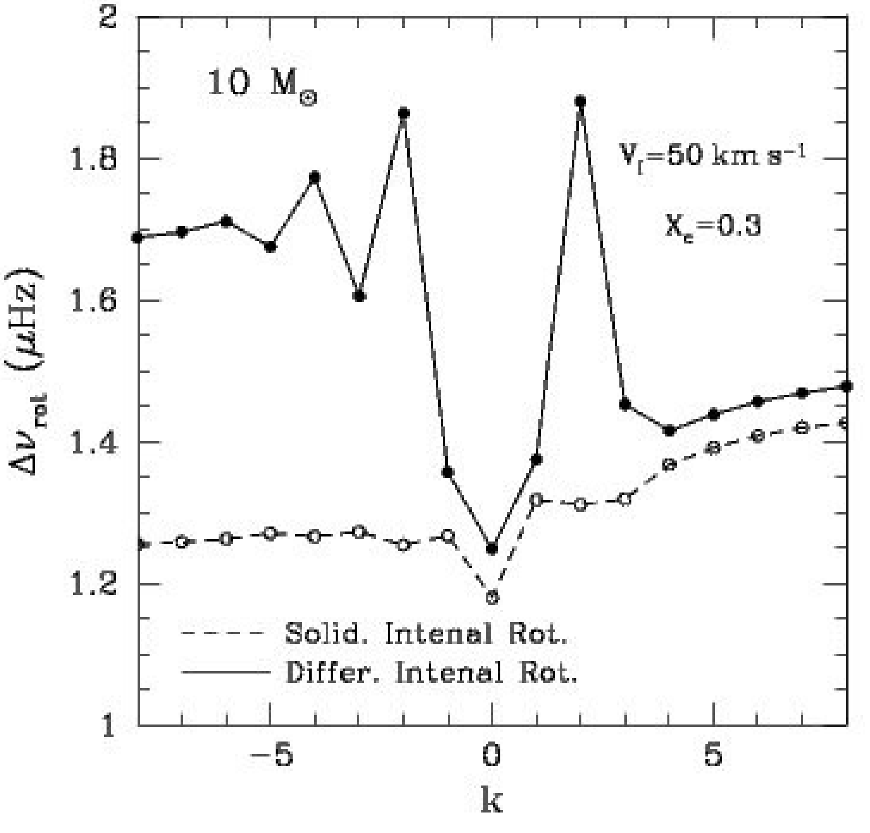
Montalban et al 2008

Rotationally induces mixing changes the structure
Hence frequencies and eigenfunctions

And therefore also splittings and asymmetries

Effect of pseudo rotationally induced mixing on splittings

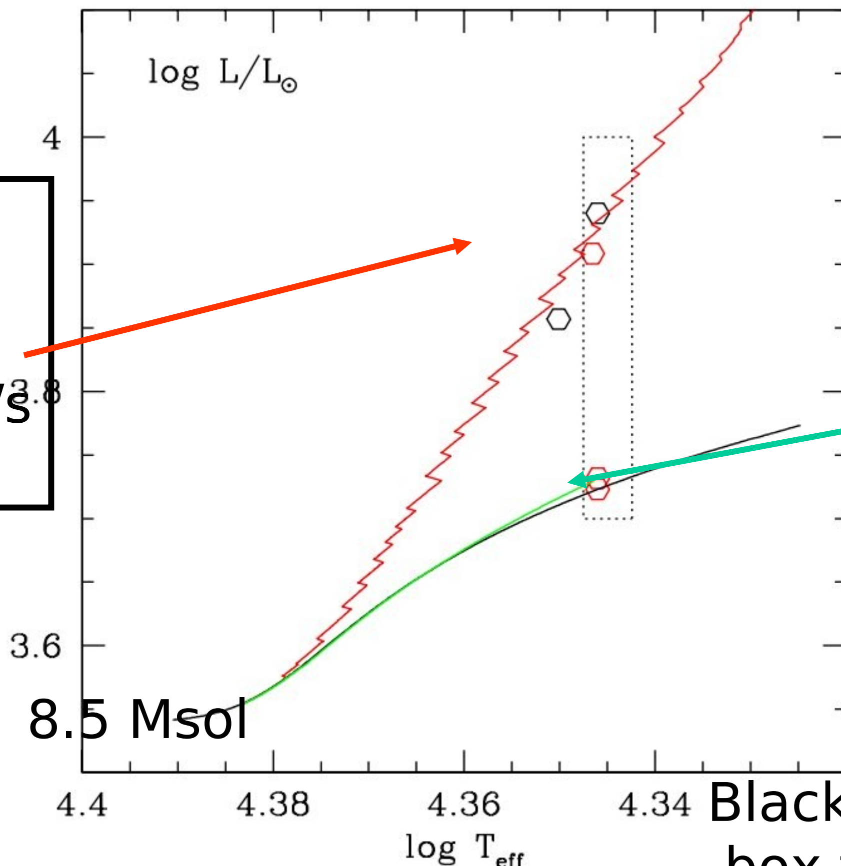




Montalban et al 2008

2-

Evolution including coupling rotationally induced mixing and angular momentum transport



V=75 km/s
Initial on pms

Vsurf= 167 km/s
Red dot_

no rot mixing
V0

V=15 km/s

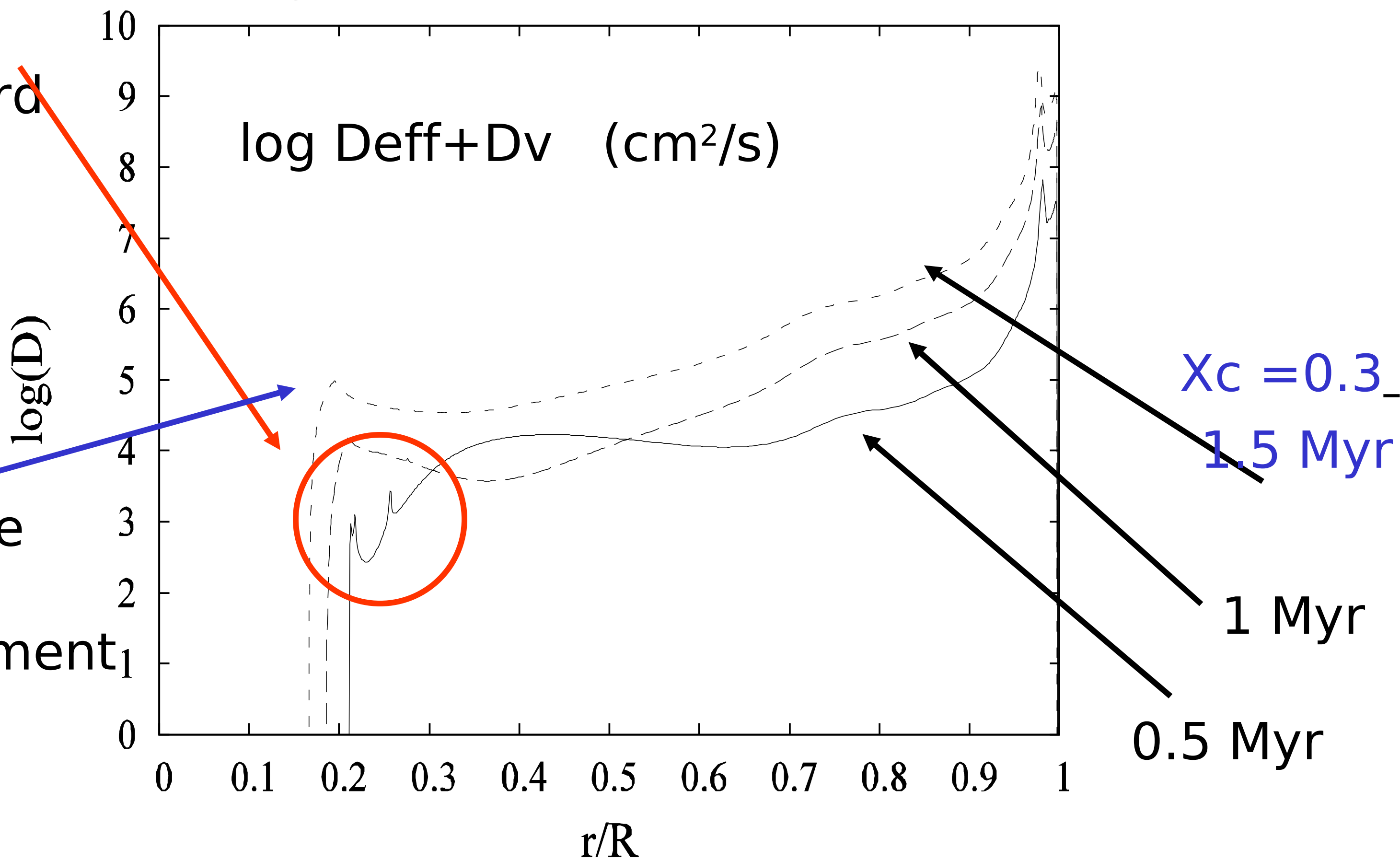
Initial on pms

Vsurf = 30 km/s
red dot model
V15

Black dotted box = error box for ϑ Oph
Black dots : HD129929, υ Eri

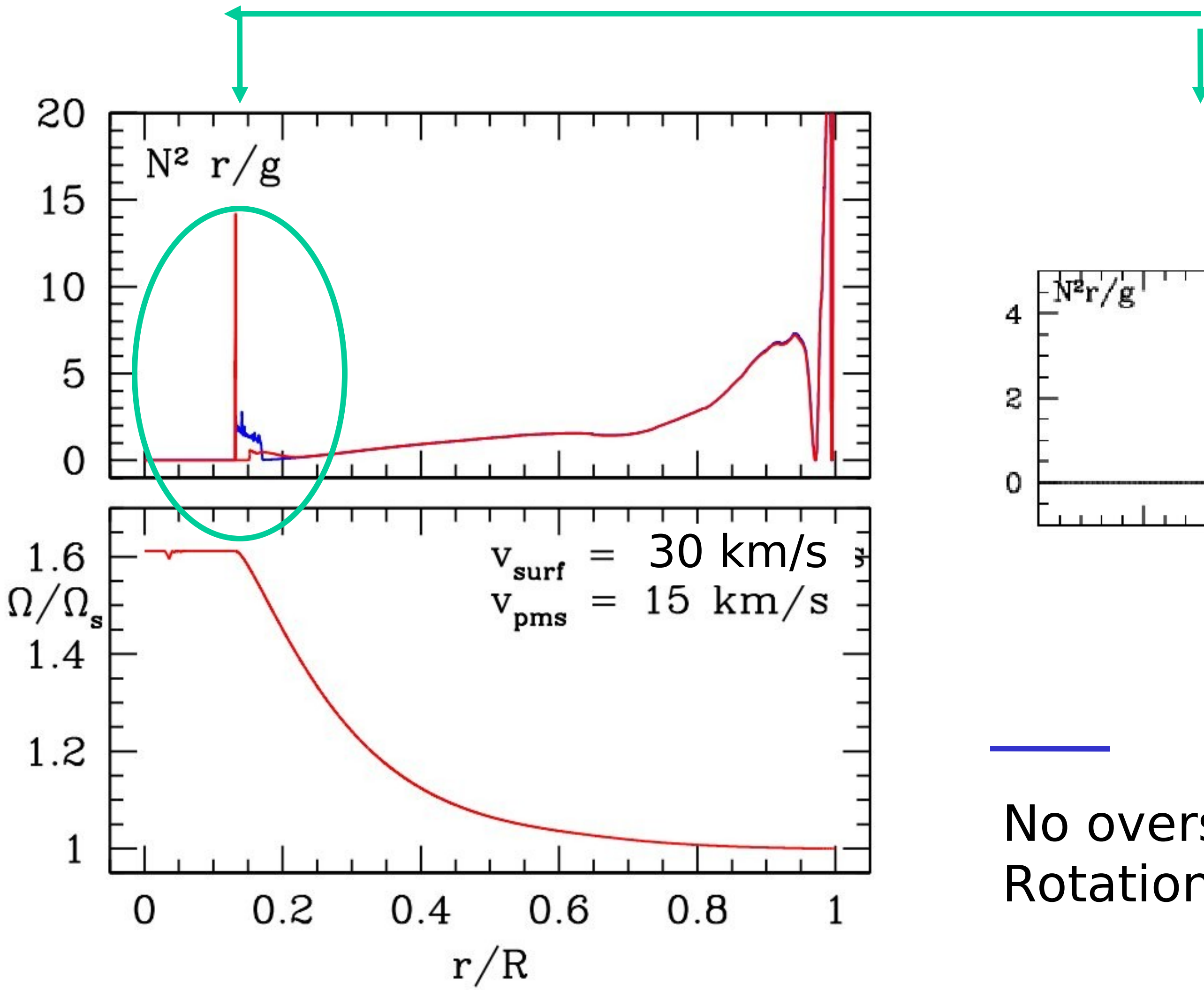
(in collab with S. Talon)

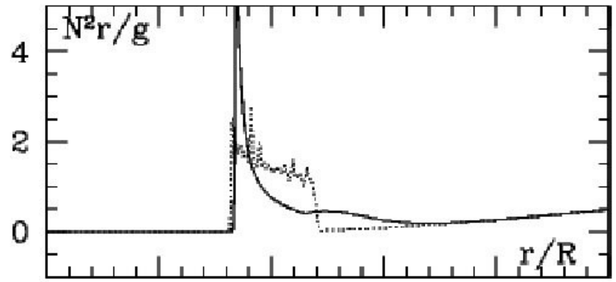
From pms, uniform to strong differential rotation
→
Relaxation toward
stationary profil



then stays on the
stationary profil
with only ajustement
due expansion
and contraction
with evolution

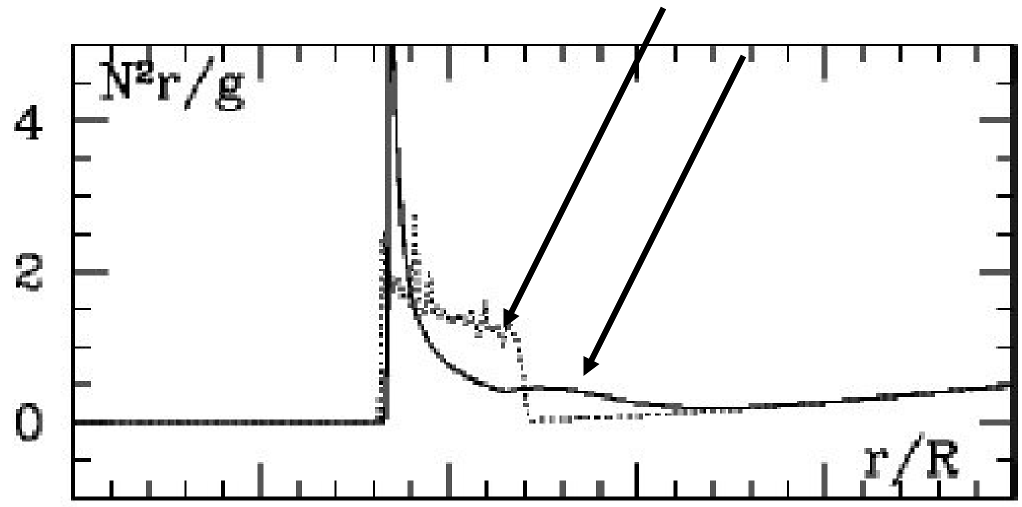
8.5 Msol V15 km initial on pms, vsurf =48.8 km/s for Xc=0.3



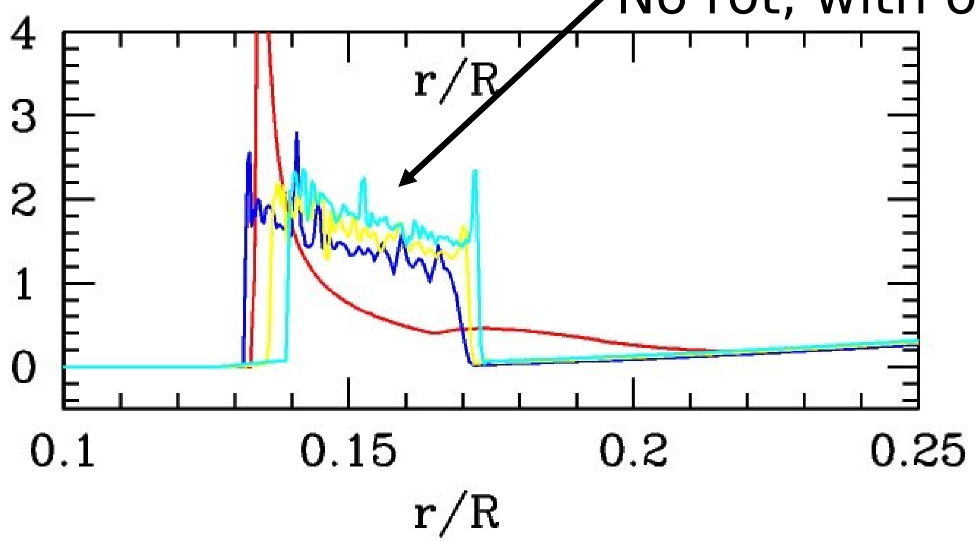


No overshoot, no
Rotational induced mixing

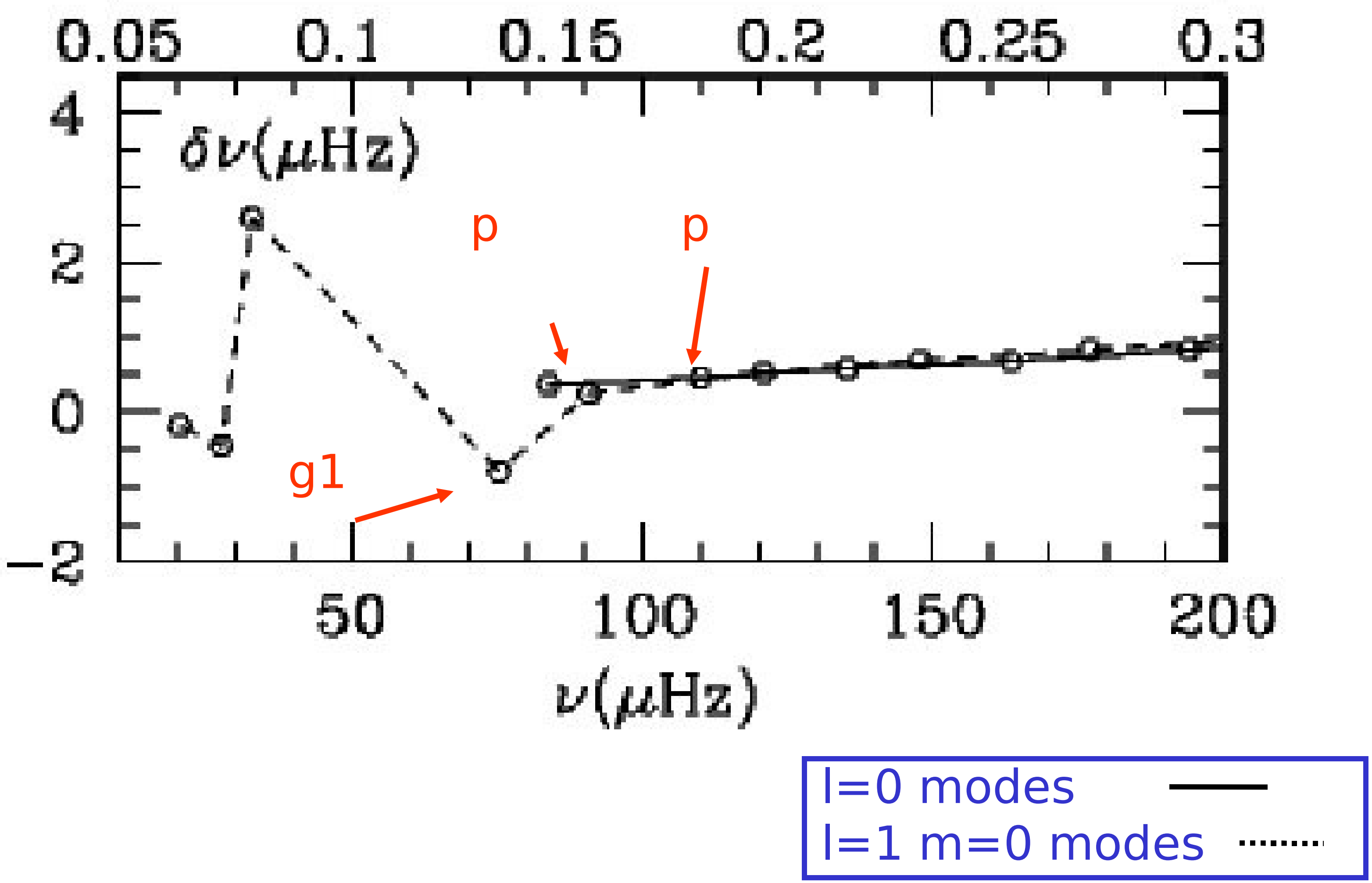
Rotational induced mixing
V surf =30 km/s



No rot, with overshoot 0.1 and 0



requency differences between V0 (v=0km/s) and V15 (30km/s) mode



gn and magnitude of δω dependent on the mdoe

How can we understand ?

$$\mathcal{L}_0\boldsymbol{\xi} - \rho_0\hat{\omega}^2\boldsymbol{\xi} = 0$$

$$\omega_0^2 = \frac{1}{I} < \vec{\xi}_0^*|\mathcal{L}_0 \vec{\xi}_0 > \quad ; \quad I =< \vec{\xi}_0^*|\vec{\xi}_0 >$$

W02 = 1/I int dr K0(r)

First order in perturbation, eigenfunction inchanged
W02(rot) – W02(no rot)= (1/I) int dr del K0

Linearized momentum equation

- $$\omega^2 \boldsymbol{\xi} = \nabla p' + \rho_0 \nabla \phi' + \rho' \nabla \phi_0$$

$$\omega^2 I = \int_V \boldsymbol{\xi}^* \cdot (\nabla p' + \rho_0 \nabla \phi' + \rho' \nabla \phi_0)\ \mathbf{d^3r}$$

After some manipulation

$$\omega^2\ (I + J) = \int_R \lambda \xi_r\ (\frac{N^2}{g/r})\ (\frac{c_s^2}{r^2})\ \rho_0 r^3 dr$$

with

$$J = \int_R (\frac{dln\rho_0}{dlnr}\frac{\xi_r}{r} + \lambda)\xi_h\ \rho_0 r^3 dr$$

and

$$I = \int_R (\xi_r^2 + \Lambda^2 \xi_h^2)\ \rho_0 r^2 dr$$

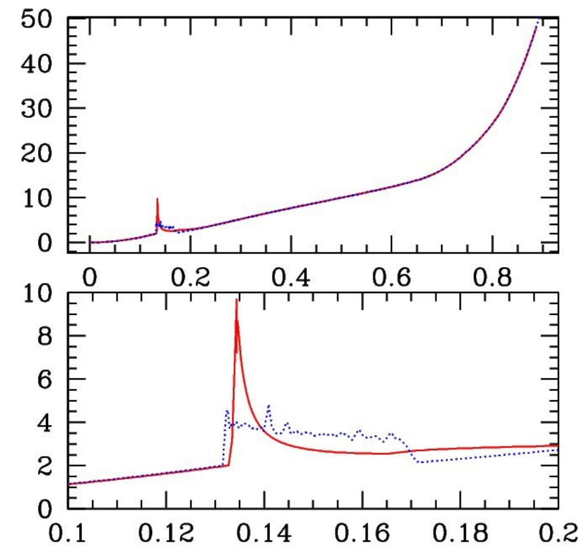
Definitions

$$N^2 = \frac{g}{r}(\frac{1}{\Gamma_1}\frac{dlnp_0}{dlnr} - \frac{dln\rho_0}{dlnr})$$

$$\lambda = \frac{1}{r^2}\frac{dr^2\xi_r}{dr} - \frac{\Lambda^2}{r}\xi_h$$

Differences between V0 and V15 come from the derivatives of the structure quantity and actually from

$$\frac{dln\rho}{dlnr}$$



then

$$\delta\frac{N^2 r}{g} \sim -\frac{dln\rho}{dlnr}$$

and

$$2\omega_0\delta\omega(I+J) + \omega_0^2\delta J = \int_R \lambda\xi_r \frac{\delta(N^2}{g/r)}\frac{c_s^2}{r^2}\ \rho_0 r^2 dr$$

$$\delta J = \int_R \delta\frac{dln\rho_0}{dlnr}\ \xi_r\xi_h\ \rho_0 r^2 dr$$

$$\delta\omega = \frac{1}{2\omega_0(I+J)}\int_R\ \delta(\frac{N^2}{g/r})\xi_r(\lambda\frac{c_s^2}{r^2} - \omega_0^2 r\xi_h)\ \rho_0 r^2 dr$$

Sign of δω