P-modes excitation by turbulent convection

Réza Samadi

(LESIA, Observatoire de Paris-Meudon, France)

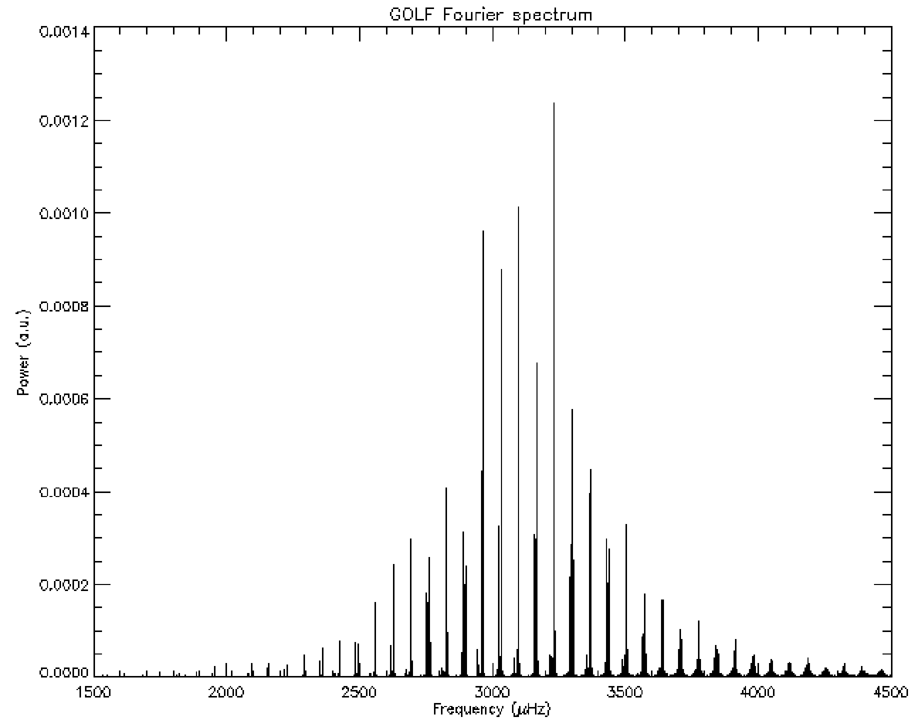
- Solar-like oscillations and associated seismic constraints
- Modeling the stochastic excitation
- Application to solar p-modes
- Excitation rates and mode amplitudes across the HR diagram
- Recent results & standing problems

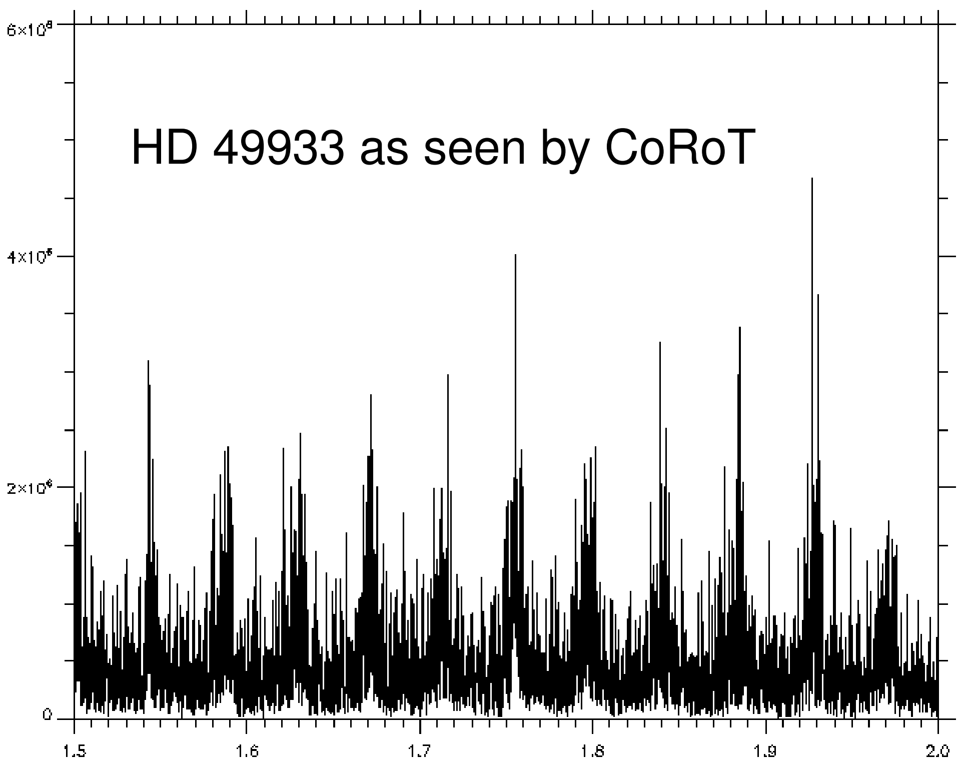
Solar-like oscillations

- The Sun :
  - more than one millions detected oscillations
  - low amplitudes : $\delta L/L \simeq 2\ 10^{-6}$

V ~ 30 cm/s

  - all modes are damped





- Similar oscillations in other stars :
- e.g. in HD 49933 and as seen by CoRoT

Mode kinetic energy and energy balance

$$E_{osc}(t) = \frac{1}{2} \int dm \; \vec{V}^2$$

We assume a linear damping : $\frac{d\vec{V}}{dt} = -\eta \; \vec{V}(t)$ η : damping rate

$$\frac{dE_{osc}}{dt}(t) = \underbrace{P}_{\text{Sources of excitation}} - \underbrace{2\,\eta\, E_{osc}(t)}_{\text{Sources of damping}}$$

Time average: $\left\langle \frac{dE_{osc}}{dt}(t) \right\rangle = 0 \;\Rightarrow\; \langle E_{osc} \rangle = \frac{P}{2\eta}$

In time average:
balance between excitation and damping

Seismic Constraints (1/3)

$$\boxed{\begin{aligned}\delta\vec{r} &= \frac{1}{2}\ A(t)\ \vec{\xi}(r)\ e^{i\omega_0 t}\ e^{-\eta t} + cc \\ v_{osc}^{\rightarrow} &= \frac{d\,\delta\vec{r}}{dt}\end{aligned}}$$

$\rightarrow$ $$\langle E_{osc}\rangle = \frac{1}{2}\ \langle A^2\rangle\ I\ \ \omega_0^2$$

Mode inertia: $$I = \int dm\ \ \xi_r^2$$

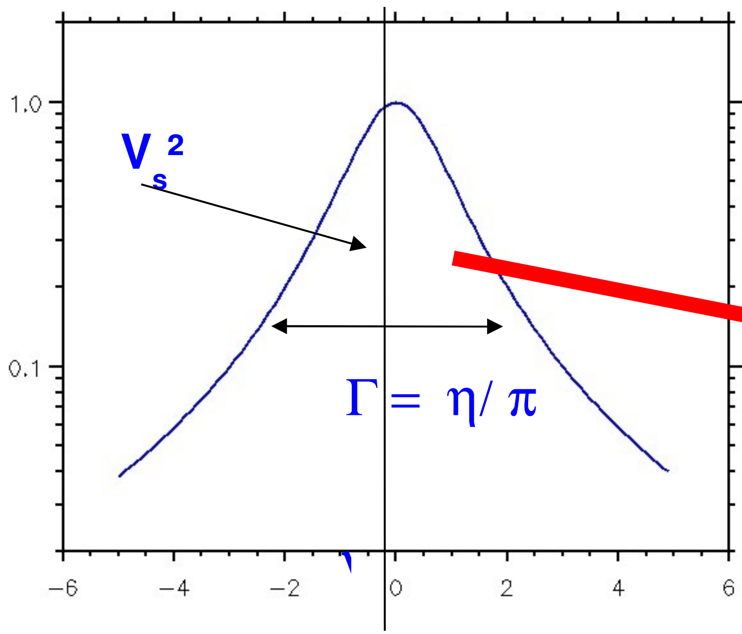
Mode *surface* velocity: $$\langle v_s^2\rangle = \langle A^2\rangle\ \ \omega_0^2\ \ \xi_s^2$$

$\rightarrow$ $$E_{osc} = \frac{1}{2}\ M\ \langle V_s^2\rangle\ = \frac{P}{4\eta}$$

Mode mass: $$M = \frac{I}{\xi_s^2}$$

$\rightarrow$ $$\boxed{P = 2\eta\ \ M\ \ \langle V_s^2\rangle}$$

Seismic Constraints (2/3)



η : damping rates ($\eta = \pi\Gamma$)

Γ: mode line-width

H : mode height

M : mode mass

$$\langle V_s^2 \rangle = \pi \; \Gamma \; H$$

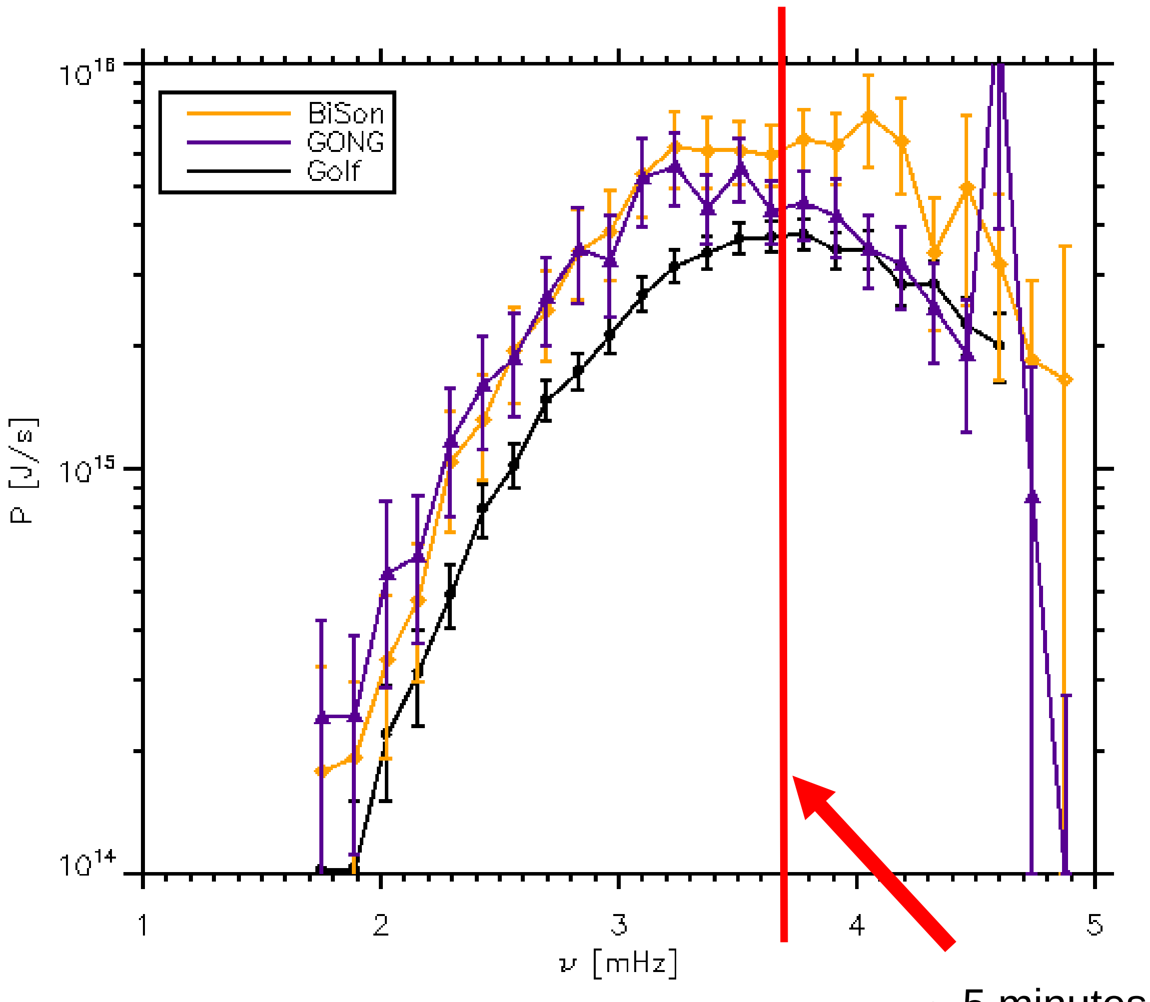
(Parseval-Plancherel relation!)

$$\boxed{P = 2 \; \pi^2 \; M \; \Gamma^2 \; H}$$

Some 'technical' problems:

- Mode mass (*M*) model dependence, depends on the measurement location => not trivial to estimate ! even for the Sun (Baudin et al, 2005)
- Kjeldsen et al (2008) : the Sun observed using stellar technique => estimate for *M*
- Strong anti-correlation between Γ and H (Chaplin & Basu 2008)

Seismic Constraints (3/3)



(Baudin et al 2005)

At high frequency:

P decreases because of a less efficient driving

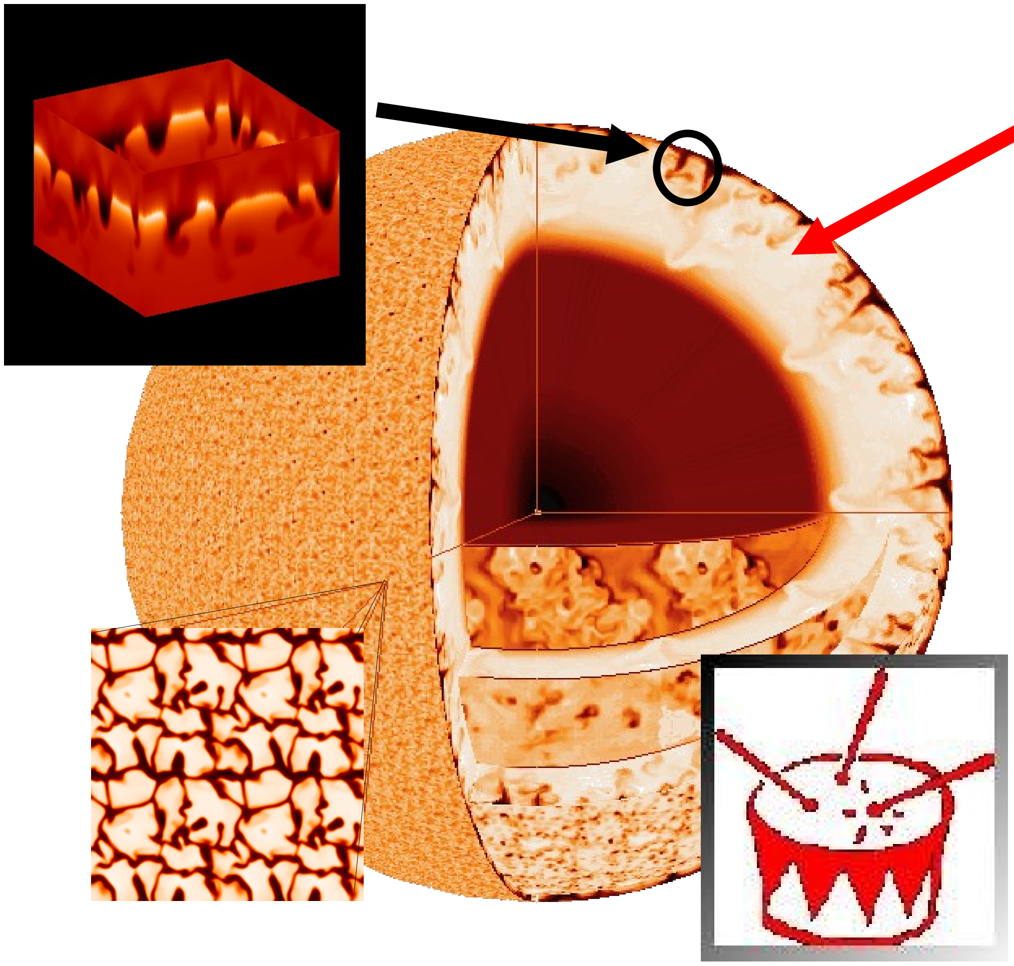
At low frequency :

P increase with ν because mode inertia decreases.

Discrepancies between the different instruments are within 1-2 sigma.

Where stochastic excitation take place?

“Stochastic Excitation” : turbulent motions force randomly the eigenmodes of the star cavity



Convective Zone :

- region of instability
- strongly turbulent medium
- many different length scales and time scales

Stars Teff < 7000 K (F and G type) :

- have an upper convective zone
- p-modes stochastically excited

The inhomogeneous wave equation (1/3)

- Continuity equation: $$\frac{\partial}{\partial t}\rho+\vec{\nabla}.(\rho\vec{v})=\vec{0}$$ (1)

- Momentum equation: $$\frac{\partial}{\partial t}\rho\vec{v}+\vec{\nabla}:(\rho\vec{v}\vec{v})=\rho\vec{g}-\vec{\nabla}p$$ (2)

- Fluctuations : $f=f_0+f1$ (Eulerian) $\delta f=f1+\delta\vec{r}.\vec{\nabla}f_0$ (Lagrangian)

- Equation of state : $p_1=c_s^2\rho_1+\alpha_s s_1+second\ \ order\ \ terms$ (3)

- Fluctuations of gravity neglected (g1=0, Cowling approximation)

- Mean stratification : $$\vec{\nabla}p_0+\langle\vec{\nabla}:(\rho\vec{v}\vec{v})\rangle=\rho_0\vec{g}$$ (4)

(1), (2), (3), (4) : $\longrightarrow$

$$\frac{\partial^2}{\partial t^2}(\rho\vec{v})-\vec{\nabla}\left(c_s^2\vec{\nabla}.(\rho\vec{v})-\alpha_s s_1\right)=-\left(\vec{\nabla}:(\rho\vec{v}\vec{v})-\langle\vec{\nabla}:(\rho\vec{v}\vec{v})\rangle\right)$$

The inhomogeneous wave equation (3/3)

Assumptions and approximations:

- We decompose the movement and fluctuations : oscillation + turbulence :

$$\vec{v} = \vec{u} + \vec{v}_{osc}$$

$$\rho_1 = \rho_t + \rho_{osc}$$

$$s_1 = s_t + s_{osc}$$

- Incompressible turbulence : $\vec{\nabla} . (\vec{u}) = 0$
- We neglect non-linear terms associated with the oscillations (e.g. V^2_{osc})
- Adiabatic oscillations : $\frac{d}{dt} \delta s_{osc} = \frac{\partial s_{osc}}{\partial t} + \vec{v}_{osc} . \vec{\nabla} s_0 = 0$
- Entropy (s_t) is a scalar passive : $\frac{d}{dt} \delta s_t = \frac{\partial s_t}{\partial t} + \vec{u} . \vec{\nabla} (s_0 + s_t) = \chi \nabla^2 (s_0 + s_t)$

The inhomogeneous wave equation (3/3)

$$\frac{\partial s_t}{\partial t} + \vec{u} \,.\, \vec{\nabla}\left(s_0 + s_t\right) = \chi \, \nabla^2 \left(s_0 + s_t\right)$$

$$\frac{\partial s_{osc}}{\partial t} + \vec{v}_{osc} \,.\, \vec{\nabla} \, s_0 = 0$$

$$\frac{\partial^2}{\partial t^2}\left(\rho \, \vec{v}\right) - \vec{\nabla}\left(c_s^2 \vec{\nabla} \,.\, \left(\rho \, \vec{v}\right) - \alpha_s \frac{\partial}{\partial t} s_1\right) = -\left(\vec{\nabla} : \left(\rho \, \vec{v} \, \vec{v}\right) - \left\langle \vec{\nabla} : \left(\rho \, \vec{v} \, \vec{v}\right)\right\rangle\right)$$

$$\Rightarrow \quad \boxed{\frac{\partial^2}{\partial t^2} \vec{v}_{osc} - L\left(\vec{v}_{osc}\right) - D\left(\vec{v}_{osc}\right) = \frac{\partial}{\partial t}\left(\vec{f}_t + \vec{\nabla}\left(h_t\right)\right) + negligible \;\; terms}$$

➪ **inhomogeneous wave equation**

L : wave operator (linear)

D : damping (linear)

$f_t = -\vec{\nabla} : \left(\rho_0 \vec{u} \, \vec{u}\right)$: Reynolds term

$\frac{\partial}{\partial t} \vec{\nabla} h_t = -\vec{\nabla}\left(\chi \, \alpha_s \nabla^2 s_t - \alpha_s \vec{u} \,.\, \vec{\nabla} s_t\right)$: entropy source term

Forced oscillators

(Goldreich & Keely (77)'s approach)

- without turbulence (u=0)⇒ ***homogeneous*** wave equation :

$$L(\delta r)+\frac{\partial^2}{\partial t^2}\delta r=0$$

L : linear operator

eigenvector : $\delta r=\vec{\xi}(r)e^{i\omega_0 t}$

ω_0 : oscillation frequency

- with turbulence (☞ forcing) ⇒ ***inhomogeneous*** wave equation :

$$L(\delta r)+\omega_0^2\delta r - D(\delta r)=S$$

$$\boxed{\delta r=A(t)\vec{\xi}(\vec{r},t)}$$

A(t) : instantaneous amplitude

damping

S : source terms

$\vec{f}_t$ Reynolds Tensor

+

$\frac{\partial}{\partial t}\vec{\nabla}h_t$ entropy source term

General solution

$$A \propto \int \int dt' d^3x \; \vec{S} \cdot \; \vec{\xi} \; e^{i\omega_0 t'}$$

ω_0 : eigenfrequency

Source terms

eigenfunction

- Stochastic process : ➪ observational constraints provides the mean square: <**A**^{2}>
- The source function (S) can be modeled **statistically**

$$\langle A^2 \rangle \propto \int dm \; \int d^3 r \, d\tau \; \vec{\xi} \cdot \langle \vec{S} \vec{S} \rangle \cdot \vec{\xi} \; e^{i\omega_0 \tau}$$

dτ : Time correlation length

dr : space correlation length

two-points correlation product (2 points space ; 2 points time)

Modeling the turbulence

$$\langle S\ \ S\rangle = \underbrace{\langle u\ \ u\ \ u'\ \ u'\rangle}_{\text{Reynolds}} + \underbrace{\langle s\ \ u\ \ s'\ \ u'\rangle}_{\text{Entropy source term}}$$

➪ 4^{th} order moments

Approximations:

- **quasi-Normal Approximation** (QNA):

$$\langle u u\ \ u' u'\rangle = 2\left(\langle u u'\rangle\right)^2$$

$$\langle s u\ \ s u'\rangle = 2\langle u\ \ u'\rangle\ \langle s\ \ s'\rangle$$

- Turbulence assumed: **isotropic, stationary, incompressible and homogeneous** (**at small scale**)

Batchelor (1953) :

$$\langle u_i\ \ u'_j\rangle(k,\omega) = \frac{E(k,\omega)}{4\pi\ \ k^2}\ T_{i,j}$$

- **Dynamic spectrum of turbulence**:

$$E(k,\omega) = E(k)\ \ \chi_k(\omega)$$

where $\int \chi_k(\omega)\ \ d\omega = 1$

E(k): spatial component

$\chi(\omega)$: frequency component = "*Eddy time-correlation function*"

Final expression

For radial modes and Reynolds stress only: (see details in Samadi & Goupil 2001)

$$P(\omega) \propto \eta \langle A^2 \rangle \propto \int dr \;\; 4\pi \;\; r^2 \left(\frac{d\xi_r}{dr} \right)^2 \; S_R(\omega_{0,} r)$$

with the source function :

$$S_R(\omega_{0,} m) = \int dk\, d\omega \;\; \frac{E^2(k)}{k^2} \chi_k(\omega_0 + \omega) \chi_k(\omega)$$

Relevant quantities:

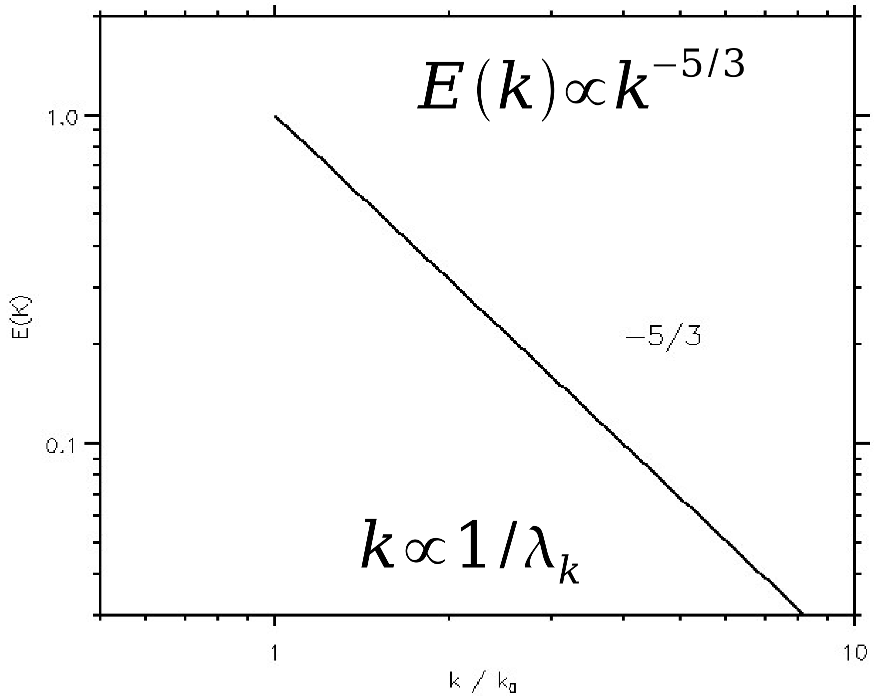
- kinetic energy : $E_c \equiv \rho_0 \langle u^2 \rangle = \int dk \;\; E(k)$
- mode compressibility : $\dfrac{d\xi_r}{dr}$

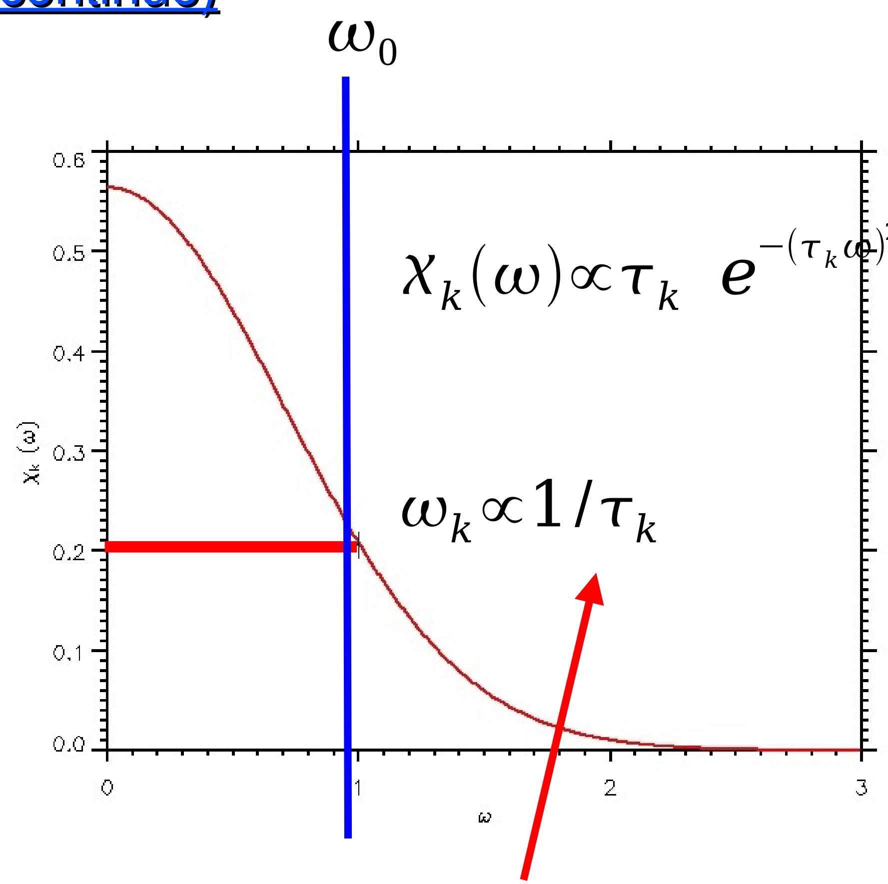
➪ controls the excitation **strength**

- eddy time-correlation : $\chi_k(\omega) \propto \tau_k \;\; e^{-(\tau_k \omega)^2}$

➪ controls the excitation **efficiency**

Relevant quantities (continue)





At fixed mode frequency (ω_0) :

efficient excitation for $\omega_0 < \omega_k \ \Rightarrow \ \tau_k \omega_0 < 1$

Eddy turn-over time : $\tau_k \propto \lambda_k / u_k$

$$E(k) \propto k^{-5/3} \ \Rightarrow \tau_k \approx \tau_\Lambda \ \left(\lambda_k / \Lambda\right)^{2/3}$$

$$\tau_k \omega_0 < 1 \ \Rightarrow \ \lambda_k < \Lambda \left(P_{osc} / \tau_\Lambda\right)$$

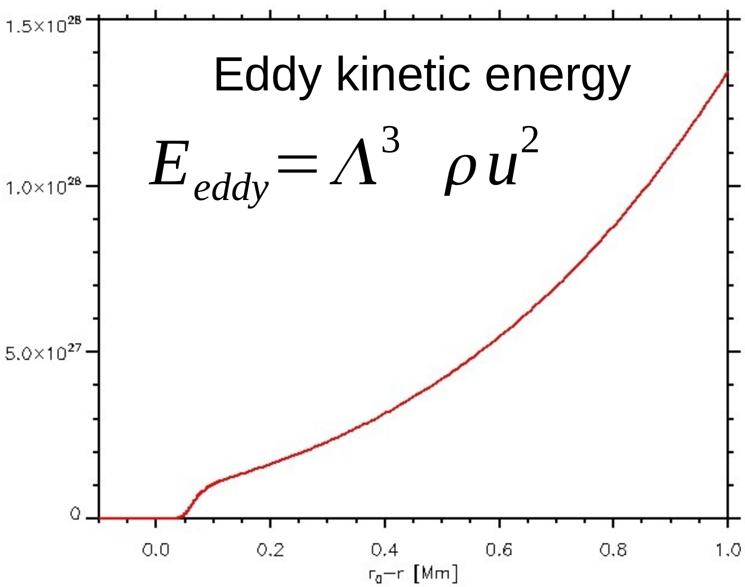
A simplified expression

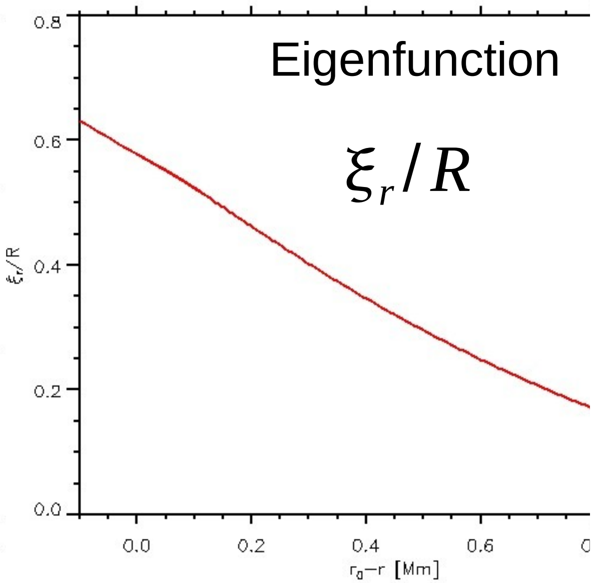
$$P \propto \frac{1}{I} \int dm \ \ \rho_0 u^4 \Lambda^3 \ \left(d\xi_r / dr\right)^2 \ \tau_\Lambda e^{-(\tau_\Lambda \omega_0)^2}$$

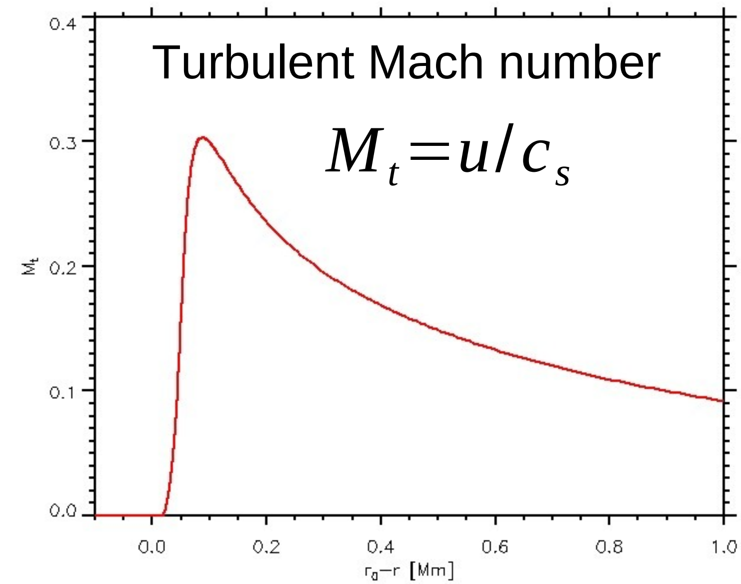
for : $P_{osc} > \tau_\Lambda$

$$E_{eddy} = \Lambda^3 \ \ \rho u^2 \qquad d\xi_r / dr = k \ \ \xi_r \approx (\omega_0 / c_s) \ \ \xi_r \qquad M_t = u / c_s$$

$$\Rightarrow \qquad P \propto \frac{1}{I} \int dm \left(E_{eddy} \omega_0\right) \left(\xi_r^2 \ \ M_t^2 \ \ \tau_\Lambda \omega_0 e^{-(\tau_\Lambda \omega_0)^2}\right)$$

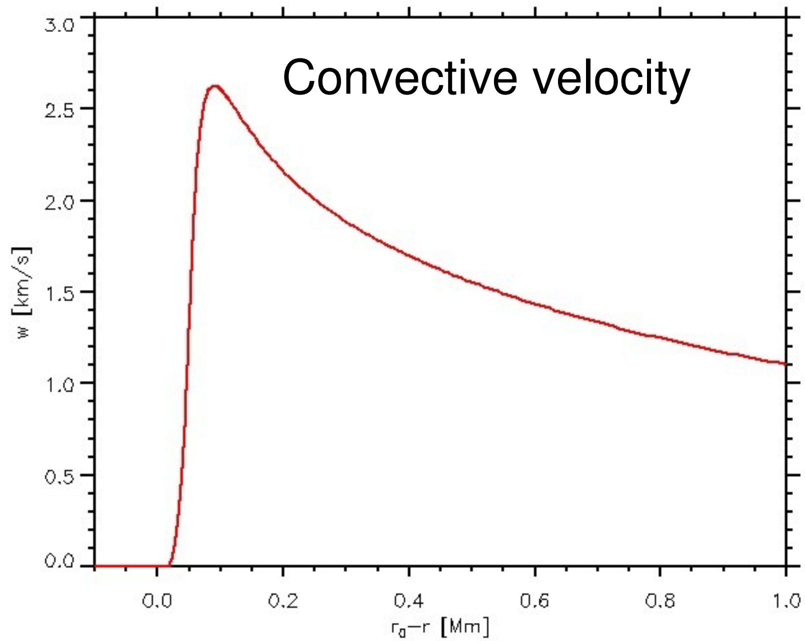






surface interior

Relevant location



u

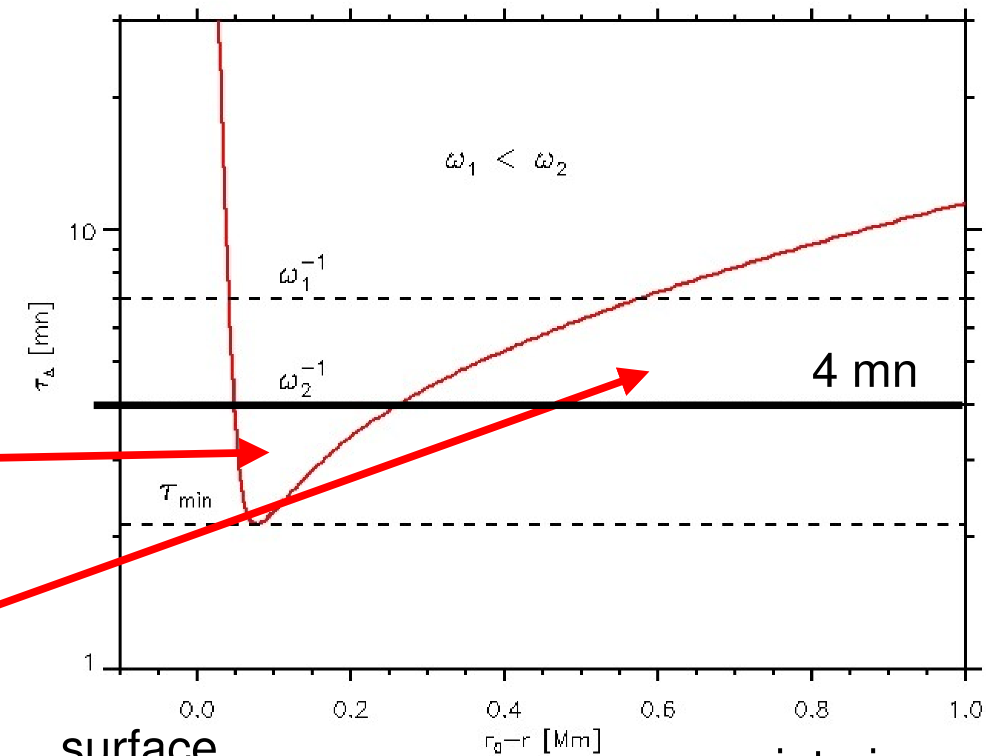
surface

interior

Upper part of the CZ :

largest u ➪ shortest τ

Eddy turn-over time: $\tau_\Lambda \approx \Lambda / u$



surface

interior

$\tau_\Lambda < P_{osc}$ ➪ **efficient** excitation

$\tau_\Lambda > P_{osc}$ ➪ **inefficient** excitation

Energy equipartition
(Godreich & Keely 1977)

$$P \propto \frac{1}{I} \int dm \ \left(E_{eddy} \ \ \omega_0 \right) \ \left(\xi_r^2 \ \ M_t^2 \ \ \tau_\Lambda \omega_0 \right)$$

in the region where :

$$\tau_\Lambda \omega_0 < 1$$

Mode damped by turbulent viscosity (Ledoux & Walraven 1958) :

$$\eta \propto \frac{1}{I} \int dm \ \ \nu_t \ \left| r \frac{d}{dr} \frac{\xi_r}{r} \right|^2$$

with the viscosity $\nu_t = \lambda \ \ u_\lambda$

$$\Rightarrow \quad \eta \propto \frac{1}{I} \int dm \ \left(\xi_r^2 \ \ M_t^2 \ \ \tau_\Lambda \omega_0^2 \right)$$

$$\nu_t = \Lambda \ \ u$$

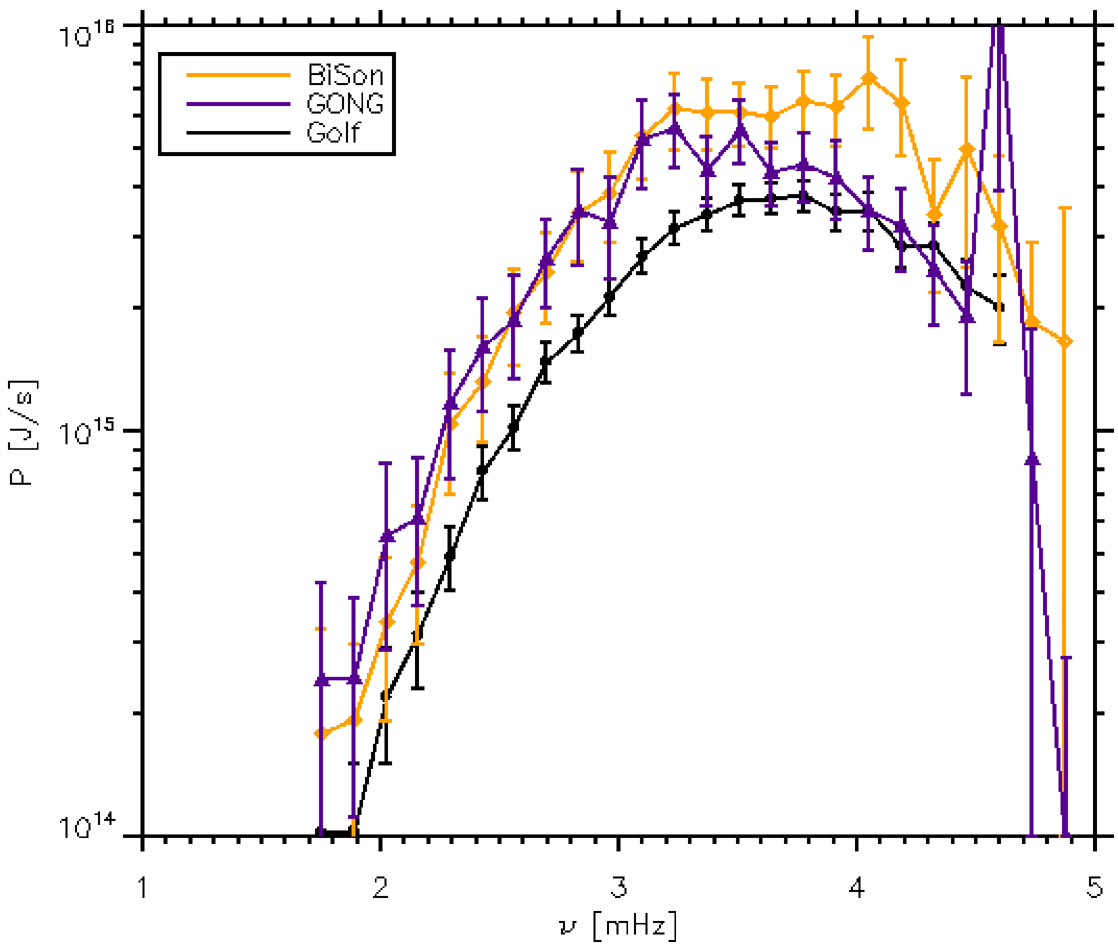
in the region where : $\tau_\Lambda \omega_0 < 1$

$$E_{osc} = \frac{P}{4\eta} \quad \Rightarrow \quad E_{osc} \propto E_{eddy}$$

Equipartition between the mode and the resonant eddies.

BUT viscosity is not the dominant source of damping

Application to the solar p modes



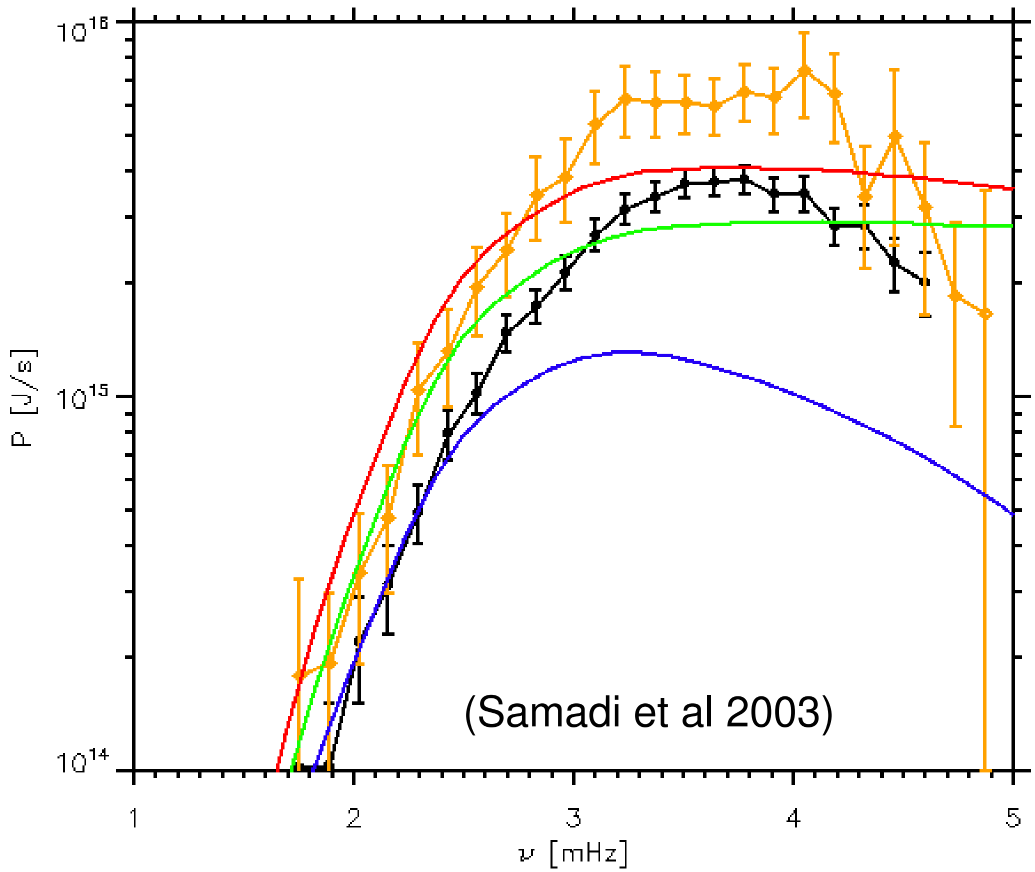
How far are the theoretical estimations ?

Can we reproduce the variation with the frequency ?

What can be learned about turbulent convection ?

Application to the solar p modes

How far are we from the observations ?



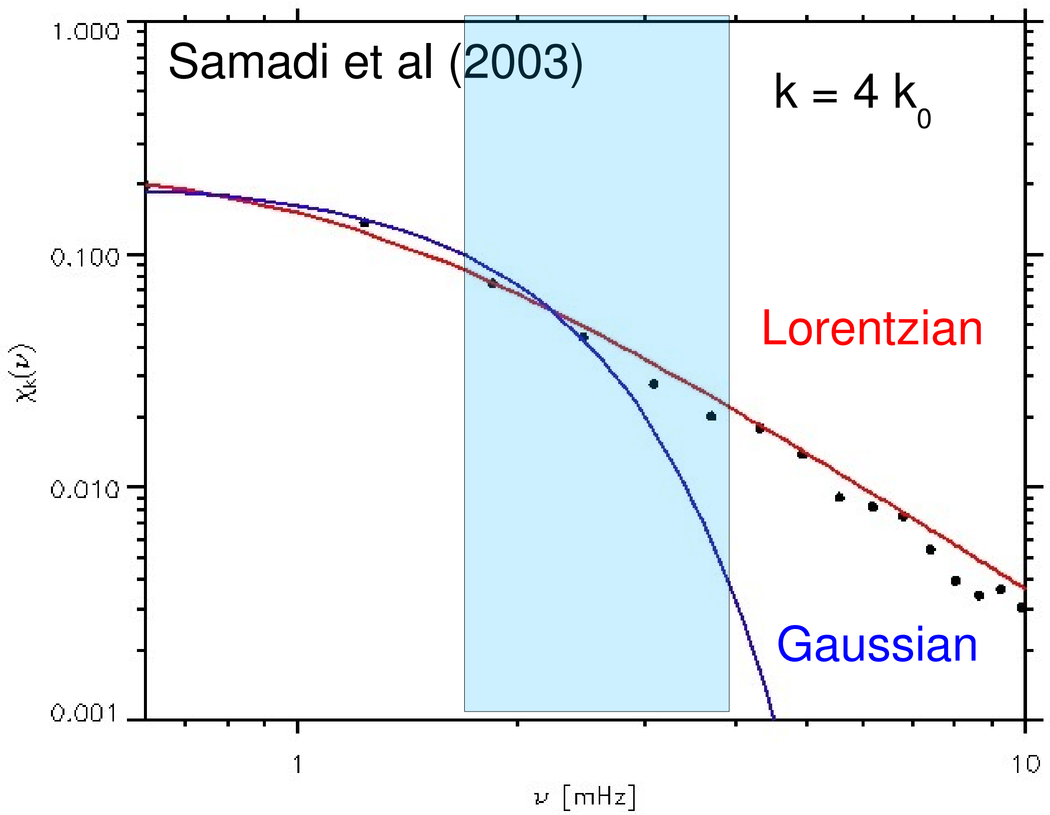
Turbulent spectrum from a solar 3D simulation, Gaussian $\chi_k(\omega)$.

Application to the solar p modes

Gaussian $\chi_k(\omega)$?

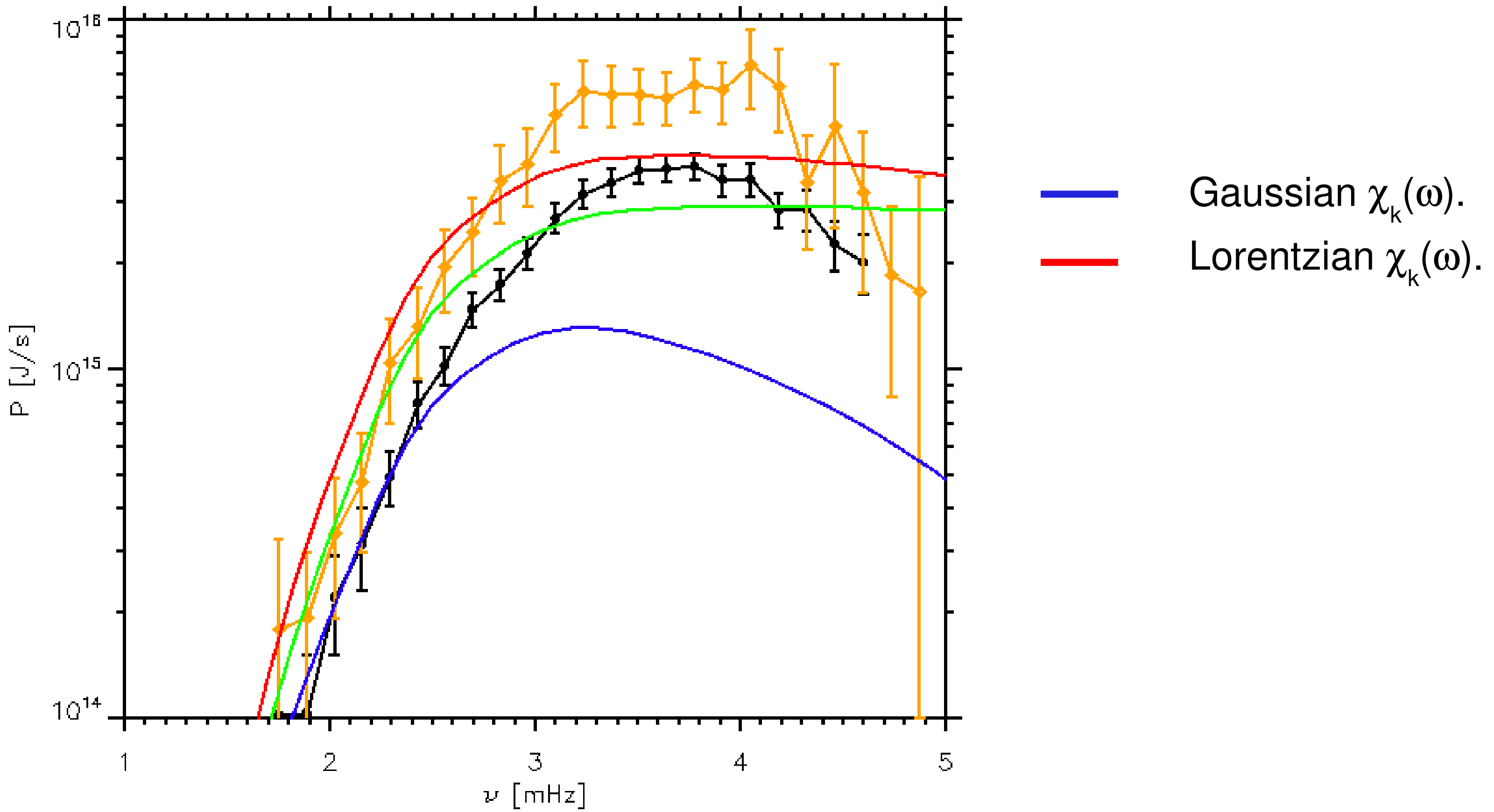
- $\chi_k(\omega)$ computed with Stein & Nordlund (1998)'s code.
- Gaussian function underestimates $\chi_k(\omega)$ above ~1.5 mHz
- Lorentzian eddy time-correlation at large scale : predicted by Gough (1977)'s time-dependent Mixing-Length theory

Eddy time-correlation function, $\chi_k(\omega)$



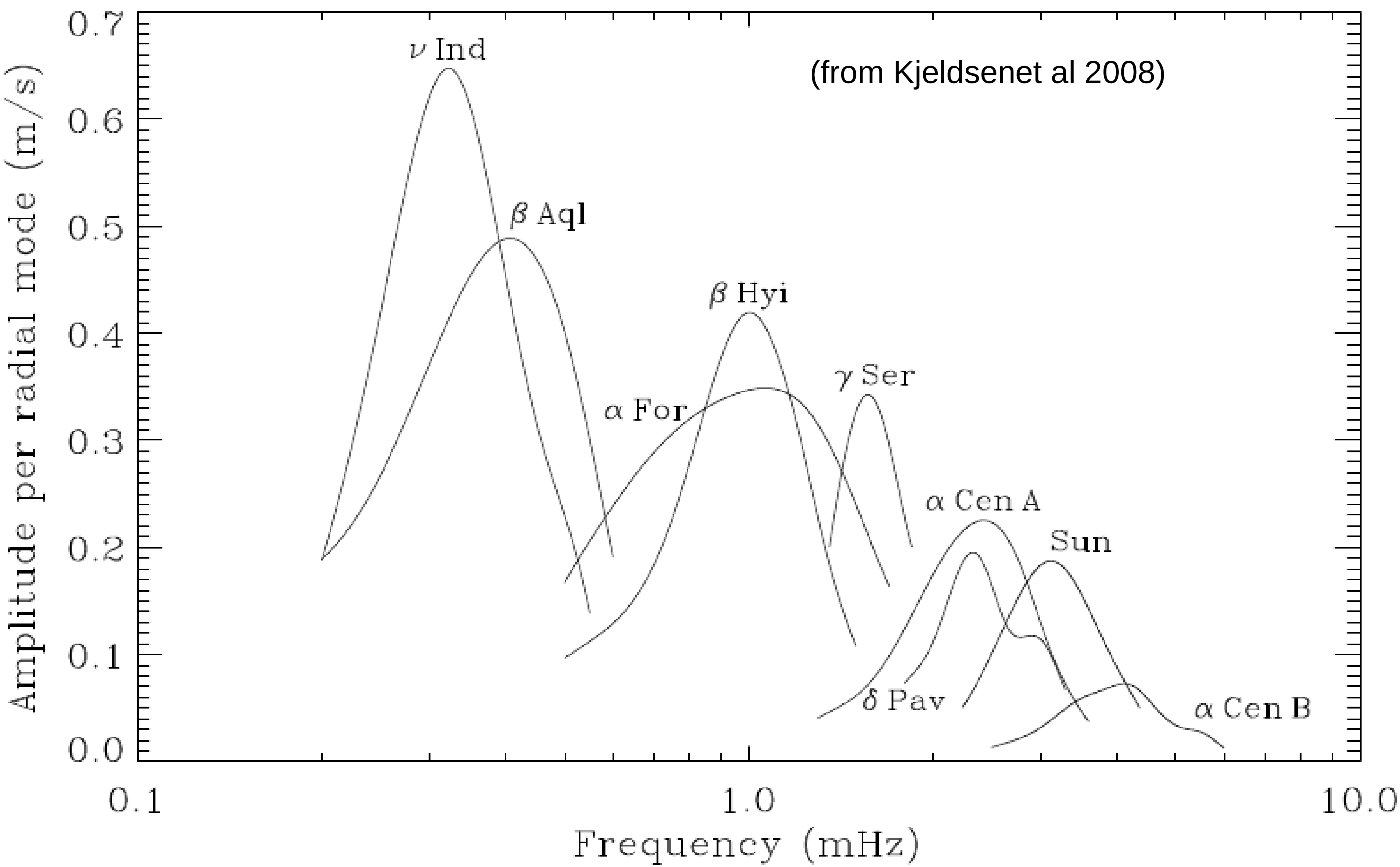
(50 mn, 25 km,253x253x163)

Application to the solar p modes



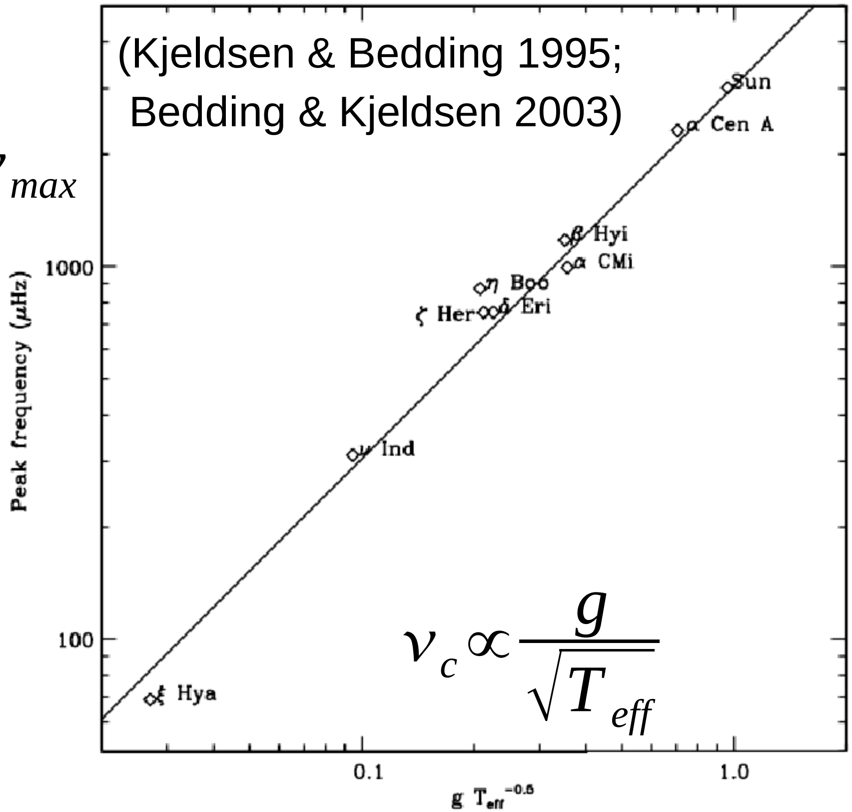
(Samadi et al 2003)

Amplitude of Solar-like oscillations across the HR diagram



Amplitude of Solar-like oscillations accross the HR diagram (1/4)

ν_{max}



(Kjeldsen & Bedding 1995; Bedding & Kjeldsen 2003)

$$\nu_c \propto \frac{g}{\sqrt{T_{eff}}}$$

Characteristic eddy turn-overtime:

$$\tau \approx \frac{\Lambda}{u} \quad \Rightarrow \quad \nu_{max} \approx \frac{u}{\Lambda}$$

Assuming fully convective envelop:

$$\sigma T_{eff}^4 = \rho C_p \delta T \; u$$

Eddies accelerated by buoyancy force:

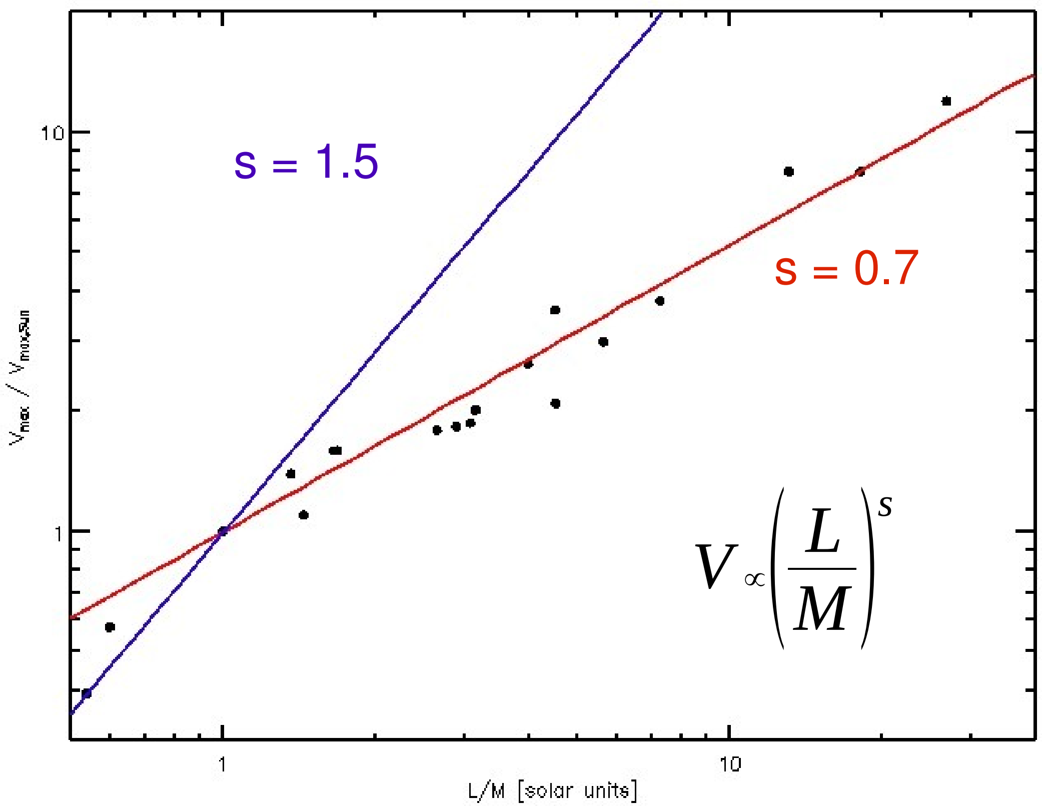
$$\rho \; u^2 = g \delta \rho \; \Lambda$$

We then derive :

$$\nu_{max} \propto g \left(\frac{T_{eff}}{\rho} \right)^{1/3}$$

$$\nu_{max} \propto g^{2/3} \; \left(R T_{eff} \right)^{1/3}$$

Amplitude of Solar-like oscillations across the HR diagram (2/4)



- Observations => slope s ~ 0.7 – 0.8
- Kjeldsen & Bedding (1995) : s = 1.0
- Houdek et al (1999) : s = 1.5

- With s=1.5 oscillation amplitudes are severely over-estimated for hot stars

$$L \propto R^2 \; T_{eff}^4$$

$$g \propto M / R^2$$

$$\frac{L}{M} \propto \frac{T_{eff}^4}{g}$$

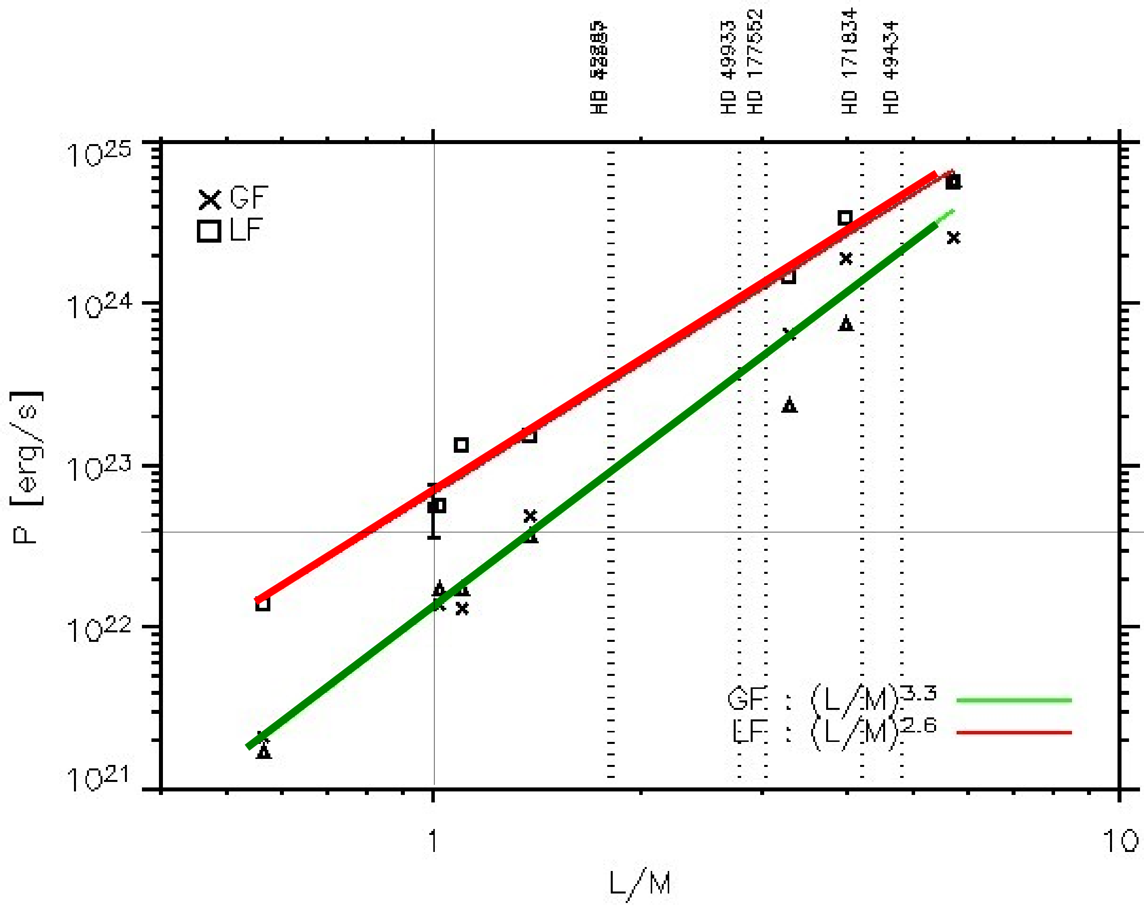
Amplitude of Solar-like oscillations across the HR diagram (3/4)

Lorentzian $\chi_k(\omega)$

$$P_{max} \propto \left(\frac{L}{M}\right)^{2.6}$$

Gaussian $\chi_k(\omega)$

$$P_{max} \propto \left(\frac{L}{M}\right)^{3.1}$$



Solar level

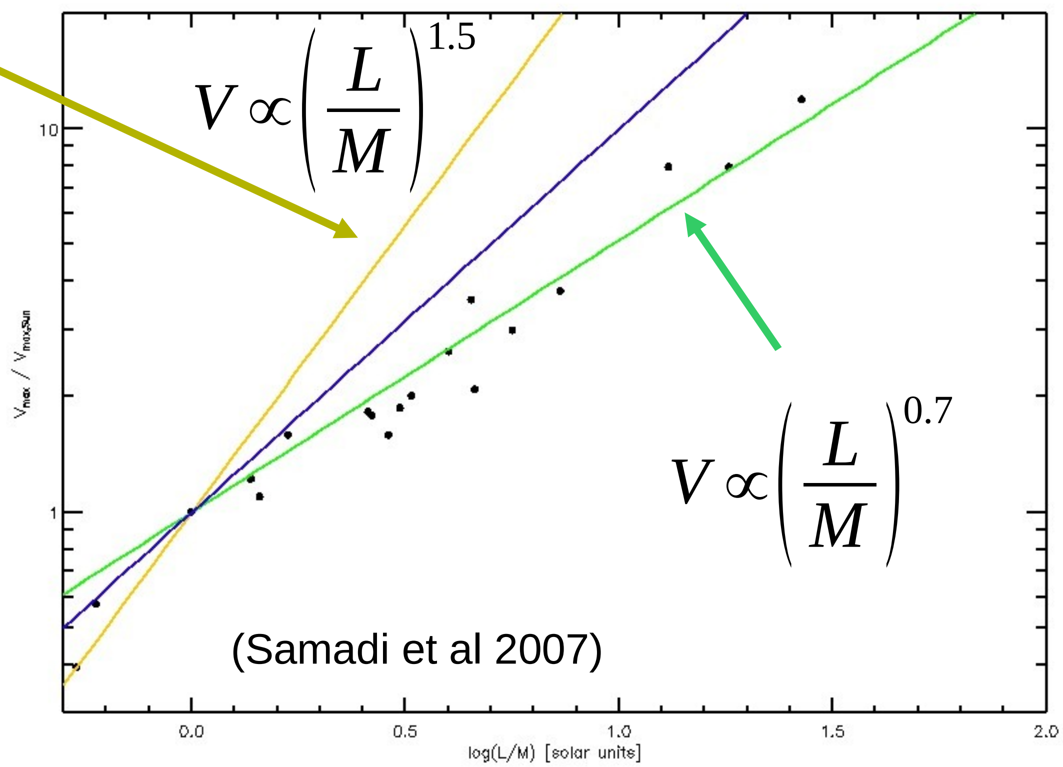
(Baudin et al 2005)

$$\frac{L}{M} \propto \frac{T_{eff}^4}{g}$$

(Samadi et al 2005, 2007)

Amplitude of Solar-like oscillations across the HR diagram (4/4)

Houdek & Gough (2002)



- Good matching using Lorentzian $\chi_k(\omega)$
- Use of a Gaussian explains part of the overestimation

But some debate:

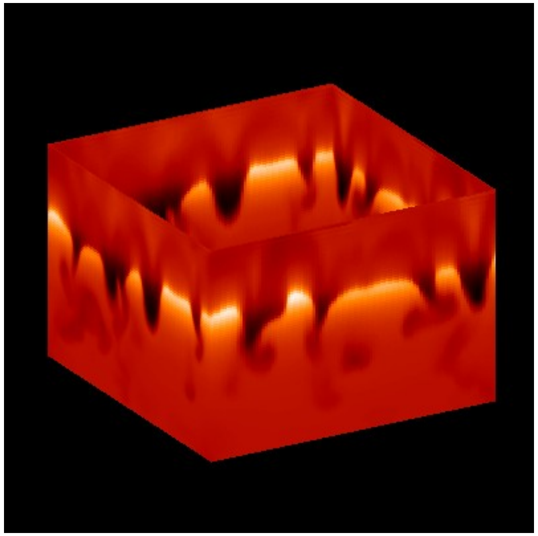
- Houdek (2006) : overestimation comes partially from under-estimation of the mode line-width (Γ)

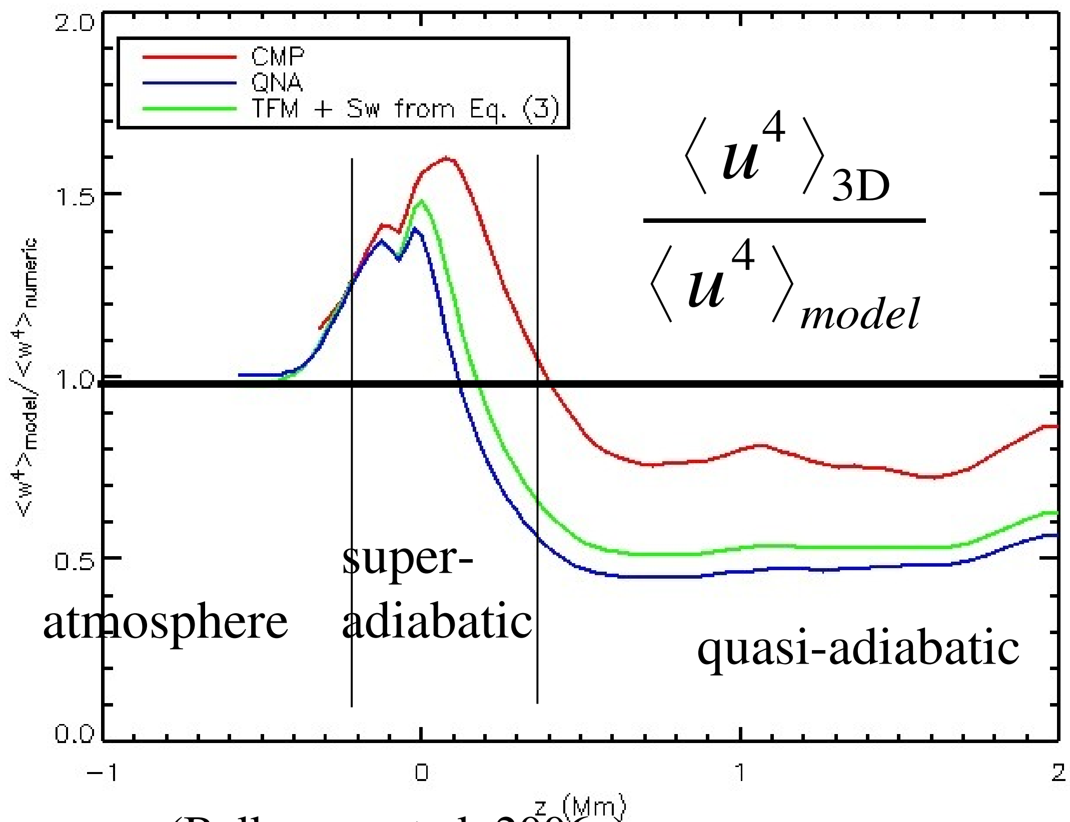
More recent results (1/5)

Quasi Normal Approximation (QNA) : $\langle u^4 \rangle = 3 \langle u^2 \rangle^2$ Assumes symmetric PDF

$\Rightarrow$ The medium is highly asymmetric

$\Rightarrow$ QNA not verified in the solar surface (see Belkacem et al 2006, Kupka & Robinson 2007)





(Belkacem et al, 2006a)

More recent results (2/5)

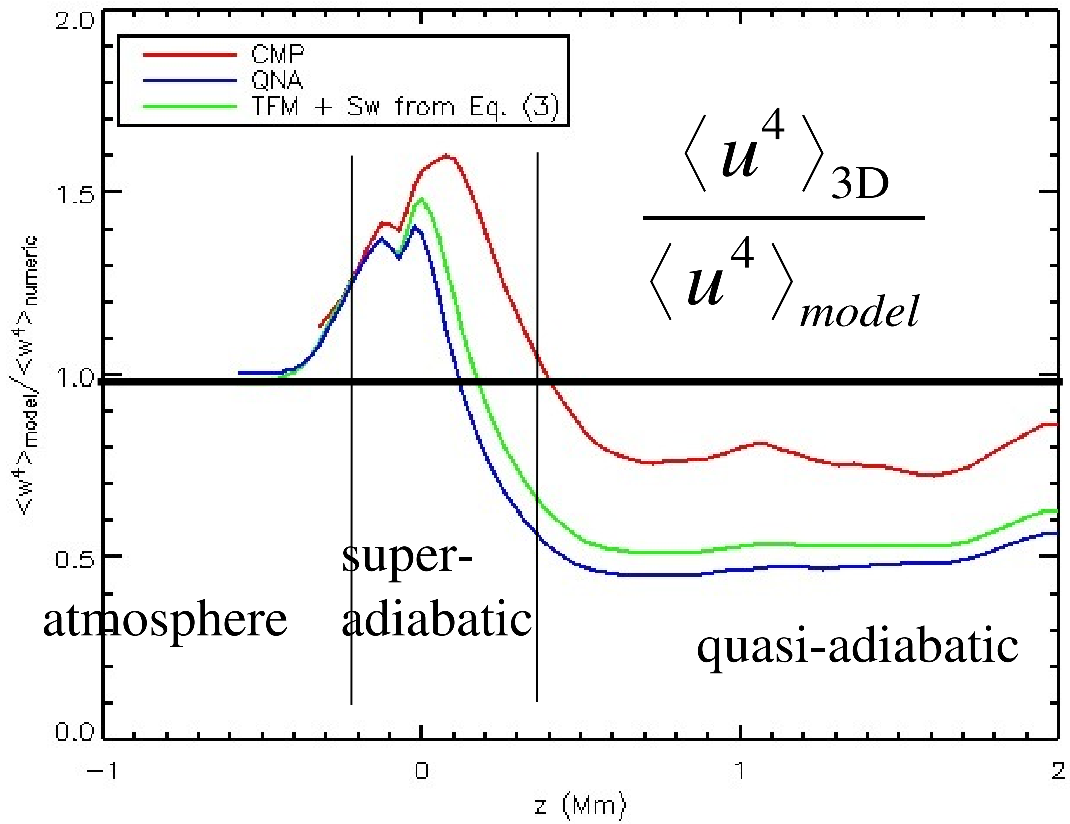
Two-scale Mass Flux Models (TFM, Gryanik & Hartmann 2002)

- quasi-laminar flow
- flow split in term of downdrafts (plumes) and updrafts (granules)

$$\langle u \rangle = a \langle u \rangle_{upflow} + (1-a) \langle u \rangle_{downflow}$$

$$\langle u^4 \rangle = 3 \left(1 + \frac{1}{3} S_w^2 \right) \langle u^2 \rangle^2$$

$$S_u = \frac{\langle u^3 \rangle}{\langle u^2 \rangle^{3/2}} = \frac{1-2a}{\sqrt{(a(1-a))}}$$



(Belkacem et al 2006a)

More recent results (3/5)

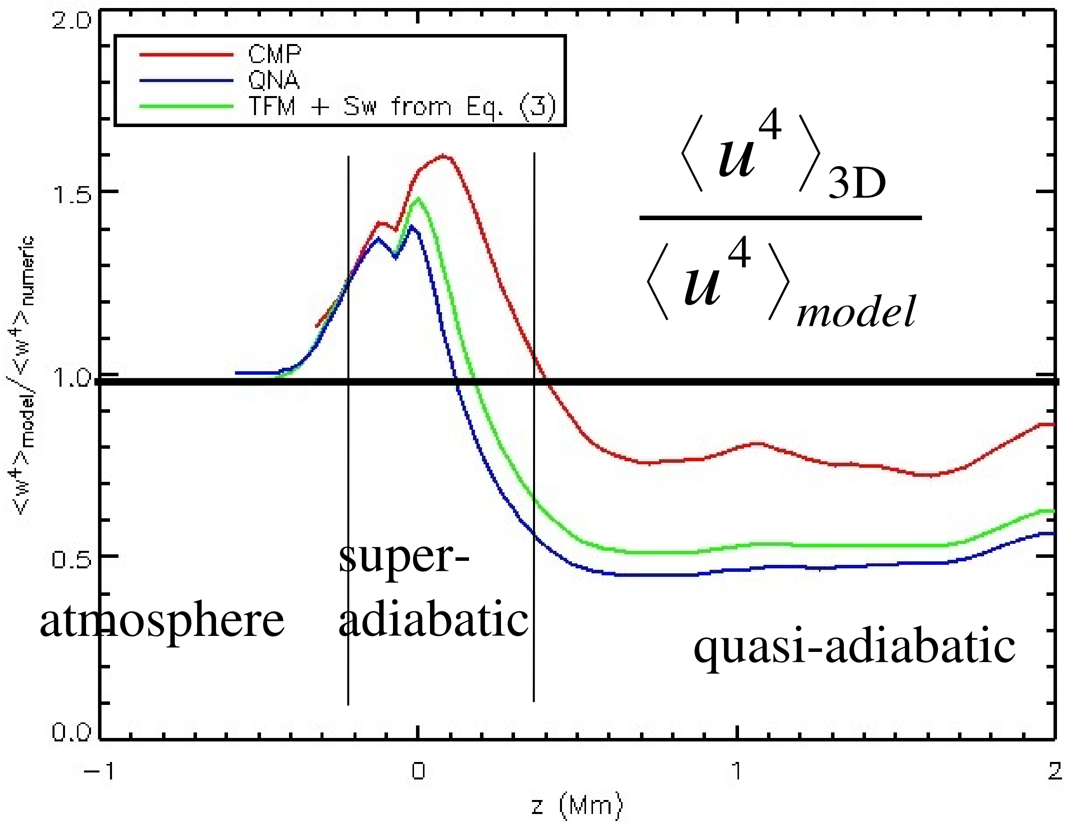
Closure Model with Plumes (CMP)
(Belkacem et al 2006a) :

• Generalised TFM: takes into account the turbulence into each flow

$$S_u = \frac{a\,\delta u}{\langle u^2 \rangle^{3/2}} \left((1-2a)(1-5a)\,\delta u^2 - 3\langle u^2 \rangle \right)$$

$$\delta u = \langle u \rangle_{upflow} - \langle u \rangle_{downflow}$$

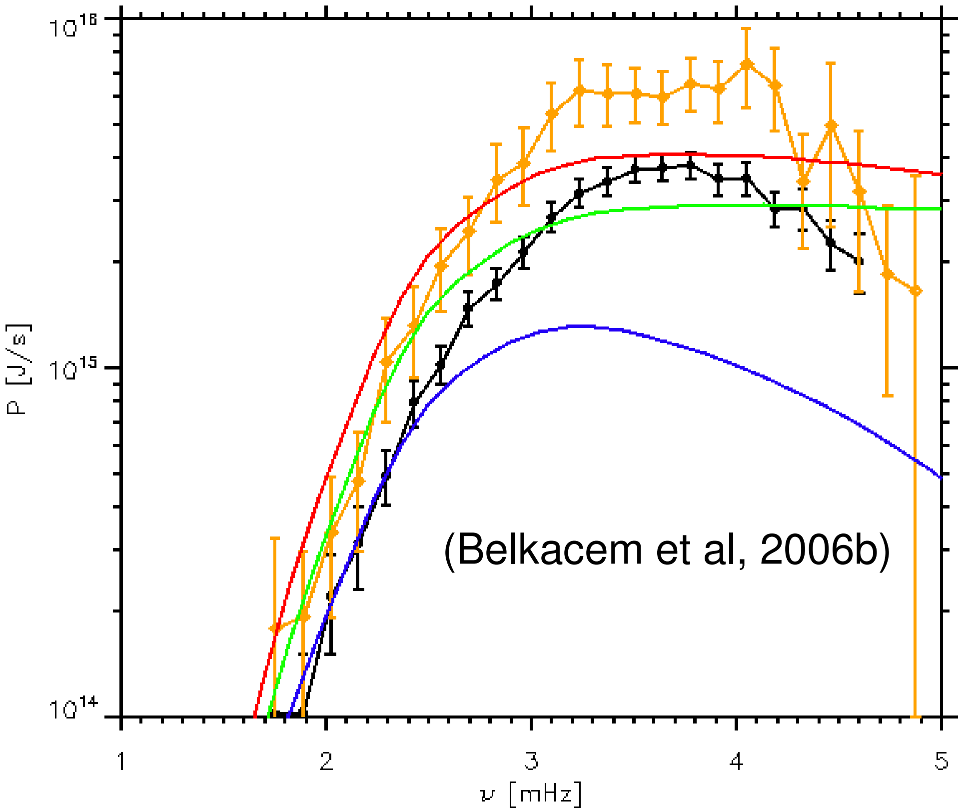
• δu obtained using Rieutord & Zahn (1995)'s plumes model



(Belkacem et al 2006a)

More recent results (4/5)

The Sun



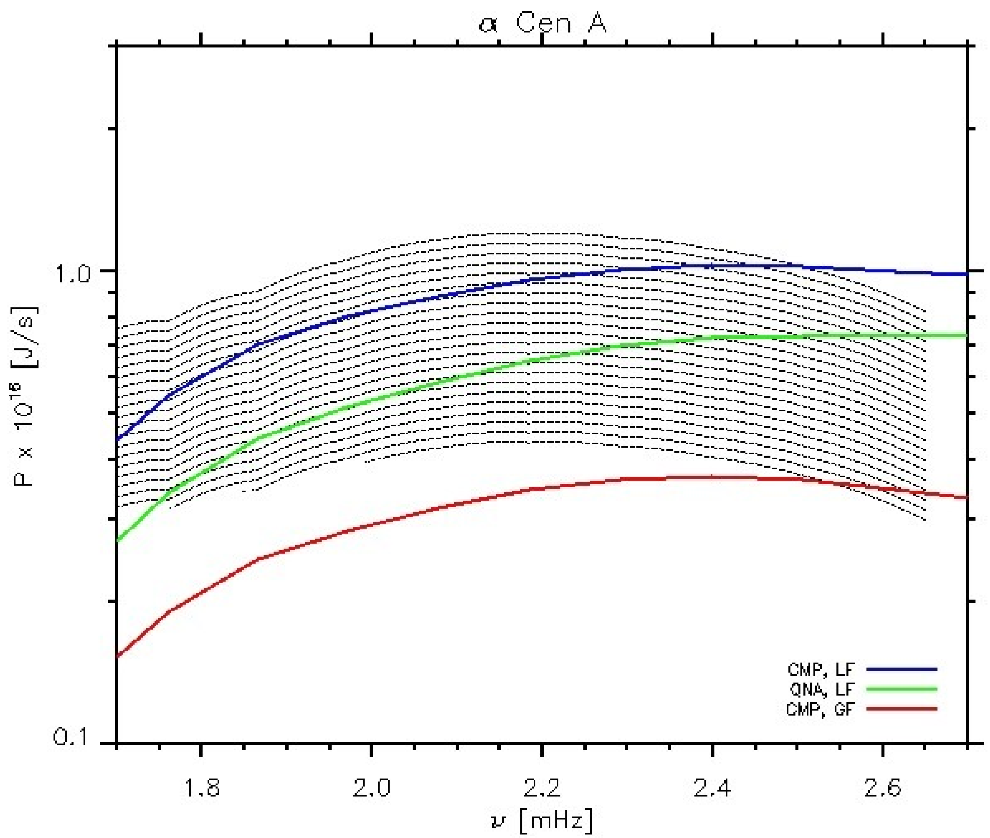
Closure model with plumes (CMP)

QNA

Gaussian $\chi_k(\omega)$.

More recent results (5/5)

Alpha Cen A



(Samadi et al 2008)

More recent results (continue...)

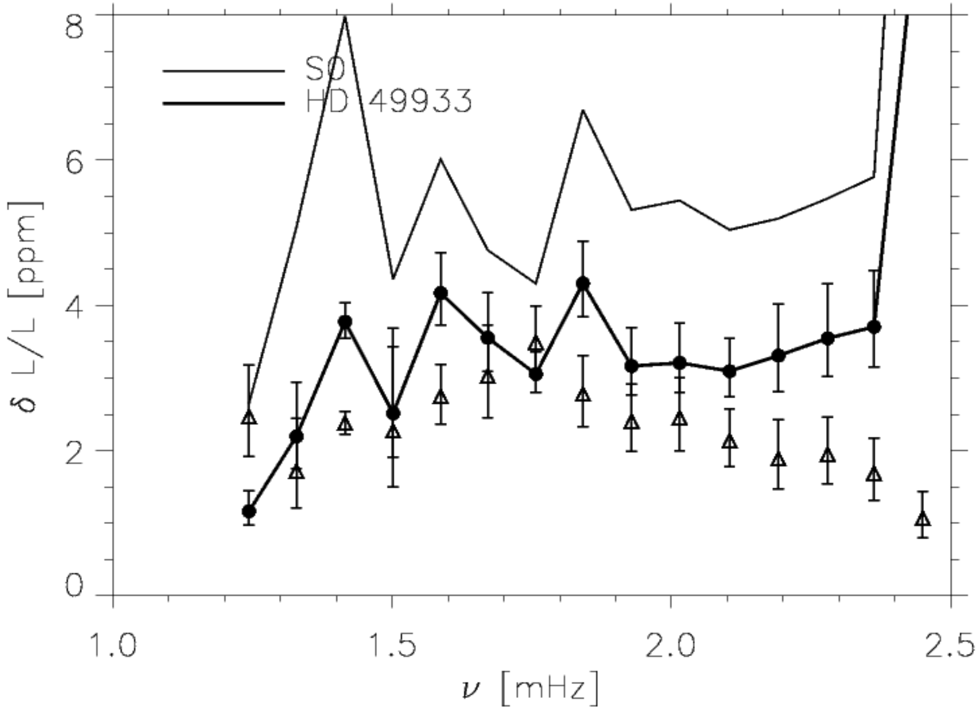
- Generalisation to non-radial mode (Belkacem formalism et al 2008a)
- Application to g-modes (Belkacem et al 2008b) $\Rightarrow$ see Kévin's talk
- Influence of the rotation (Belkacem et al, in prep.)

Standing problems

- Entropy treated as a passive scalar => crossing terms vanish (no interferences between the entropy and the Reynolds stress)
- Scale length separation (oscillation / turbulence) : $k_{osc} << k_t$

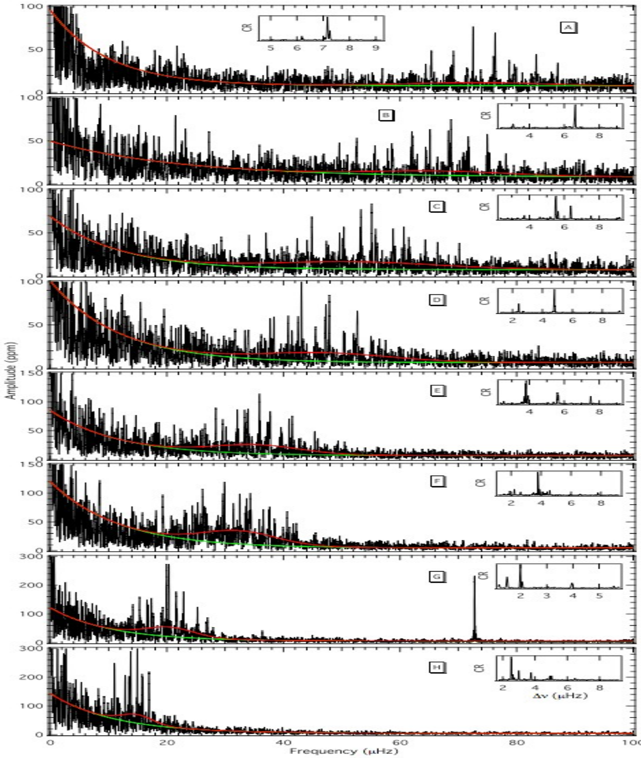
Perspectives

Solar-like oscillations in star with low or high surface metal abundances, redgiants, β Cephei, γ Doradus ...



HD 49933 (CoRoT)

(Samadi et al in, prep)



T. Kallinger and the giant team

Amplitude of Solar-like oscillations accross the HR diagram (4/5)

$$P \propto \frac{1}{I} \int dm \, \rho \, u^4 \Lambda^3 \; \left(\omega_0 \, \xi_r / c_s\right)^2 \; \tau \; e^{-(\tau \omega_0)^2}$$

$$\tau \omega_0 \sim 1$$

$$F_{kin} = F_c \left(g \Lambda / c_p T\right)$$

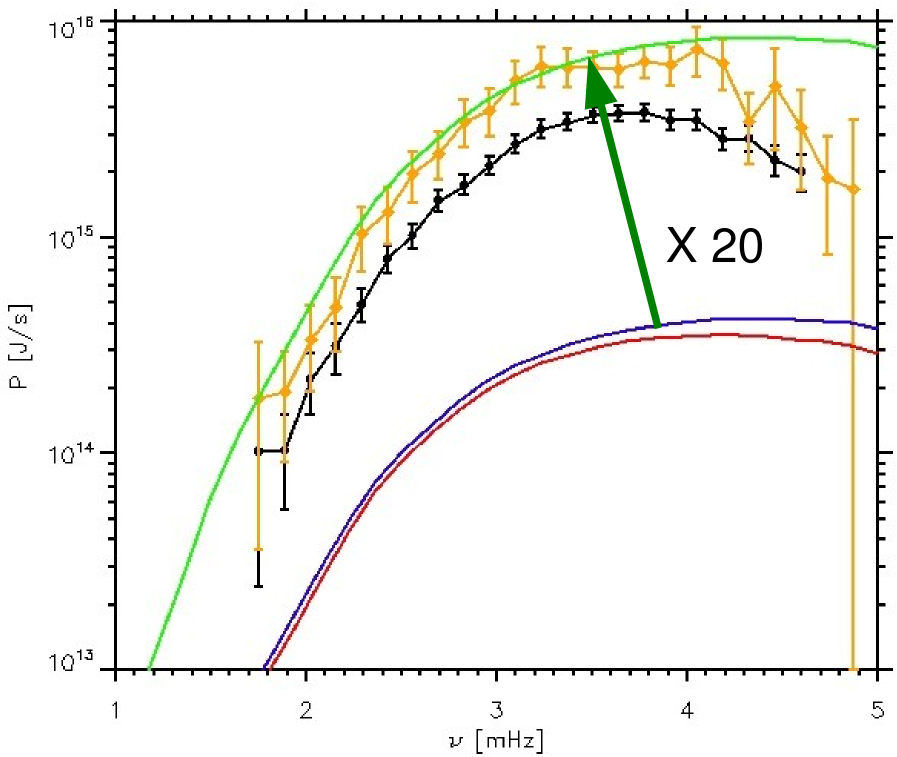
$$\Lambda \propto T / g$$

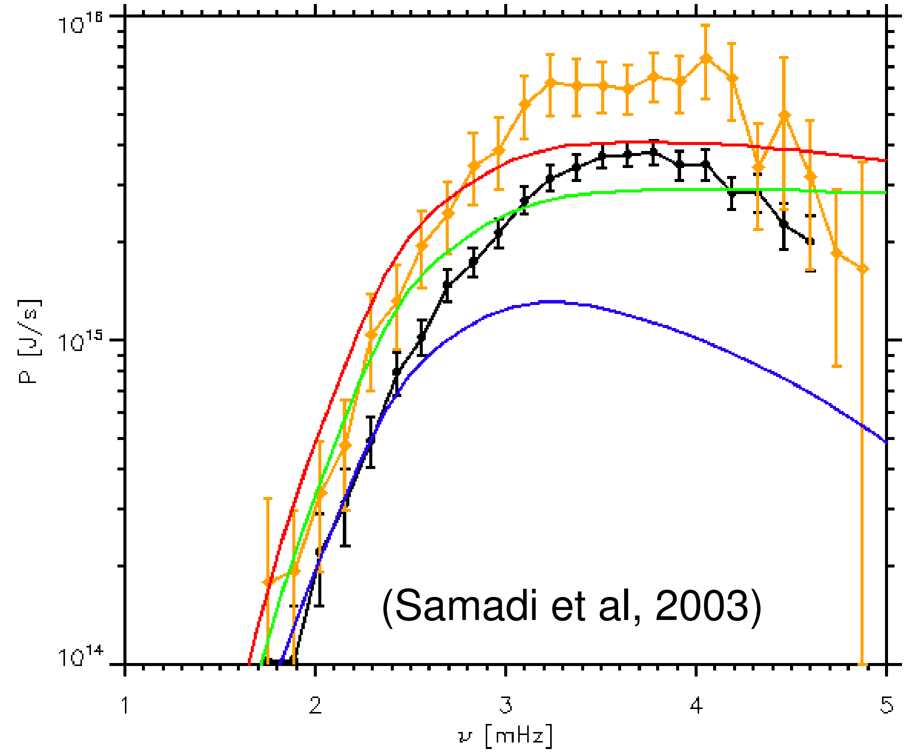
$$P \propto \frac{1}{I} \int d\, m \;\; \xi_r^2 \; \left(T_{eff}\right)^{23/12} \; g^{-2} \; \rho$$

$$\bar{\rho} = \frac{M}{R^3}$$

$$P \propto \frac{1}{I} \int d\, m \;\; \xi_r^2 \; \left(T_{eff}\right)^{23/12} \; g^{-3} \; M^{1/3}$$

Application to the solar p modes





Calculation as in Chaplin et al (2005) :

- Kolmogorov spectrum, no entropy contribution, Gaussian $\chi_k(\omega)$.
- Entropy contribution, Kolmogorv spectrum, Gaussian $\chi_k(\omega)$.
- blue x 20

- Turbulent spectrum from a solar 3D simulation, Gaussian $\chi_k(\omega)$.