Observing and modelling stellar magnetic fields. 1. Basics

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Stellar Magnetic Fields Introduction

- Do stars have magnetic fields?
- Does it matter if they do?
- This workshop will explore how magnetic fields may influence stellar structure, star formation and evolution
- As a first step, we look at the question of how to observe magnetic fields in stars
 - How to discover if they are present
 - What information we can obtain about field strength, structure, and related phenomena at the surface of the star from observations

Topics of my lectures

- To observe stellar magnetic fields, and to interpret or model observations, there are several kinds of background that need to be discussed
 - Behaviour of atoms in a magnetic field
 - Structural effects of magnetic field on stellar atmosphere
 - Radiative transfer in presence of magnetic field
 - Techniques to solve inverse problems
- Then, with these tools, what are some of the observed characteristics of stellar magnetism?
- What hints do these observations give us about the origin, nature, and importance of magnetic fields in stellar evolution?

Atom in a magnetic field

 Consider the Hamiltonian of an atom in a magnetic field (Gaussian cgs units; atom in L-S coupling)

$$H = -\frac{\hbar}{2\mathrm{m}}\nabla^2 + V(r) + \xi(r)\boldsymbol{L}\cdot\boldsymbol{S} + \left[-\frac{e}{2\mathrm{mc}}\boldsymbol{B}\cdot(\boldsymbol{L}+2\boldsymbol{S}) + \frac{e^2}{8\mathrm{mc}^2}\boldsymbol{B}^2r^2\sin^2\theta\right]$$

- First 3 terms are kinetic energy, electronic potential, spin-orbit coupling with $\xi(r) = (1/2m^2c^2)(1/r)(dV/dr)$
- Last two terms are magnetic energy terms derived from magnetic vector potential
- For fields up to B~10 MG (1 kT), magnetic terms are small compared to Coulomb potential. Fine structure and field treated by perturbation theory

Magnetic field regimes

Here is that Hamiltonian again

$$H = -\frac{\hbar}{2\mathrm{m}} \nabla^2 + V(r) + \xi(r) \mathbf{L} \cdot \mathbf{S} + \left[-\frac{e}{2\mathrm{mc}} \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S}) + \frac{e^2}{8\mathrm{mc}^2} B^2 r^2 \sin^2 \theta \right]$$

(Perturbation theory) regimes:

- Quadratic magnetic term << linear term << spin-orbit term: (linear) Zeeman effect
- Quadratic magnetic term << linear term and spin-orbit term << linear term: Paschen-Back effect
- Quadratic magnetic term >> linear term and quadratic term >> spin-orbit term: quadratic Zeeman effect
- See Schiff 1955, Quantum Mechanics, Secs 23 & 39

Fields of various regimes

- May estimate size of magnetic terms by taking L~ħ, r~Bohr radius a₀, V~Ze/r. We find
 - For normal atoms and B < 50 kG (5 T), most atomic lines are in Zeeman regime
 - Fine structure splitting varies greatly from one level to another, so a few lines may be in Pachen-Back limit at much smaller B than normal (e.g. Fe II 6147 and 6149 A).
 Paschen-Back splitting occurs in H and Li below 30 kG
 - Above about 100 kG quadratic term becomes important.
 Quadratic Zeeman effect is observed in lines of H
 - Above about 10 MG magnetic terms become comparable to Coulomb term, perturbation methods no longer work. Must solve structure of atom in combined field

Zeeman effect

 In weak-field (Zeeman) limit, atomic energy level is only slightly perturbed by $(e/2mc) B \cdot (L+2S)$ In L-S coupling (light atoms), J and m, are good quantum numbers. Magnetic moment of atom is aligned with J. Energy shift of level proportional to $B \cdot J$, so there are 2J+1 different magnetic sublevels $E_i = E_{i0} + g_i \left(e/2mc \right) Bm_I \hbar$ where g_i is (dimensionless) Lande factor (in L-S): $g_i = 1 + [J(J+1) + S(S+1) - L(L+1)]/[2J(J+1)]$ Spectral lines arise as (allowed) differences between sublevels of lower and upper energy levels

Subcomponent separation due to Zeeman effect

$$E_{i} = E_{0i} + g_{i} \left(\frac{e}{2mc}\right) B m_{i} \hbar$$

$$v_{ij} = \frac{\left(E_{i} - E_{j}\right)}{h} = v_{0} + \left(\frac{e \hbar B}{2mch}\right) \left(g_{i} m_{i} - g_{j} m_{j}\right)$$

$$\lambda_{ij} = c / v_{ij} = \lambda_{0} + \Delta \lambda_{z} = \lambda_{0} + \left(\frac{e \lambda_{0}^{2} B}{4\pi mc^{2}}\right) \left(g_{j} m_{j} - g_{i} m_{i}\right)$$

$$\Delta \lambda_{z} \left(\mathring{A}\right) = 4.67 \, 10^{-13} \, \lambda_{0}^{2} B \left(g_{j} m_{j} - g_{i} m_{i}\right)$$

Note that splitting is (1) proportional to B and to λ², and (2) is about +/- 0.012 A (pi-sigma separation) for 5000 A and 1 kG

Example: Zeeman line components

Zeeman components for sodium D lines

green: pi components, red & blue: sigma components



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Zeeman patterns

- <u>Allowed</u> transitions have $\Delta m_J = 0$ (π), -1 or +1 (σ). Only some combinations of sublevels produce lines!
- If spacing of upper sublevels is same as spacing of lower sublevels, only three line components appear ("normal Zeeman effect")
- Usually spacing of upper sublevels is different from spacing of lower sublevels, and several lines of each of $\Delta m_1 = 0, -1$ or
 - +1 are present ("anomalous Zeeman effect")
- A few transitions have no splitting ("null lines")
- Splitting of each level determined by g_i values. Best values are from experiments (see Moore, Atomic Energy Levels, NBS Publication 467), or from atomic energy level calculations, but L-S coupling values are often good

Paschen-Back Effect

- This regime has few astronomical applications most fields in non-degenerate stars are too weak to push lines into Paschen-Back range
- A few pairs of levels have very small fine-structure separation, and their Zeeman patterns are distorted by "partial" Paschen-Back effect, e.g. Fe II 6147-49 A
- In Paschen-Back regime, **L** and **S** decouple, so *J* is not a good quantum number. Now m_L and m_s become good quantum numbers, so perturbation energy becomes $(e/2\text{mc}) B(m_L + 2m_s)\hbar$
- With this perturbation, all lines are split by same amount. Only three components ($\Delta m = -1, 0, 1$) are seen

Zeeman and Paschen-Back splitting

SPLITTING OF SPECTRAL LINES IN SODIUM



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Polarisation and properties of Zeeman components

- Another property of Zeeman line components that turns out to be very important in practice is their *polarisation*
- "Natural" light may be thought of classically as made up of short wave trains that have no particular phase or plane of polarisation
- Linearly polarised light has the electric field vector confined to a single plane (left figure)
- Circularly polarised light has an electric field vector which at any instant is spiral in form (right)





Polarisation of Zeeman components

Polarisation of Zeeman line components depends on orientation of field to line of sight
For field transverse to line of sight, Δm_j = 0 (π)

components in an *emission* line source are polarised linearly, parallel to field lines, while $\Delta m_j =$ -1 or +1 (σ) components are perpendicular to field • For field parallel to the line of sight, $\Delta m_j = 0$ components *vanish*, while the $\Delta m_j = -1$ or +1 components have opposite circular polarisations • Intermediate field orientations: elliptical polarisation

Can we detect & measure stellar fields with the physics discussed so far?

- First discovery of field in a sunspot by Hale (ApJ 1908)
- Hale observed splitting of spectral lines in spectra of Sun obtained with long slit covering a sunspot
- Guessed that splitting is due to Zeeman effect
- Verified that effect is magnetic by demonstrating (with circular polarisation analyser) that line components are circularly polarised
- From separation of line components, estimate that field of sunspot is about 3000 G
- In the first detection of a stellar magnetic field, polarisation properties of Zeeman effect played a crucial role
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Hale's detection of sunspot field

Left image: view of sunspot on slit plane (slit is vertical line)
Right: spectrum (dispersed horizontally) with Zeeman splitting at location of spot



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Sunspot magnetism

Hale's discovery opened study of sunspot fields

- Sunspots typically have fields of 1- 3 kG
- Sunspots tend to come in pairs of opposite polarity
- In one hemisphere, north polarity usually leads and south trails, while in opposite hemisphere polarities are reversed
- During (roughly) 11-yr sunspot cycle, spots are formed closer and closer to equator, finally die out
- When new cycle starts, leading north polarity has moved to the opposite hemisphere: real sunspot cycle is 22 yr
- Sunspots regions are sites of most intense solar activity flares, filaments & prominences
- Solar activity generally follows number and size of sunspots

Hale's Polarity Law:

The polarity of the leading spots in one hemisphere is opposite that of the leading spots in the other hemisphere and the polarities reverse from one cycle to the next.



Cycle 21

Cycle 22

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Variations of sunspot latitude and number during sunspot cycle





Detection of fields in other stars

- For nearly 40 years, no further fields found. Why?
- Sunspot fields could be detected because
 - Sun is extremely bright: high dispersion possible
 - Sunspot spectral lines not broadened by rotation
 - Sunspot fields are quite strong
- Zeeman splitting in stars (typically 0.01 A per kG) can be masked by even small v sin i of a few km/s
- Zeeman splitting had not been found in any stars
- To improve chances of detection, H W Babcock exploited polarisation of Zeeman effect, using high dispersion spectrograph of Mt Wilson 100-in telescope

Polarisation in stellar spectral lines

- In figure, left column is for field along line of sight, right is for field transverse to sight line
- Top row shows splitting of some line in lab
- Next row shows effects on unpolarised absorption line – little change in shape



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Polarisation in stellar spectral lines

- Third row: absorption in left and right circular polarisation (left), two orthogonal linear polarisation directions (right)
- Bottom row is difference spectra (net polarisation vs wavelength) – note large (1st order) effect in circular polarisation



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Polarisation analysis of spectra

- Babcock searched for magnetic fields by obtaining circularly analysed spectra of various kinds of sharp-line stars
 - Used a quarter-wave plate followed by a calcite block (in effect a linear polarisation beamsplitting analyser) to obtain two circularly polarised spectra side by side.
 - Detected significant Zeeman splitting among "peculiar A" stars
 - He thought he found fields in other (sharp-line) main sequence stars, but these have not been confirmed

Calcite block and quarter waveplate





FIG. 2.—a and b are double spectra of the magnetic variable 53 Cam (No. 28), taken near the phases of maximum negative and positive field intensity, respectively. The two stellar spectra on each plate are taken through the differential analyzer for right- and lefthanded polarization. Note the differential offsets of the lines, which are proportional to the field intensity, and which are of opposite sign for a and b. The lines (note particularly Eu II λ 4205) are much sharper in b. The double spectrogram c of the ultra-sharp-line, weak-field, "manganese star" ι CrB (No. 57) illustrates the sharpness of the lines (w=0.07). The resolution of Mg II λ 4481 into its two components about 0.2 A apart in the spectrum of ι CrB is well shown in d, which is enlarged about sixty times from the original plate.

Field measurements in Ap stars simple approximations

- Objective is to deduce some value of magnetic field strength from spectrophotometry
- Two simple approximations have been used for many measurements:
 - Weak line limit
 - Weak field limit
- Weak line limit is approximately correct for most spectral lines, although it leads to somewhat different results from strong and weak lines
- Weak field limit is good up to about 1 kG for most metal lines, or up to at least 10 kG for Balmer lines

Weak line approximation

In an optically thin gas, the separation between the position of the spectral line as seen in right and in left circularly polarised light is given by the mean separation of the sigma components multiplied by the cosine of the angle θ between the field and the line of sight

 $2\Delta\lambda_{Z}(A) = 2 \times 4.6710^{-13} \lambda_{0}^{2} z B \cos\theta$

where z is a suitable average Lande factor

 The weak line approximation consists of assuming that this is correct for spectral lines even if they are strong enough to be saturated

Field measurement in Ap stars weak field limit

 If the field is very weak, so splitting is small compared to (local) line width, the separation of the two circularly analysed components is as shown below. A measurement of net circular polarisation V in the line wing yields a field estimate through $V(\lambda) \approx 4.67 \, 10^{-13} \, z \, \lambda^2 (dI/d \, \lambda) \langle B_z \rangle$ where all wavelengths $I(\lambda) + \frac{dI}{d\lambda} 2\Delta\lambda$ are measured in A. $I(\lambda)$ 2Δλ (cf Landstreet 1982, ApJ 258, 639; Bagnulo et al 0 2002, A&A 389, 191) λο WAVELENGTH

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Typical observed characteristics of a magnetic Ap star

- Mean longitudinal field varies periodically, almost always approximately sinusoidally
- Usually the observed field reverses sign
- The star also usually varies in light (+/- a few 0.01 mag), and spectral line shapes and strengths vary, with same period as <B_>
- The period may be anywhere from 0.5 d to many years, and is inversely correlated with v sin i (large v sin i <=> short period)
- Clearly variations are due to rotation, and show that the star is not axisymmetric

Typical Ap stars variations (HD 184927)

Top: <B_z
 variations
 Middle: He line strength (equivalent width)

 Bottom: brighness variations in Stromgren u band



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Basic model of magnetic Ap the oblique (dipole) rotator

- Sinusoidal variations of <B_z> (including sign reversal) suggest that we are seeing rotation of a roughly dipolar field, inclined to the rotation axis of the star
- As star rotates we usually see first one pole and then the other
- Spectrum variations suggest that chemical composition of atmosphere varies over the surface: the star has "abundance patches"
- Abundance patches are probable cause of light variations

Oblique rotator

- Arrows show field vectors of roughly axisymmetric field
- Coloured bands could show abundance variations over surface
- Star rotates about axis not aligned with dipole axis



Parameter choice for simplest model

- Simplest model requires us to specify inclination of rotation axis *i*, "obliquity" β of field axis to rotation axis, and polar field strength of dipole.
- Observations only provide two constraints (say, largest and smallest values of <B_z>), so the model will only be unique if we can get *i* from *v* sin *i* and stellar radius
- If we can measure average magnitude of field on stellar surface ("mean field modulus") as well as longitudinal field component, we get two more constraints and can fit a 4-parameter model

Measuring the mean field modulus

- If star has small rotation velocity (less than few km/s) and a large field (some kG), the Zeeman splitting of (at least some) spectral lines may be measurable
- Example of effect in HD 94660 (Mathys)
- Splitting is proportional to total magnitude of field, so we obtain a surface-averaged value of
- This also usually varies roughly sinusoidally
- With both <B_z > and we can fit i, β, and field strength at two poles (e.g. with a decentred dipole, or dipole + quadrupole) without detailed computations of line profiles

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Zeeman splitting in HD 94660



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Better Ap field models

Results of such measurements and modelling:

- Clean splitting of spectral lines shows that field is fairly homogeneous over surface, not spotty as on sun
- Typically larges value of |<B_z>| is about /3 or /4, showing that field is not much more complicated topologically than a dipole

- Fits to variations of $\langle B_2 \rangle$ and $\langle B \rangle$ are reasonably good

 Show that in general the two magnetic poles are not equally strong, which is probably connected with the (at first surprising) observation that variable spectral line strengths are not similar at the two extrema of <B_z > even if these are equal and opposite. If polar fields are different, chemistry could be too.

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Fit to <Bz> and in HD 187474

 Fit of simple dipole plus quadrupole model (lines) to observed variations of <B_> (top left) and (lower right) (cf Landstreet & Mathys 2000, A&A 359, 213)



Summary

- The atomic physics of the Zeeman effect has provided us with tools to detect magnetic fields in stars.
- Such fields have been known for many years on the Sun and on Ap stars
- The solar field is strongly structured on a small length scale, and variable on a short time scale.
- In contrast, the fields of the Ap stars seem to be structured rather simply, and are constant in time except for the changing appearance due to stellar rotation.