

A story of the Sun oblateness: from Princeton, 1966 to Pic du Midi, 2009.

**Jean-Pierre Rozelot¹, Sandrine Lefebvre²,
Cilia Damiani⁴, Nayyer Fazel^{1,3}**

¹OCA, GEMINI Department, 06130 Grasse, FRANCE

²CEA-Saclay, Sap/DAPNIA, 91191 Gif-sur-Yvette, FRANCE

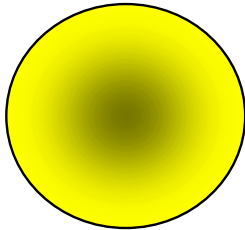
³The University of Tabriz, Faculty of physics, Department of Theoretical Physics
and Astrophysics, P.O. Box 55134-441, Maragha, IRAN

⁴LESIA, Observatoire – Paris Meudon, FRANCE.

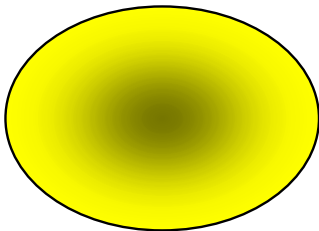
Once upon a time....



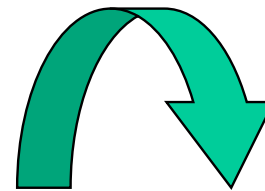
A long time ago: a disk



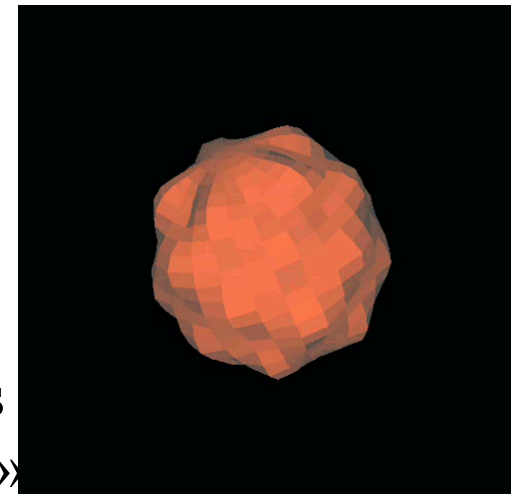
Antiquity: a sphere



XVIII th century:
a spheroid



To-day: the Heloid



Asphericities
« bashed-up »

Rotation of the Sun and Stars

Will the observations allow a precise measurement of ϵ ?

→ Stars



VLT – ESO
CHARA-USA

→ Sun



Turret dome (50-cm) Pic du Midi

Some balloon flights (SDS)

Paradoxe !

The flatness of stars are better determined today
than those of the Sun...

*See numerous examples in S. Lefebvre et al., 2005.
Memorie della Società Astronomica Italiana", Vol.76, p. 961.*

Observations. Paradoxe!

- Altair: 2001 ($1.7 M_{\odot}$)

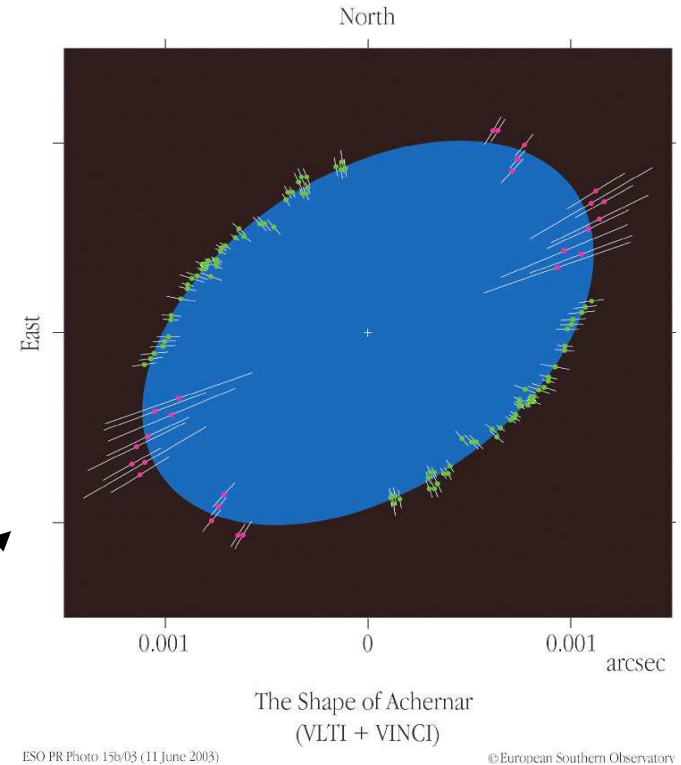
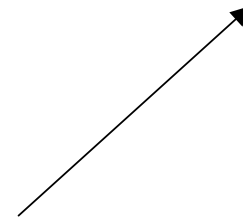
$$\Delta r = 424 \pm 79 \text{ microas}$$

About theoretical prediction

- Achernar: 2003 ($6 M_{\odot}$)

$$\Delta r = 910 \pm 50 \text{ microas}$$

Far from the theory (more oblate)



Observations. Paradoxe!

- Altair: 2001 ($1.7 M_{\odot}$)

$$\Delta r = 424 \pm 79 \text{ microas}$$

About theoretical prediction

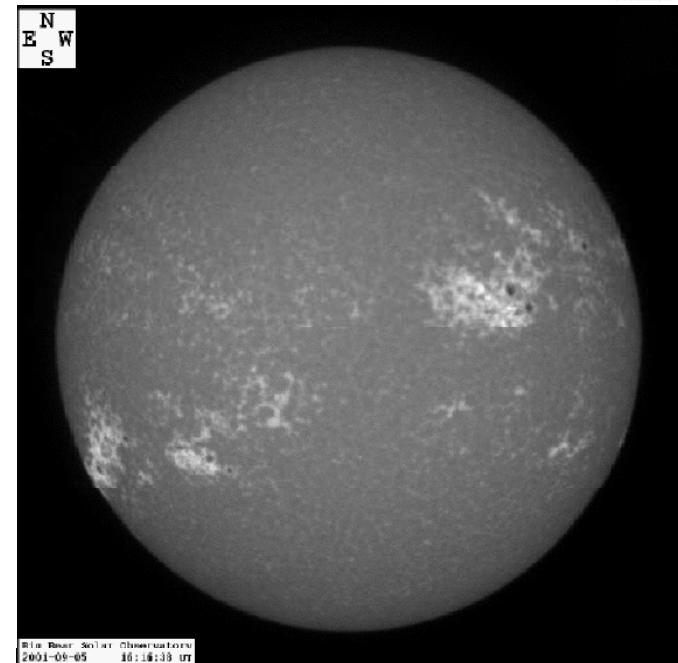
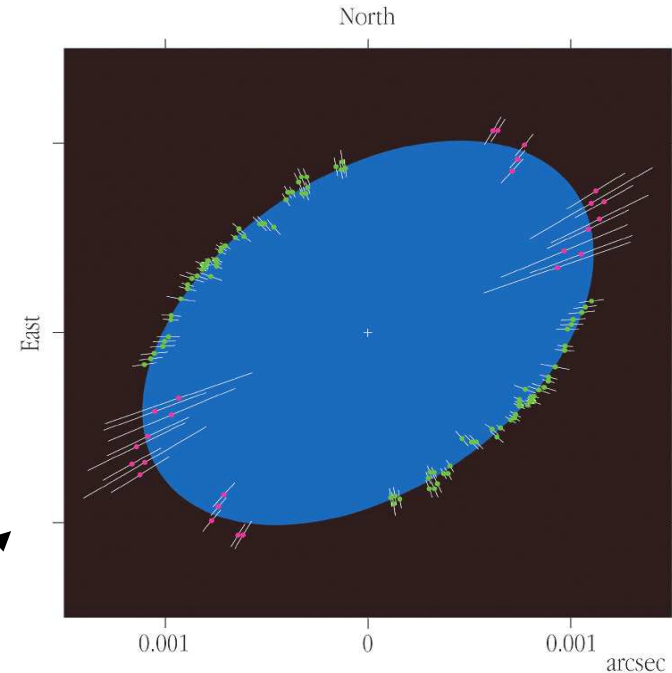
- Achenar: 2003 ($6 M_{\odot}$)

$$\Delta r = 910 \pm 50 \text{ microas}$$

Far from the theory (more oblate)

- **Sun:** $\epsilon \cong 8.86 \cdot 10^{-6}$

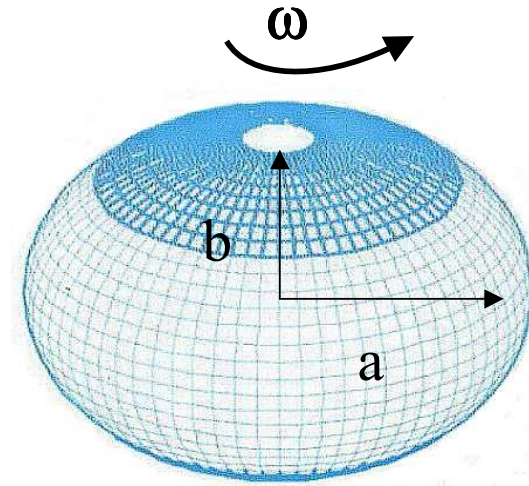
less than $1.1 \cdot 10^{-5}$



Asphericities

To first order $\rightarrow$ oblateness

$$\varepsilon = (a-b)/a$$
$$\Delta r = (a-b)$$



Question: how to determine ε as a function of ω

Answer given by Clairaut (1733),

for a uniform velocity rate ω

and a uniform density distribution from the core to the surface ρ

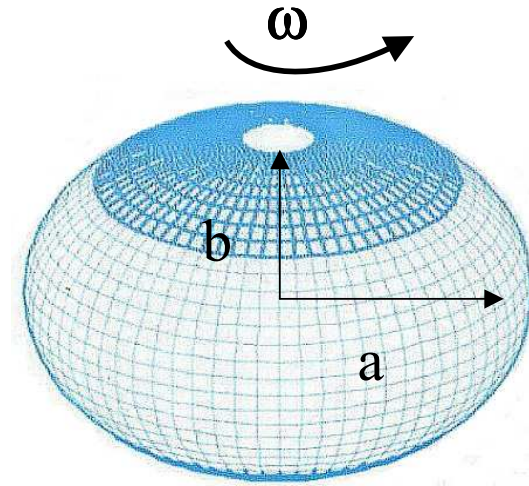
Asphericities

$$\varepsilon = (a-b)/a$$

$$\Delta r = (a-b)$$

Si le corps est en **rotation uniforme**
 Et la densité de la forme $\rho = r^{-n}$

Milne (1923),
 Chandrashekar (1933)



$$\varepsilon = (0.5 + 0.856 \rho_m/\rho_c) \omega^2 R/g$$

$$\left\{ \begin{array}{l} \rho_m = \text{densité moyenne,} \\ \rho_c = \text{densité du cœur,} \\ R = \text{rayon de la sphère initiale,} \\ g = \text{gravité à la surface.} \end{array} \right.$$

Asphericities

- Deviations to sphericity

The asphericities occur when a fluid body,
inhomogeneous in density is in **non-uniform rotation**.

It is the case of the Sun → surface deformations

Questions:

- can we quantify these asphericities?
- what do they teach us?

Solar limb astrometry

What questions?

- Does the solar radius change with solar cycle?
- Knowledge of radius changes and irradiance or luminosity changes constrain the solar cycle mechanisms... a long debated problem
- What is the Sun's shape and is this consistent with solar system limits on its gravitational potential and the internal rotation rate?
- Low frequency limb oscillations (p-modes, g-modes, f-modes) sample the deepest parts of the solar interior

Asphericities

- Let us start with the Clairaut equation (for an ideal fluid in motion):

$$\rho \ddot{\mathbf{x}} = \rho \mathbf{g} - \nabla p$$

For hydrostatic equilibrium $\ddot{\mathbf{x}} = 0$ and bearing in mind $\mathbf{g} = -\nabla \phi$,

So that

$$0 = -\rho \nabla \phi - \nabla p$$

To get the full gravity potential, one must add the rotational one:

$$\nabla p = -\rho \nabla \phi + \omega^2(r, \theta) \vec{v}$$

→ magnetic field

Asphericities

Consider asphericities as small perturbed quantities

$$\phi(r, \theta) = \phi_0(r) + \phi_1(r, \theta)$$

with

$$\phi_1(r, \theta) = \sum_{n=0}^{\infty} \phi_{1,2n}(r) P_{2n}(\cos \theta)$$

Adding the Poisson's equation: $\nabla^2 \phi = 4\pi \rho G$

Together with boundaries conditions at $r = 0$ and at the surface:

Eq. (Clairaut) gives

$$J_{2n} = \frac{R_{\odot}}{GM_{\odot}} \phi_{1,2n}(R_{\odot})$$

Asphericities

- where J_{2n} are dimensionless coefficients satisfying

$$\phi_{\text{out}}(r, \theta) = -\frac{GM_{\odot}}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{R_{\odot}}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right]$$

For $n = 1$ J_2 is the quadrupole moment

For $n = 2$ J_4 is the hexadecapole moment

For $n = 3$ J_6 , and so on

And so on...

→ J_n have a physical meaning.

They told us how much the matter deviates from a pure sphere

(Note that the axial symmetry in the solar case imposes n even.)

Solar Asphericities

$$r = R_{sp} \left[1 + \sum_{n=2(\text{even})}^{\infty} c_n P_n(\cos(\theta)) \right] = R_{sp} [1 + c_2 P_2 + c_4 P_4 + \dots]$$

$$\phi_{out}(r, \theta) = -\frac{GM_{\odot}}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{R_{\odot}}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right]$$

- Link between J_{2n} and c_{2n}



FIG. 29.—Telescope viewed from the front. The door (facing south) is normally closed when the instrument is operated. The rectangular observing port is $25^{\circ} \times 50^{\circ}$. Parts shown: (1) Center photocell housing, (2) Viewing eyepiece, (3) Secondary mirror.

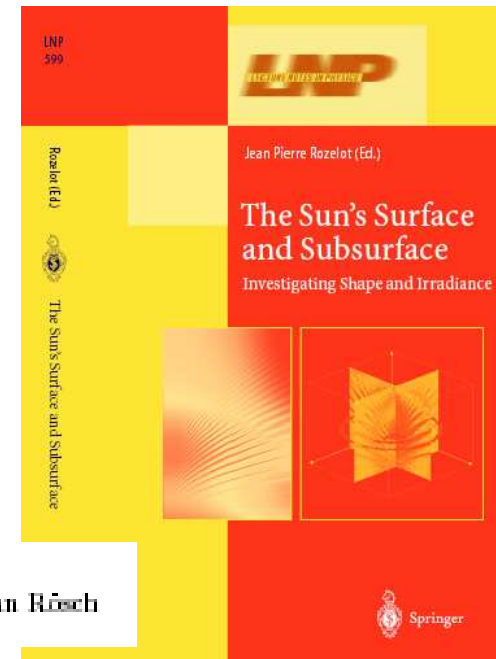
THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES No. 241, 27: 131–182
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THE OBLATENESS OF THE SUN*

R. H. DICKE AND H. MARK GOLDENBERG†

Joseph Henry Laboratories, Physics Department, Princeton University, Princeton, N.J. 08540

Pionners



In Honor of Jean Rösch



Solar Shape Measures Gravitational Potential and Solar Rotation

$$\omega = \omega_0 + \omega_2 \mu^2.$$

lowers from the equation of hydrostatic support:

$$-\vec{\nabla} p = \rho \vec{\nabla} \phi + \rho \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

represent the pressure, density, gravitational potential in non-spherical forms yields:

$$-\vec{\nabla} p' = \rho' \vec{\nabla} \phi_0 + \rho_0 \vec{\nabla} \phi' + \rho_0 \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$(R_{\text{equator}} - R_{\text{pole}}) / R_{\text{pole}} = \frac{-3}{2} J_2 + \frac{\Omega^2 R^3}{2GM}$$

Magnetic fields and non-rotational velocity flows complicate this result...

Some theoretical estimates of J_n

	J_2	J_4	J_6	J_8
$B_{cr} = 0$	-2.300×10^{-7}	6.2924×10^{-7}	-1.416×10^{-8}	5.049×10^{-13}
$B_{cr} = 0.1$ Tesla (Durney, 1998)	-2.613×10^{-7}	6.2922×10^{-7}	-1.4164×10^{-8}	5.049×10^{-13}
$B_{cr} = 0.3$ Tesla	-3.24×10^{-7}	6.2911×10^{-7}	-1.4164×10^{-8}	5.049×10^{-13}

Fazel, N., Rozelot, J.P. & Ajabdshirizadeh, A. New Astronomy, 2008

Solar oblateness

- First measurements at Princeton (1965)

R. Dicke & M. Goldenberg

→ Results were $\Delta R = 44$ mas, about 10^{-5} , 10 times greater than theory can predict

→ A lot of papers in Nature, Science, etc

→ Main concern: 44 mas were an estimate satisfying a new cosmological theory (Brans-Dicke).

- Then measurements moved to Sacramento Peak (Kuhn)

● Taken again in France in 1996 at Pic du Midi, first by Röscher, then by Rozelot up to now. (full coverage of one solar cycle)

- Space measurements

Science motivation

- Solar shape and to 1st order, oblateness, still very difficult to measure
- But exciting!
- Revival of interest due to dedicated space missions:

- Balloon flights (SDS)

(Courtesy: S. Sabatino)



Solar Disk Sextant (SDS) experiment

Balloons Flights (Yale University). Measurements at 40 km

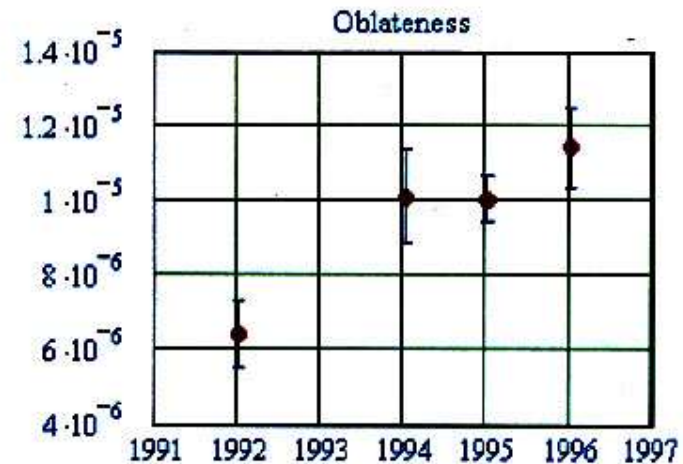
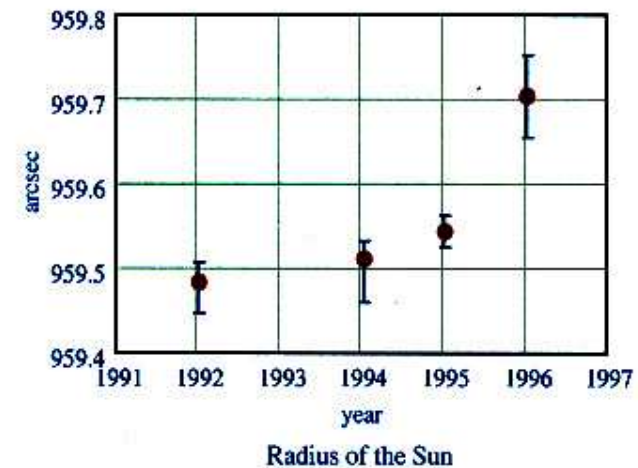
S. Sofia and B. Cacín:

Rotation law:

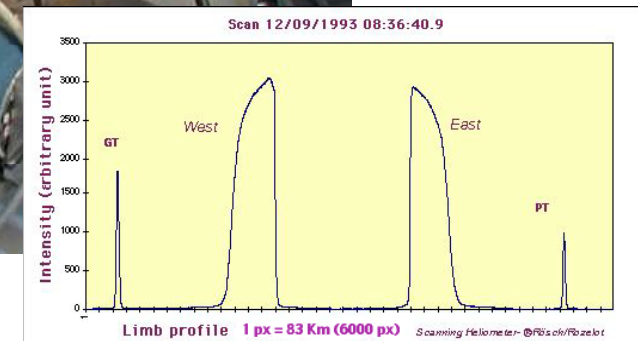
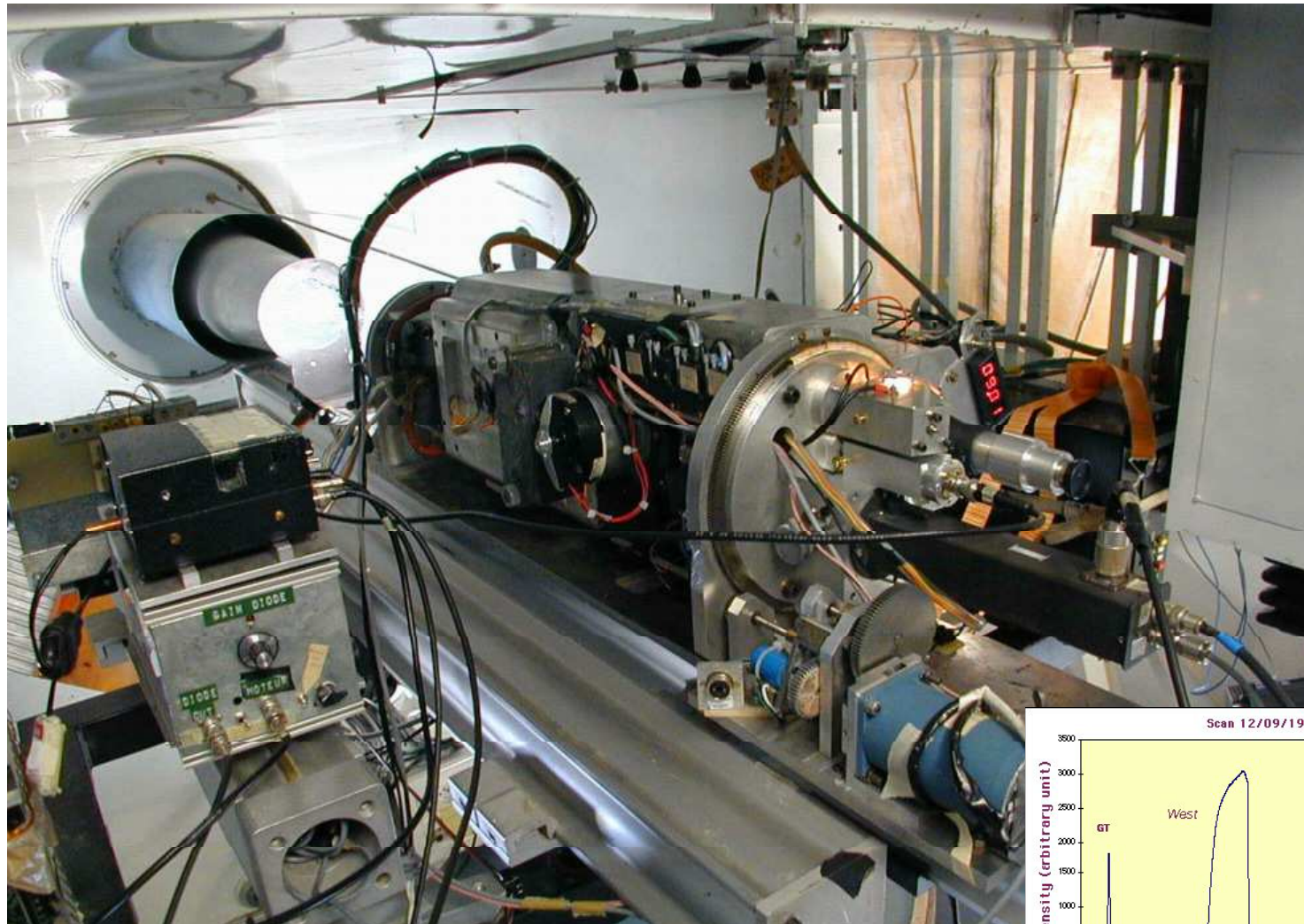
$$\omega_0 = 2.840 - 0.400 \sin^2(\theta) \text{ } \mu \text{ rad s}^{-1}$$

$$J_2 = 1.84 \cdot 10^{-7}$$

$$J_4 = 9.83 \cdot 10^{-7}$$



Scanning Heliometer (Pic du Midi)



Solar limb shape distortions

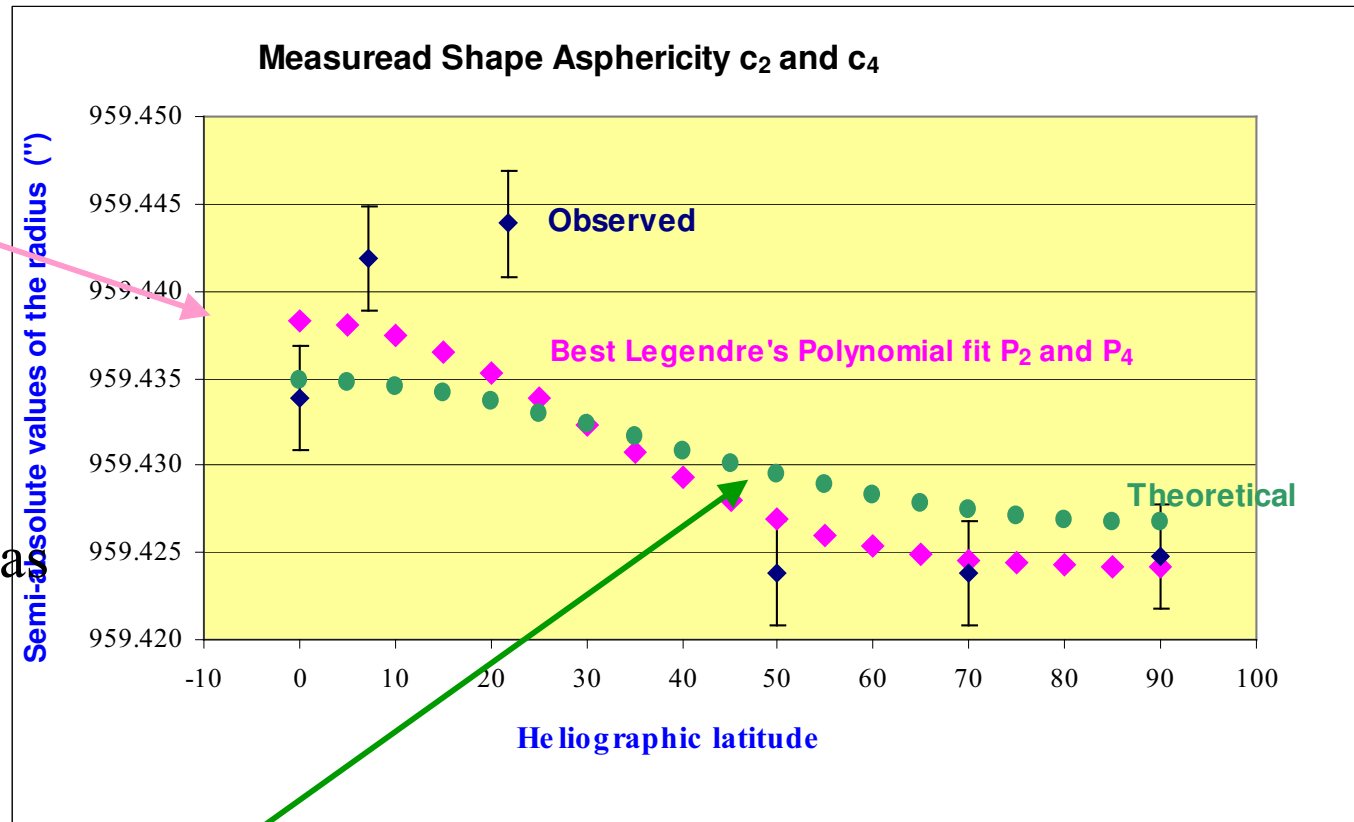
$$R(\theta)|_{\rho=\text{cste}} = R_0 [1 + \sum_n c_n * P_n(\theta)]$$

Best fit data

$$c_2 = -1.0 \pm 0.5 \cdot 10^{-5}$$

$$c_4 = 3.4 \cdot 10^{-6}$$

$$(\Delta r)_{\text{max}} = 21 \text{ m}$$



Armstrong & Kuhn, 1999: $c_2 = -5.27 \cdot 10^{-6}$

(Theory)

The 150-foot tower

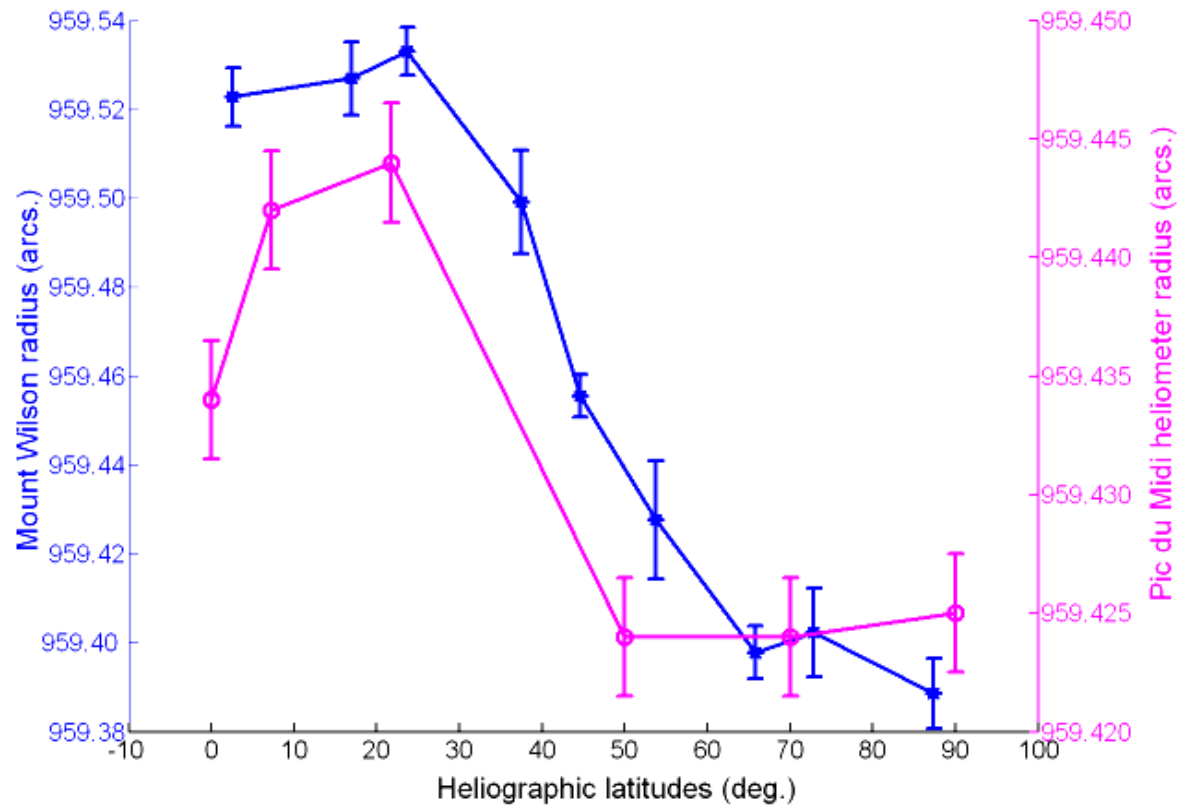
- Mount Wilson Observatory
 - 150-foot tower
 - The Sun is directed into a spectrograph by a servo-controlled coelostat and double objective lens at the top of the tower



History of Mt. Wilson solar radius measurements

- LaBonte & Howard 1981 (*Science* 214, 907)
 - Observations: 1974 – 1981
 - Spectral line: Fe I $\lambda = 525.0$ nm
 - No significant variations with the solar cycle
- Ulrich & Bertello 1995 (*Nature* 377, 214)
 - Observations: 1982 – 1994
 - Spectral line: Fe I $\lambda = 525.0$ nm
 - Solar radius varies in phase with the solar cycle
 - Amplitude of variations: ~ 0.4 arcsec.
- Ulrich, Bertello, Lefebvre, Rozelot, **2007** (*ApJ* 649, 444)
 - Period investigated: 1974 – 2003
 - Spectral line: Fe I $\lambda = 525.0$ nm
 - Latitudinal distortions

Observational Results



Does the oblateness vary in time?

- Certainly, yes, but needs new Space Missions

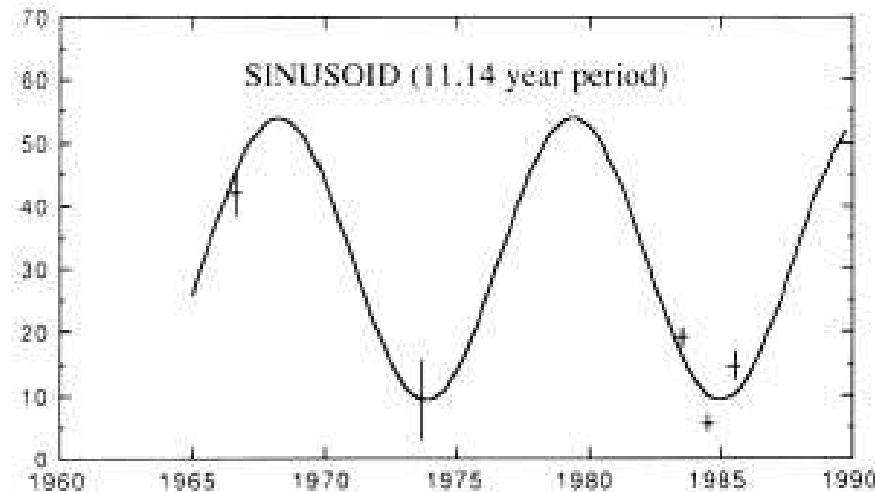


FIG. 1. Weighted least-squares fit of sinusoidal curve of solar cycle period to the four plotted solar oblateness points of 1966, 1983, 1984, and 1985. The Hill-Stebbins point of 1973 is also plotted but not fitted. The ordinate is the solar oblateness in arc milliseconds.

IS THE SOLAR OBLATENESS VARIABLE? MEASUREMENTS OF 1985

R. H. DICKE and J. R. KUHN

Joseph Henry Laboratories, Physics Department, Princeton University

AND

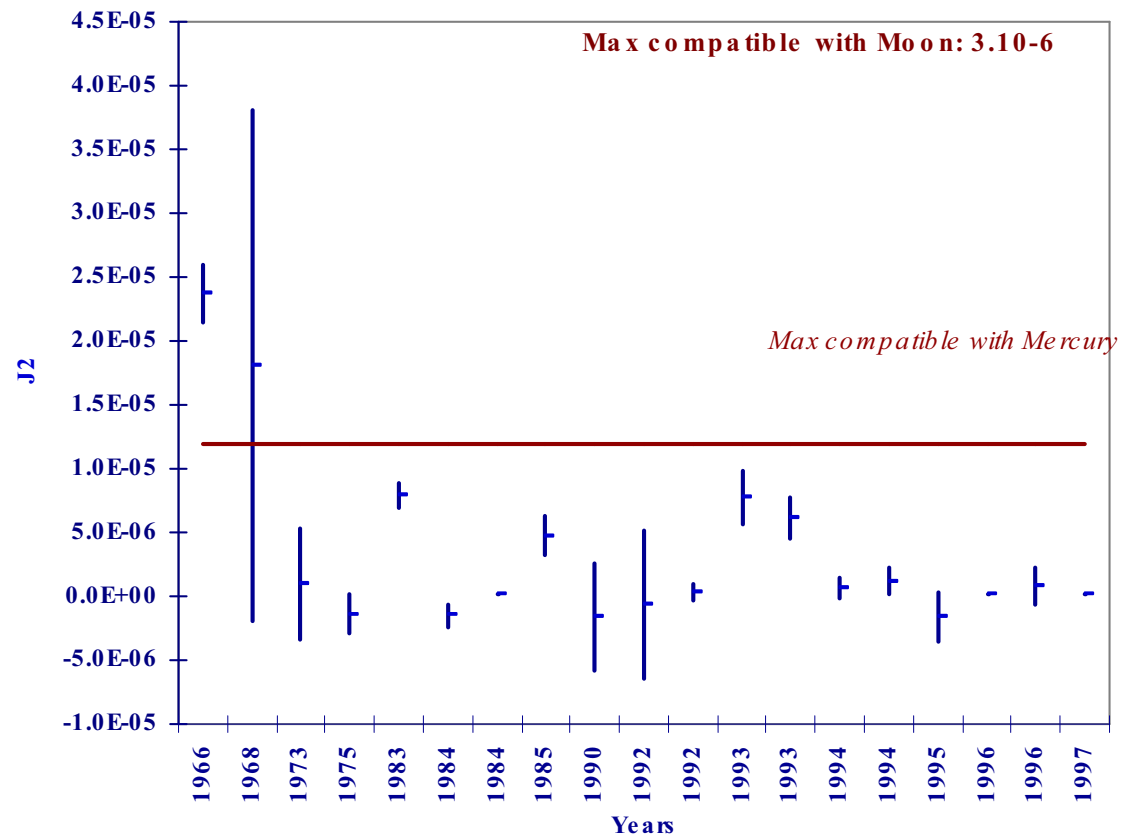
K. G. LEBRECHT

Solar Astronomy, California Institute of Technology

Received 1986 November 10; accepted 1986 December 17

THE ASTROPHYSICAL JOURNAL, 318:451-458, 1987 July 1

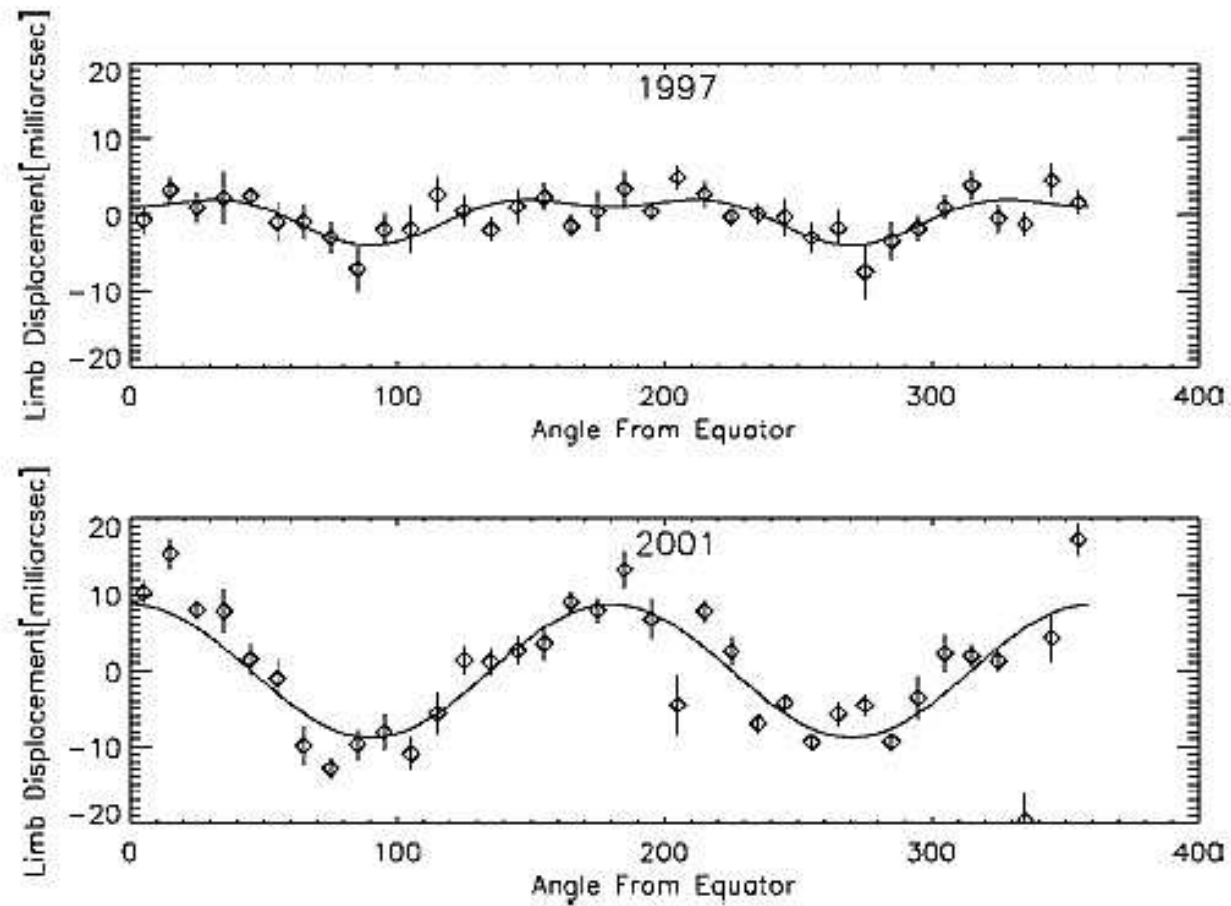
OBSERVED VALUES OF J2



Roze lot, J.P. and Rö ch, J.: Proceedings of the 8th European Solar Meeting, 1996, Thessaloniki, Greece, 1996, 172, 11 and up-graded (1998)

Oblateness

Space measurements: MDI on SOHO (Kuhn et al.)



Oblateness

Space measurements:

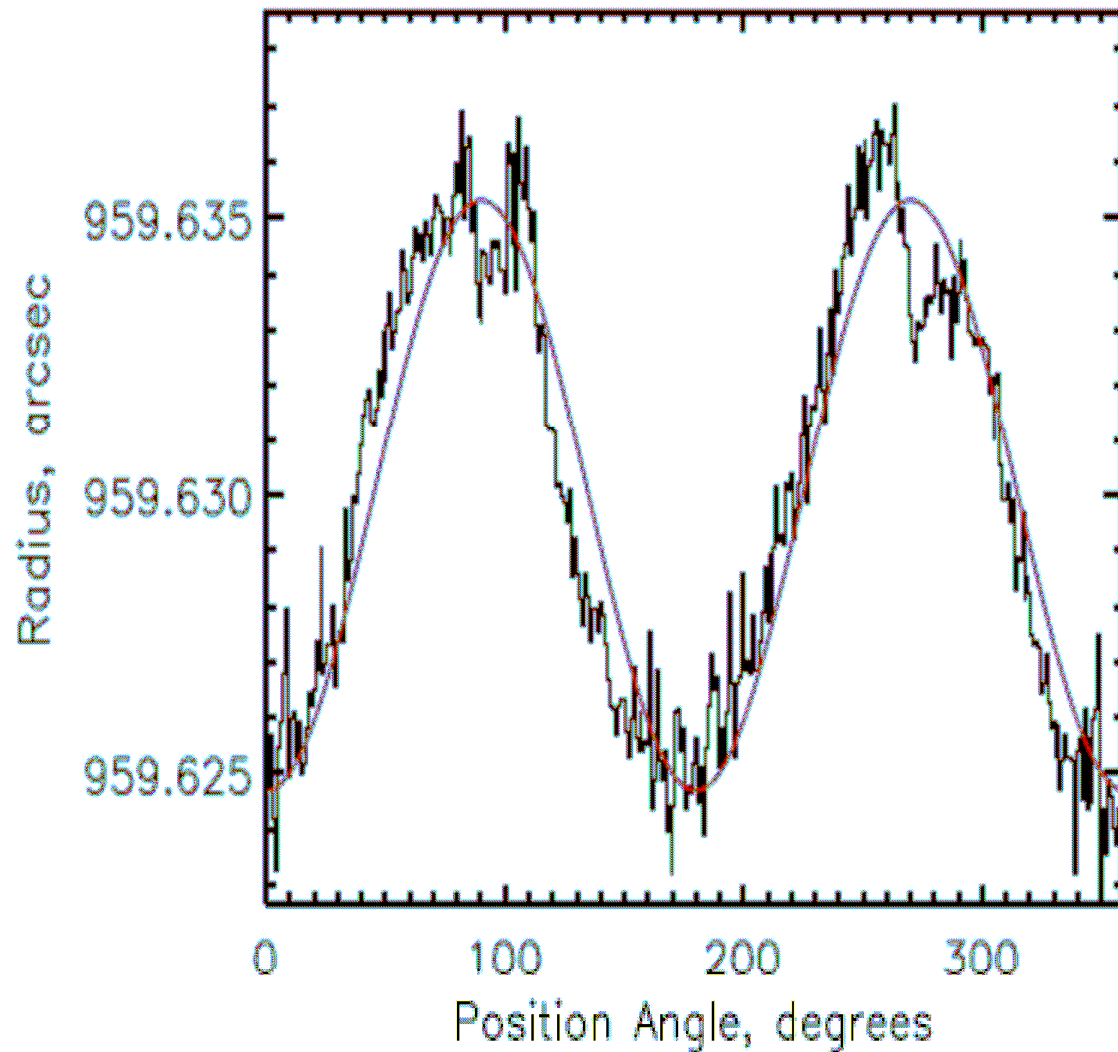
RHESSI

(Hudson et al.),

Science, October, 2008

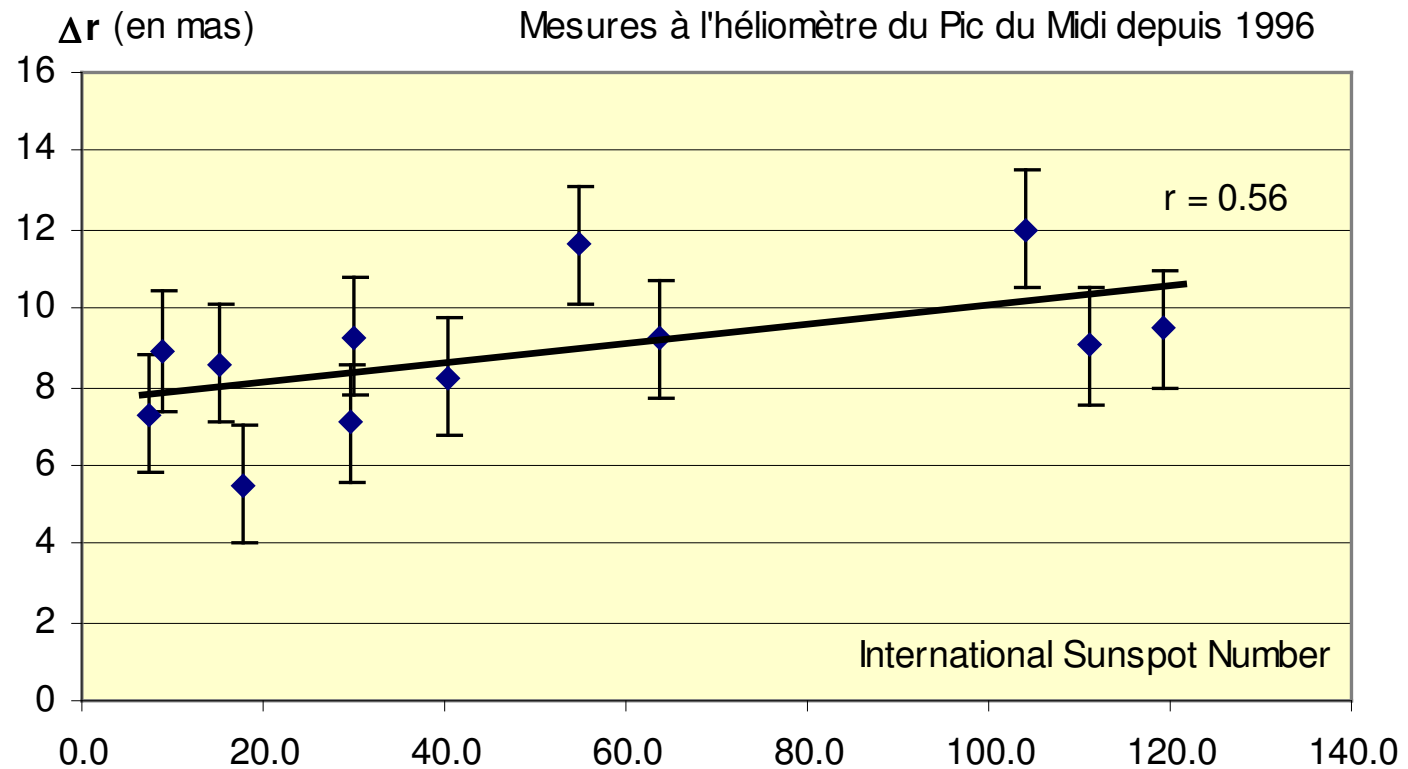
the red line shows
 10.74 ± 0.44 mas.

$$J_2 = (1.25 \pm 1.0) \times 10^{-7}$$



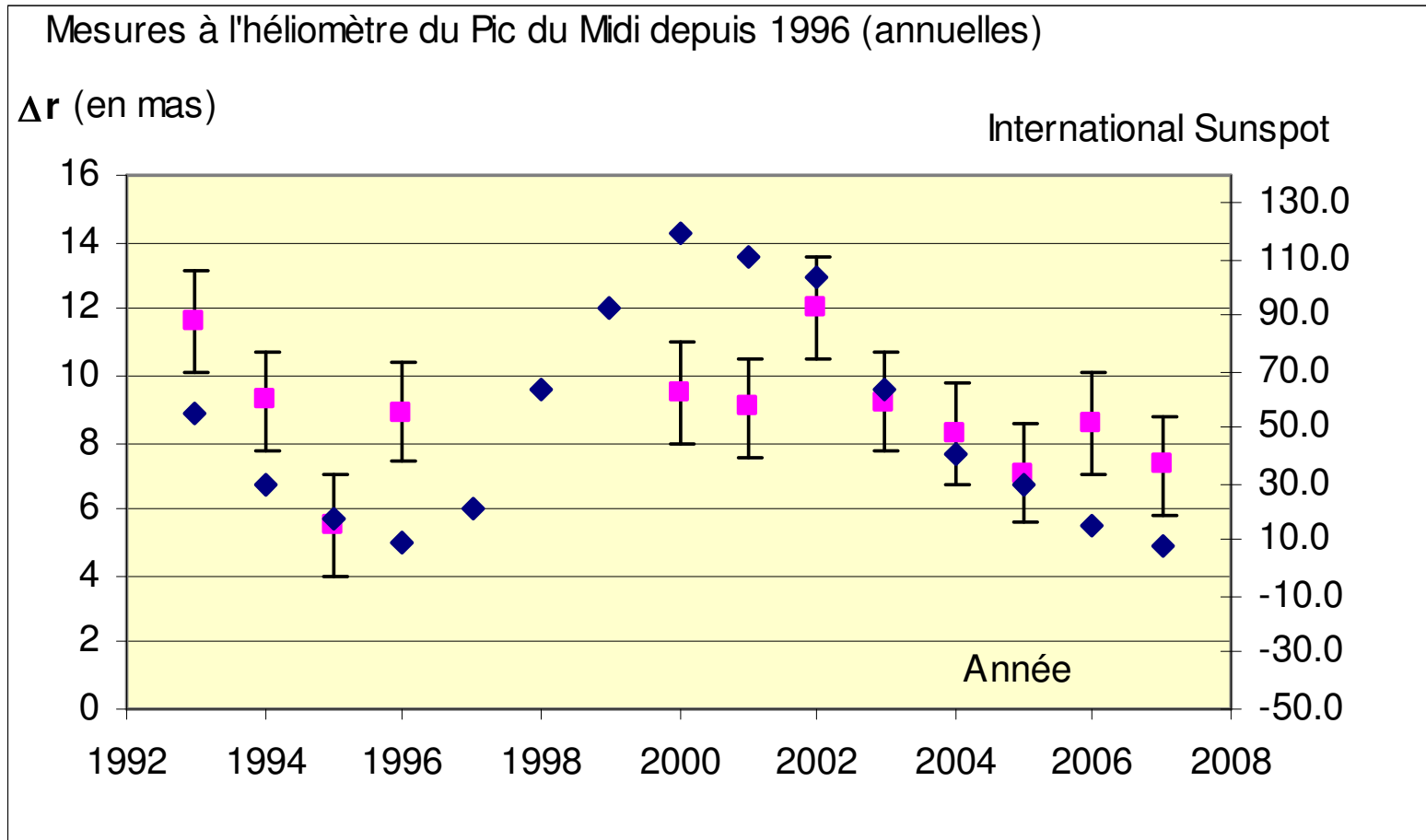
Oblateness

Aplatissement et activité solaire



Mesures de l'aplatissement solaire au moyen de l'héliomètre situé dans la coupole
Jean Rösch de l'observatoire du Pic du Midi en fonction de l'activité solaire. L'aplatissement est en phase avec le cycle.

Oblateness



Mesures de l'aplatissement solaire au moyen de l'héliomètre situé dans la coupole Jean Rösch de l'observatoire du Pic du Midi (échelle de gauche) et indice d'activité solaire (échelle de droite), en fonction du temps. Il est difficile de nier une variation phasée avec le cycle. Si l'on veut être très puriste, on pourra dire que l'aplatissement est 8.5 ± 3.5 mas. Auquel cas, on aura montré que les mesures sol à l'héliomètre sont parfaitement conformes avec la théorie (qui donne 8.2 mas dans le cas solaire), justifiant la validité du processus observationnel et d'analyse (décrit dans Rozelot et al., 2008).

Oblateness

Helioseismic changes in « limb shape » exist

(see also, Lefebvre, Kosovichev, Rozelot, 2006, 2007: ApJ

- **These measurements sample different height in the solar atmosphere and do not show the same phase**

- **It is likely that the outer atmosphere is changing differentially with the solar cycle**

- This can be caused by changes in the turbulent pressure (which is >10% of the total pressure in the outer layers of the Sun)
- This is not directly a magnetic effect (magnetic corrections if underestimated would change the shape further in the “wrong” direction)
- The measurements support the notion that convection is changing during the solar cycle

Asphericities: not only a book-case

Cross roads of:

1. Celestial mechanics
2. Physics of the sub-surface
3. Alternative tests of the GR

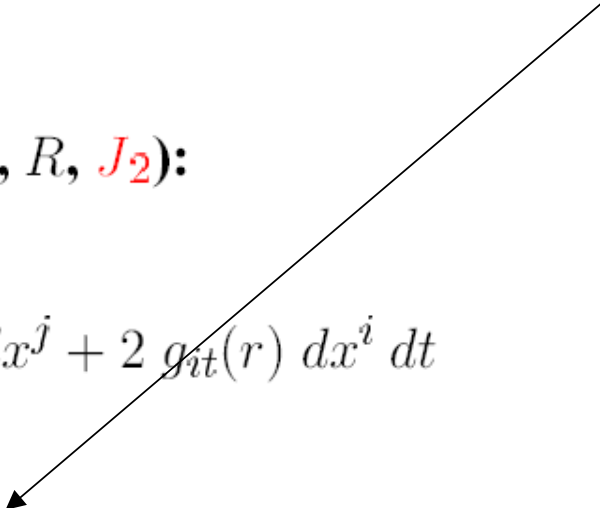
Consequences (on GR)

... in isotropic coordinates, (r, θ) ,
 a static metric of gravitational source (M, R, J_2) :

$$ds^2 = g_{tt}(r) c^2 dt^2 + g_{ij}(r) dx^i dx^j + 2 g_{it}(r) dx^i dt$$

with

$$g_{tt}(r) \simeq + \left[1 + 2 \alpha \left(-\frac{G_N M}{r c^2} \right)^1 \left\{ 1 - 2 J_2 \frac{R^2}{2 r^2} (3 \cos^2 \theta - 1) \right\} \right. \\ \left. + 2 \beta \left(-\frac{G_N M}{r c^2} \right)^2 + \frac{3}{2} \epsilon \left(-\frac{G_N M}{r c^2} \right)^3 + \dots \right]$$

$$g_{ij}(r) \simeq -\delta_{ij} \left[1 - 2 \gamma \left(-\frac{G_N M}{r c^2} \right)^1 \left\{ 1 - 2 J_2 \frac{R^2}{2 r^2} (3 \cos^2 \theta - 1) \right\} \right. \\ \left. + \frac{3}{2} \delta \left(-\frac{G_N M}{r c^2} \right)^2 + \dots \right]$$


Consequences

In pure GR:

$$\Delta v = 42.981 [1 - \mathcal{F} J_2]$$

In PPN Relativity:

$$\Delta v = \Delta v_{0,GR} \left[\frac{1}{3}(2\alpha^2 + 2\alpha\gamma - \beta) - F(\alpha) J_2 \right]$$

Observations constraint β and γ as

$$\gamma = 0.9996 \pm 0.0017 \quad \beta \in [0.9993; 1.0006]$$

Asphéricités: une problématique relativiste

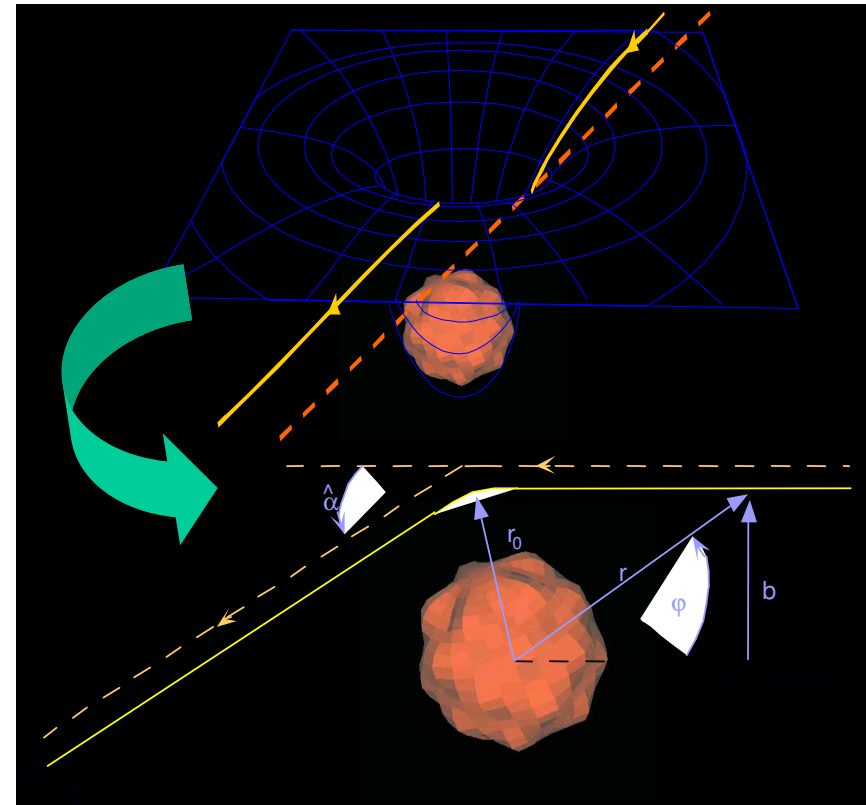
2. Astrométrie relativiste

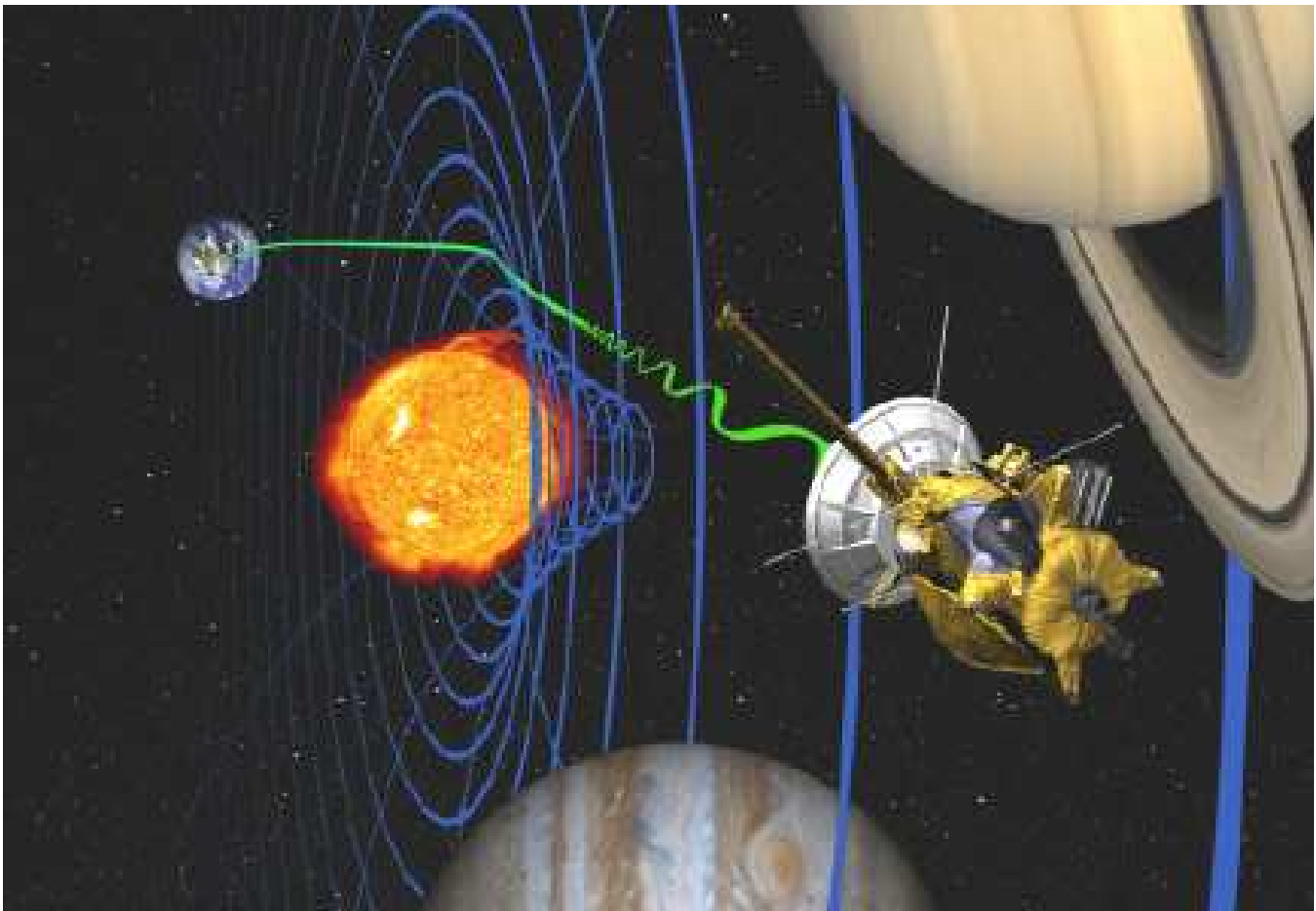
Due to **relativistic light deflection**, precise astrometry in the solar neighborhood will require a more precise knowledge of J_2 and $J_\odot$:

$$\hat{\alpha}_{\text{grazing, source at infinity}} = + (2(1+\gamma) + 2(1+\gamma)J_2) \frac{GM}{R c^2} \pm 2(1+\gamma) \frac{G|\vec{J}|}{R^2 c^3} + \Theta\left(\frac{1}{c^4}\right)$$

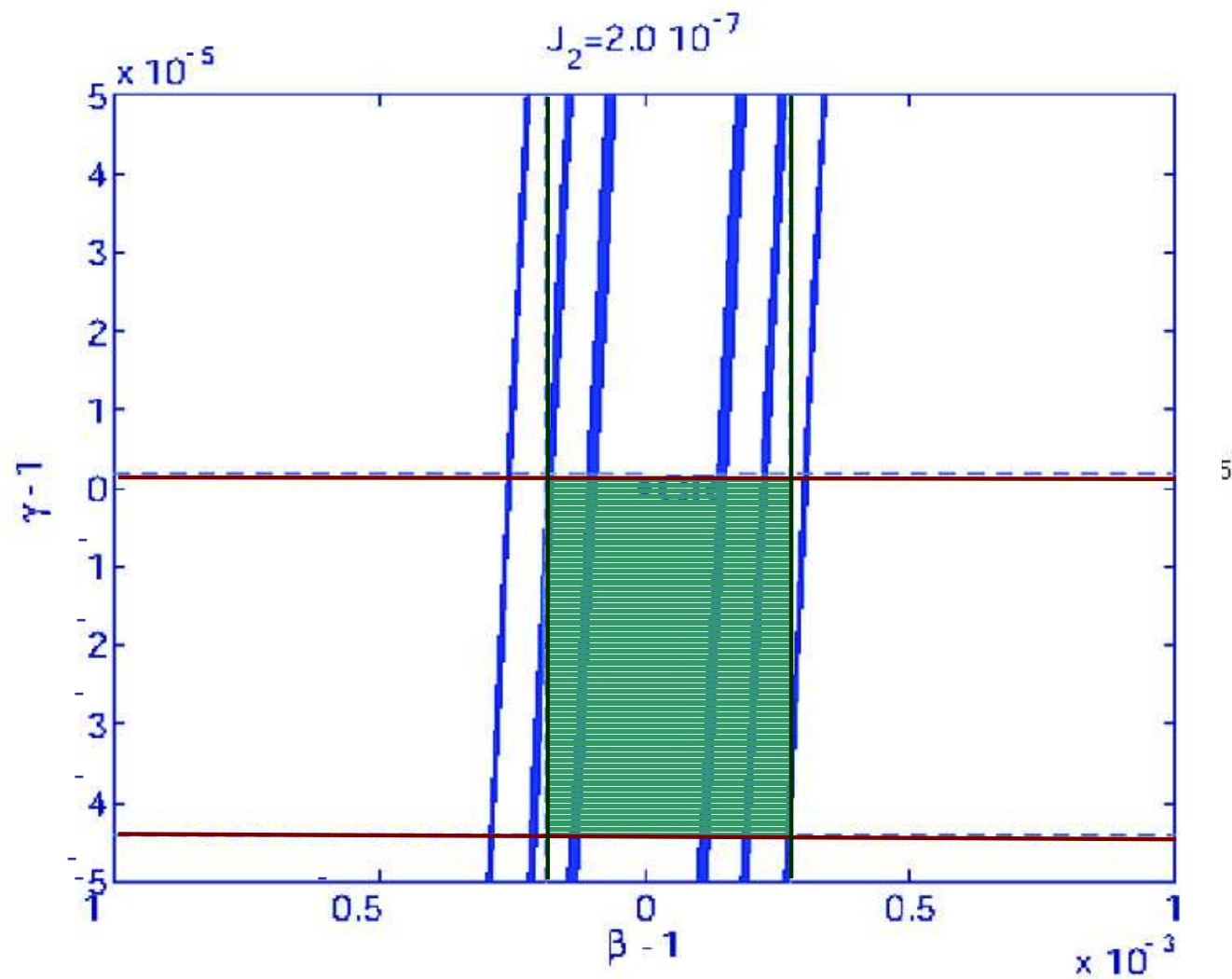
Post-Newtonian parameter encoding the amount of curvature of space-time per unit rest mass

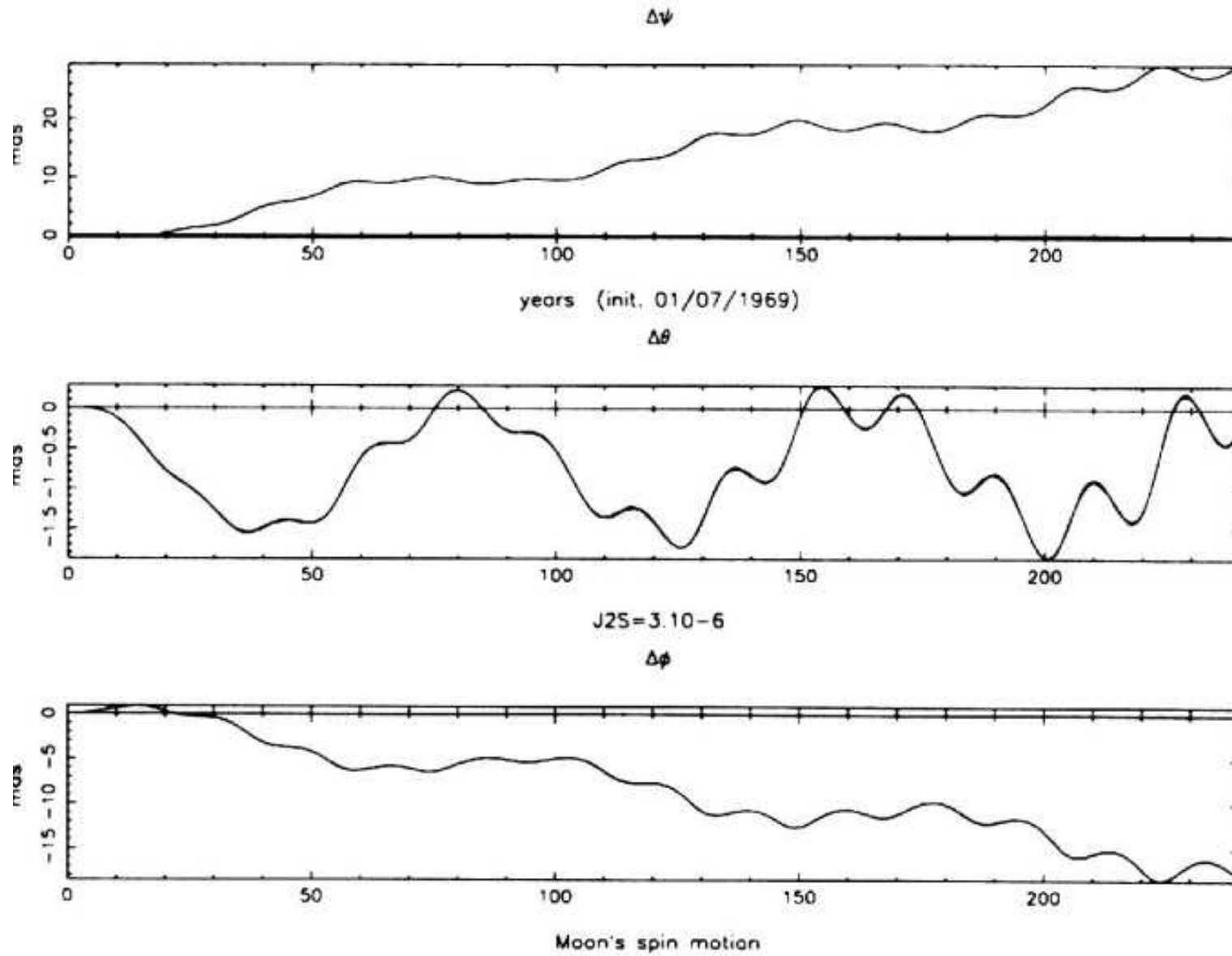
Space-time being curved by the solar body and inducing light deflection measured by an angle α .





J2 and RG

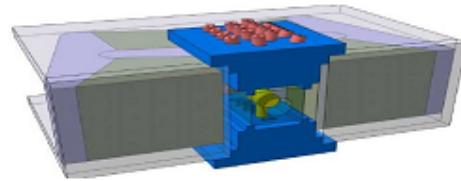




Rozelot & Bois, 1999, J. Ast. Soc Pac.

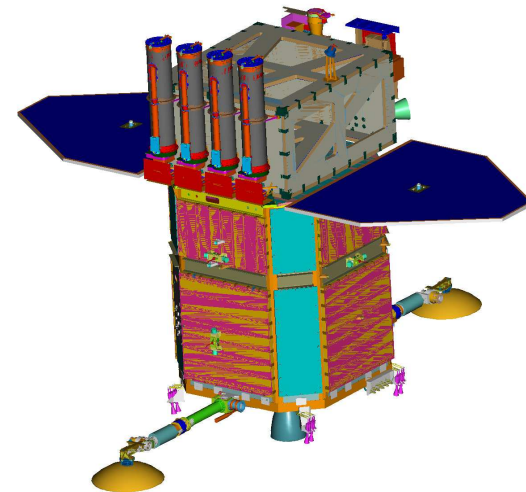
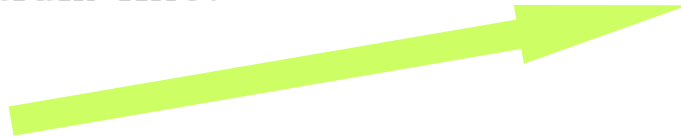
Observations spatiales futures

GOLF-NG



GOLF-NG
instrument.
Resonance
spectrometer in the
D1 sodium line.

SDO

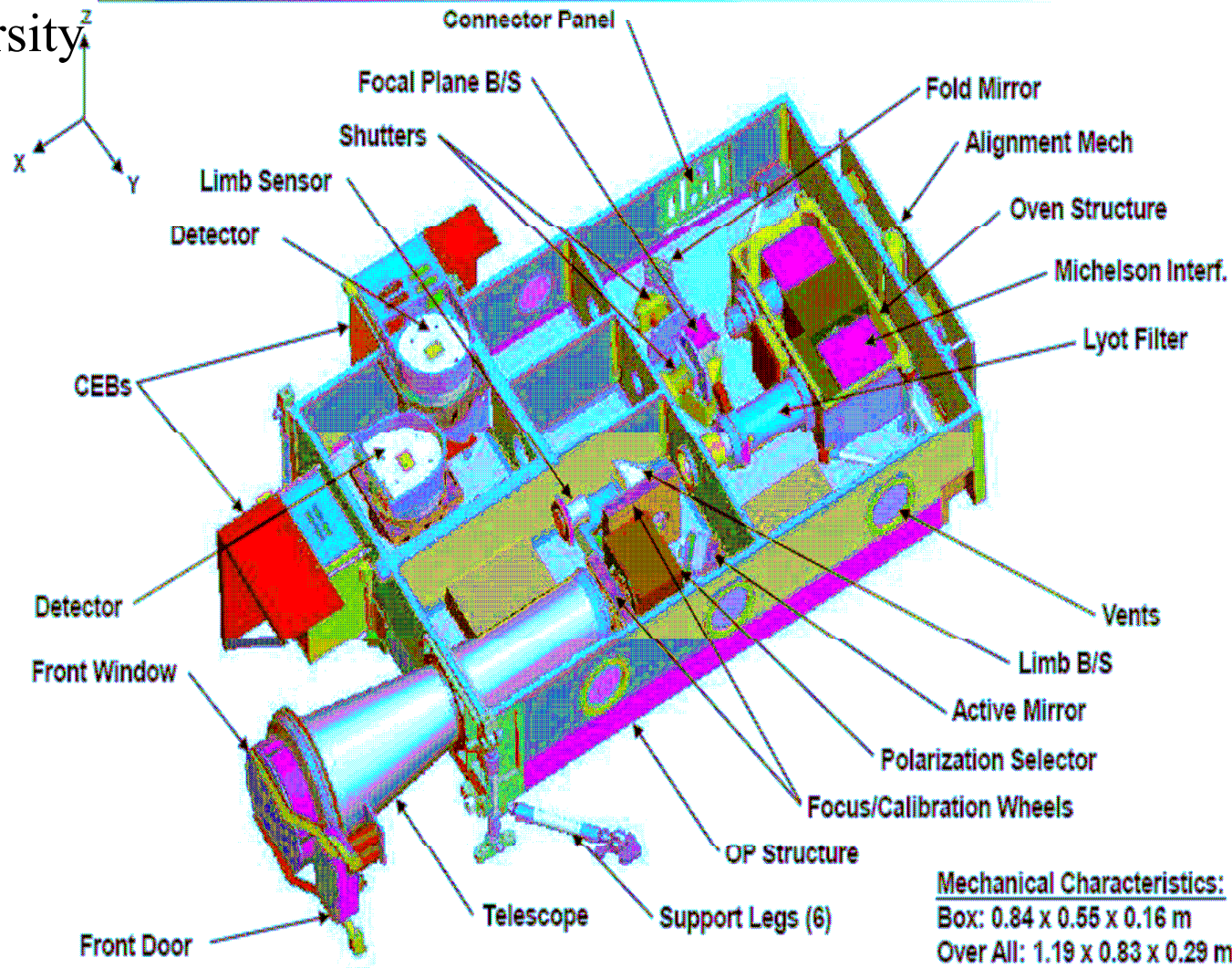


GAIA
BEPPI-COLOMBO

DynaMICCS

From
Stanford
University

Instrument Overview – HMI Optics Package (HOP)



Mechanical Characteristics:

Box: 0.84 x 0.55 x 0.16 m

Over All: 1.19 x 0.83 x 0.29 m

Mass: 39.25 kg

First Mode: 63 Hz

Solar shape is variable

Various techniques lead to the same conclusions

Solar cycles is correlated with astrometric oblateness

Signal amplitude of a few mas

Shape of Sun: oblateness + hexadecapole changes

Low frequency oscillations → Sasha (on the 23rd) → **Leptocline**

En 1958, les États-Unis lancent les premières sondes, Pioneer 1, 2 et 3 vers la Lune : trois échecs. En 1959 avec Luna 1, l'URSS réussit son premier lancement et approche la Lune à 6 000 km ; les premiers clichés de sa face cachée seront pris à la fin de cette même année. Quarante ans plus tard, il ne se passe pas de semaine sans que les médias nous annoncent le lancement réussi d'un satellite ou d'une sonde.

Concernant l'observation du Soleil, plusieurs satellites ont effectué des missions : Skylab, SMM, Ulysses et Yohkoh, avant le lancement, en décembre 1995, de Soho.

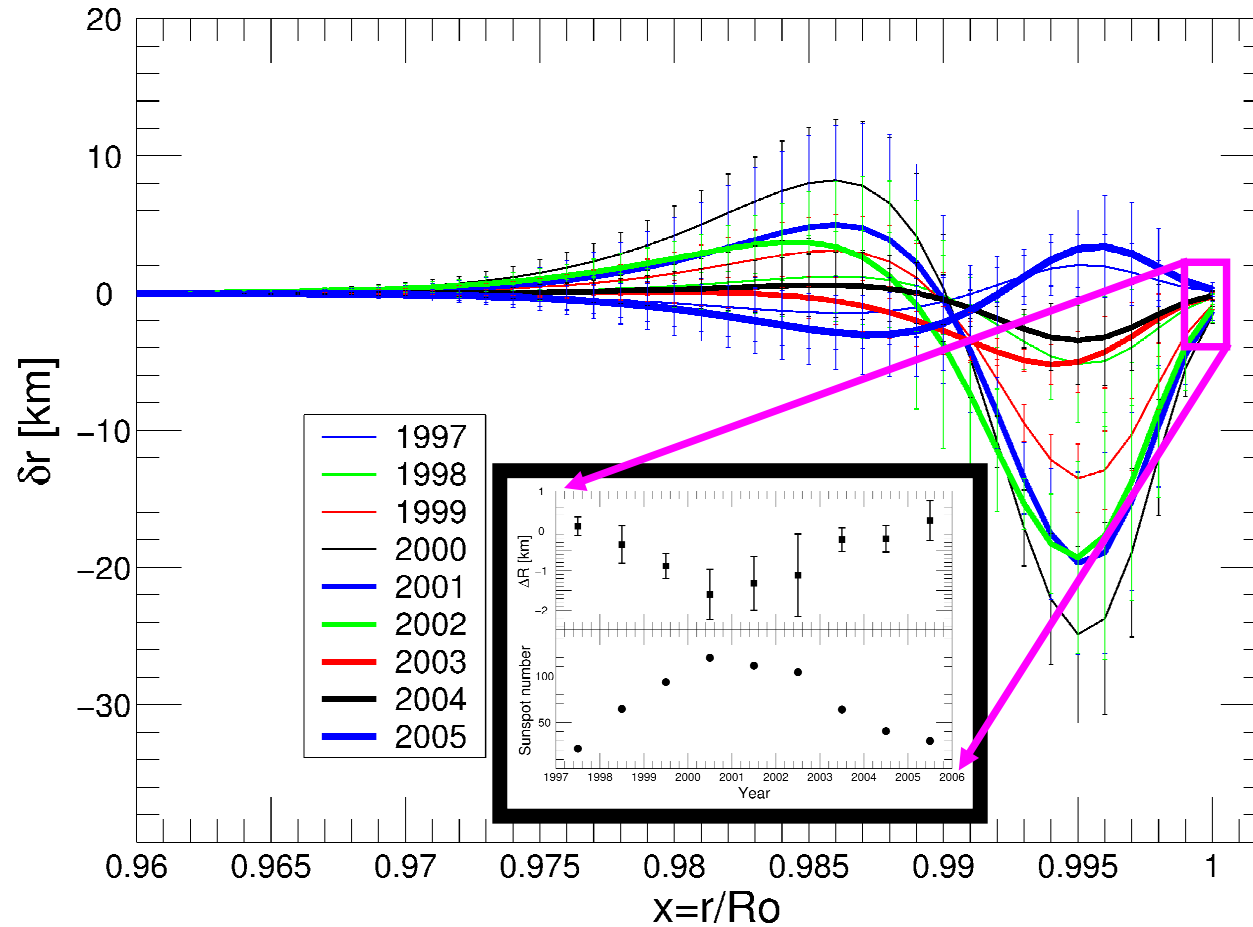
Fruit d'une coopération internationale entre l'Agence Spatiale Européenne et la NASA, Soho a été construit en Europe par un consortium industriel emmené par Matra. L'instrumentation est majoritairement européenne.

Soho, conçu pour étudier la structure interne du Soleil, son atmosphère externe étendue ainsi que l'origine du vent solaire, a parfaitement fonctionné jusqu'au 25 juin 1998 quand, à la suite d'une erreur de manipulation, il fut considéré comme "perdu" pour l'observation. Le contact a été rétabli en septembre 98 et Soho continue aujourd'hui à envoyer en continu des milliards d'informations sur le Soleil.

Soho fait vivre en direct le quotidien solaire, images toujours impressionnantes et surtout riches en informations et en surprises, comme la spectaculaire flambée d'activité du 8 octobre 1996, qui provoqua une énorme projection coronale à 3,5 millions de kilomètres de la surface du Soleil.

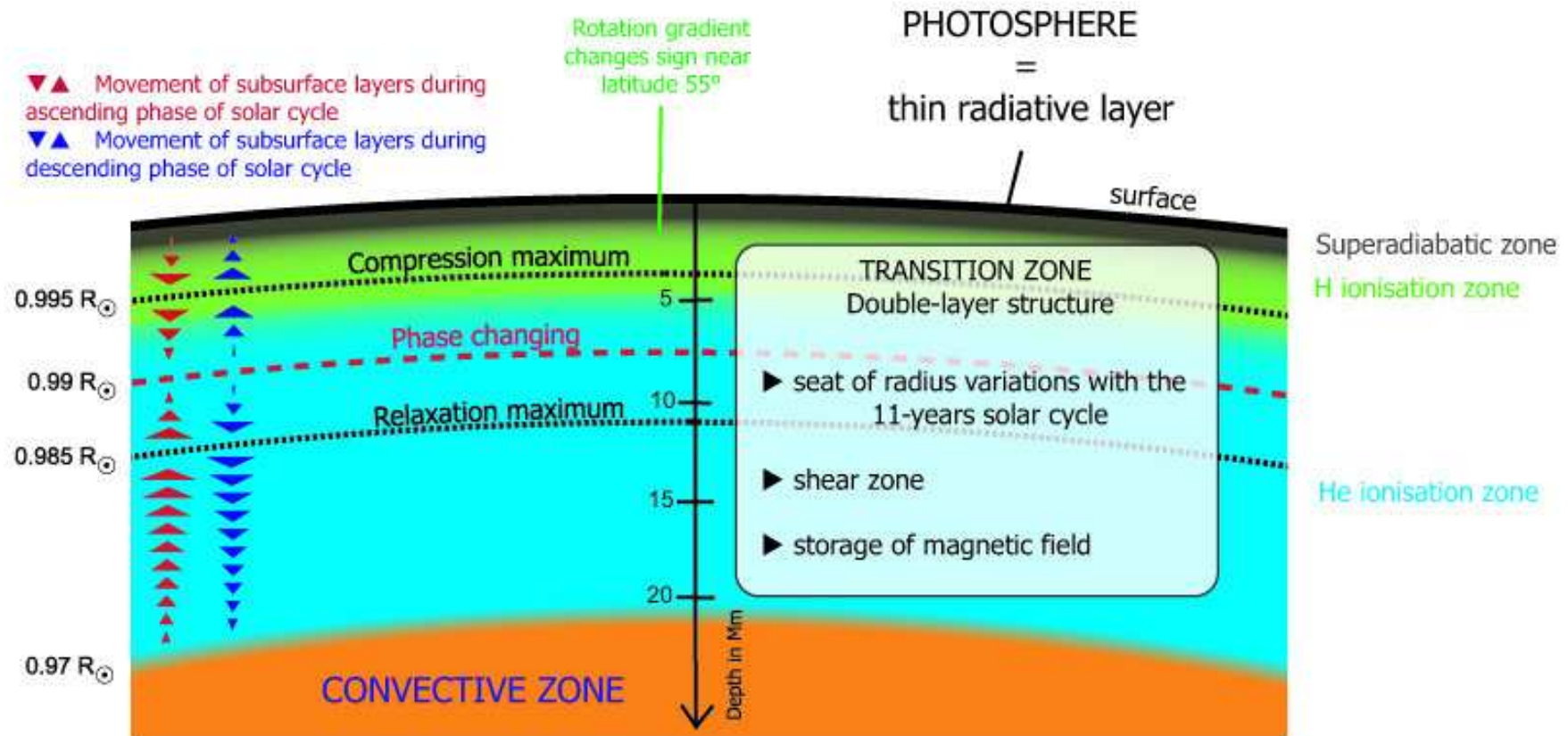
Les observations de Soho ont permis de confirmer celles effectuées au Pic par Jean Rösch et Jean-Pierre Rozelot à la coupole tournelle : le Soleil s'agitait, non pas uniformément mais en fonction de l'activité des cycles, l'agitation étant d'autant plus importante que l'activité du Soleil est intense.

Radius variation in subsurface layers



Lefebvre and Kosovichev, 2005, ApJ 633, L149

Sub-surface layer



Solar limb astrometry demonstrates

Solar radius variability

Solar shape variability

Low frequency oscillations → See papers Sandrine Lefebvre, A. Kosovichev and J.P. Rozelot, ApJ, 2005, 2006

Can be done in the future through **dedicated space instruments**:

SDO

Dynmiccs

Etc...