Observing and modelling stellar magnetic fields. 2. Models

John D Landstreet Department of Physics & Astronomy University of Western Ontario London, Canada West

In the previous episode....

- We explored the splitting of energy levels and spectral lines by the Zeeman (and Paschen-Back) effect, and found a number of useful tools
- We looked at methods of extracting useful estimates of longitudinal field averaged over the visible stellar

hemisphere from circularly polarised spectra of magnetic Ap stars, and we saw that in cases of large enough field and small enough v sin i we could also estimate the mean value over the hemisphere of

- Finally we looked at simple multipole field models that could reproduce the (limited) data from <B_z > and observations over a stellar rotation
- Now we consider a much more powerful modelling method.

Why build models of spectra??

- Some information is available "directly" from spectrum (e.g. radial velocity, v sin i, presence of Fe or Eu, hemispheric average magnetic field strength).
- Simple models can be made to fit these simple data.
- Beyond simple results, modelling is needed for
 - quantitative information, such as abundance of various chemical species, or distribution of magnetic field over surface
 - To test ideas about what is present on (or near) an unresolved star
- Basic idea: predict spectrum corresponding to hypothesis, and compare prediction with observed spectrum.
- This process is typically iterated to convergence if possible!
- Discrepancies provide hints about missing physics in the model!

Basic process of (forward) spectrum modelling

- Two essential components are required:
 - structure of outer layers of star from which radiation can escape (a "model atmosphere" - T(r), p(r), etc)
 - outgoing radiation field, usually described by "specific intensity" $I_{v}(\theta)$ of radiation (units: ergs/s-cm²-Hz-sterad)
- Structure is computed for most stars assuming that atmosphere is "thin" (plane-parallel), horizontally uniform, in hydrostatic equilibrium, and in a thermal steady state between hot stellar interior and cold exterior space
- Radiation emitted is computed from atmosphere model using "equation of radiative transfer"

Equation of radiative transfer: a simple introduction

- Most complex and obscure element of this modelling process is equation of radiative transfer
- We start by looking at a very simple form of this equation, to build intuition, and then extend it to polarised radiation
- Basic idea is to look at specific intensity $I_{v}(z, \omega)$ of radiation

passing through small volume of gas in a specific direction, and compute changes due to absorption and emission:

 $dI_v = (-\kappa_v I_v + j_v) ds$ where κ_v is the absorption coefficient (cm²/cm³) and j_v is the emissivity per unit volume (ergs/s-cm³-Hz-sterad)

EOT: simple introduction (2)

• Consider a stellar atmosphere with *ds* at an angle θ to the vertical, so $ds = dz/\cos \theta = dz/\mu$. Now define (monochromatic) optical depth $d\tau_{\mu} = -\kappa_{\mu} dz$. Then

$$\mu \frac{dI_{\nu}}{d\tau_{\nu}} = I_{\nu} - \frac{j_{\nu}}{\kappa_{\nu}} = I_{\nu} - S_{\nu}$$

This is the equation of transfer (EOT).

- Some important points
 - This form ignores all scattering processes
 - In this approximation, the EOT describes the variation of radiation along a single ray. Each ray is independent of all others, and each has its own EOT
 - If we start a ray deep inside an object and follow it to the surface, this equation predicts the emerging light $I_{ij}(0, \omega)$

EOT: simple introduction (3)

- In "local thermodynamic equilibrium" (LTE), S_v can be approximated as the Planck function $B_v(T)$.
- If we multiply the equation of transfer by the integrating factor $exp(-\tau_v/\mu)$, and assume that S_v is a known function of τ_v , the EOT can be integrated.

$$I_{\nu}(\tau_{\nu}) = e^{\tau_{\nu}/\mu} \int_{\tau_{\nu}}^{\infty} S_{\nu}(\tau_{\nu}') e^{-\tau_{\nu}'/\mu} d\tau_{\nu}'/\mu$$

- With lower limit 0, we get specific intensity emerging from surface.
- Now suppose that $S_v(\tau_c) = S_o(1 + \beta \tau_c)$, for τ_c of a particular (continuum) frequency and nearby frequencies. Integrating:

$$I_{c}(0) = S_{0}(1 + \beta \mu) = S_{c}(\tau_{c} = \mu)$$

EOT: simple introduction (4)

- Now suppose that $\kappa_v = \kappa_c + \kappa_{line} = (1 + \eta_v) \kappa_c$ where η_v is a function of frequency but not depth.
- Then $\tau_{\nu} = (1+\eta_{\nu})\tau_{c}$ and $S_{\nu}(\tau_{\nu}) = S_{0}[1+\beta \mu \tau_{\nu}/(1+\eta_{\nu})]$ so finally we get an expression for surface intensity $I_{\nu}(0) = S_{0}[1+\beta \mu/(1+\eta_{\nu})]$
- This shows that if we know how to write the line absorption coefficient (normally as a Voigt profile), then we see that as we get near line centre the emergent intensity drops, while in the nearby continuum it has the continuum value found above.
- The point: in this particular case the EOT is a solvable first order, linear DE with a driving term. (Of course, there are a lot of more complex situations!)

The Stokes vector

- Now to described *polarised* light mathematically we use the Stokes parameter description: [I, Q, U, V]
- I is the total intensity of light in the beam
- For Q and U, measure the intensity of the beam through perfect linear polarisers (polarising analysers) orientated at 0, 45, 90, and 135 degrees. $Q = I_0 I_{90}$, $U = I_{45} I_{135}$.
- Measure the intensity of the beam thorough two perfect circular polarisers. $V = I_{right} I_{left}$.
- These four quantities adequately describe the polarisation state of a light beam
- [I, Q, U, V] are functions of frequency and direction
- Q, U, V are sometimes normalised to I (Q -> Q/I, etc).

Equation of radiative transfer polarised radiation

$$\mu \frac{dI}{d\tau_{v}} = \eta_{I} (I - B_{v}) + \eta_{Q} Q + \eta_{V} V$$

$$\mu \frac{dQ}{d\tau_{v}} = \eta_{Q} (I - B_{v}) + \eta_{I} Q - \rho_{R} U$$

$$\mu \frac{dU}{d\tau_{v}} = \rho_{R} Q + \eta_{I} U - \rho_{W} V$$

$$\mu \frac{dV}{d\tau_{v}} = \eta_{V} (I - B_{v}) + \rho_{W} U + \eta_{I} V$$

- For polarised radiative transfer, EOTs are written in terms of Stokes parameters
- First order linear DEs like 1D EOT, but now four coupled equations, one for each Stokes parameter
- No scattering is included
 In LTE, B is Planck function
- Factors η describe absorption, factors ρ describe retardation (anomalous dispersion)

Coupling coefficients in EOTs

• Define η_p , η_l , η_r as the ratios of total (Voigt line profiles + continuum) opacity, in pi, right sigma, and left sigma Zeeman components to the continuum opacity. Then

$$\eta_I = 0.5 \eta_p \sin^2 \psi + 0.25 (\eta_l + \eta_r) (1 + \cos^2 \psi)$$

$$\eta_Q = 0.5 \eta_p - 0.25 (\eta_l + \eta_r) \sin^2 \psi$$

 $\eta_V = (\eta_r - \eta_l) \cos \psi$

Projection of field defines vertical (Q), and ψ is the angle between field and vertical.

- Similar expressions involving Faraday-Voigt function are required for the retardance terms
- Note that η_{Q} and η_{V} are differences of Zeeman opacity coefficients, like Q and U. They act to introduce polarisation.

Comments on polarised EOTs

- For given atmospheric structure, we can start the 4 polarised EOTs with unpolarised black-body radiation at great depth, follow many rays (many directions, frequencies) to surface and compute (discrete approximation of) emergent flux
- Equations describe effects of polarised absorption and retardation on outflowing radiation
- For non-degenerate stars, polarisation is introduced essentially by polarised Zeeman line components
- Cannot accurately predict intensity or polarisation of emergent stellar spectrum without solving these equations – even intensity spectrum is substantially different from what is computed with a single unpolarised equation of transfer

Analytical solutions

- As for the unpolarised equation of transport, there is an analytic solution to the simplest case of the polarised equations assuming a linear source function and a normal Zeeman triplet (worth playing with to build intuition)
- Simplest form was derived by Unno (1956, PASJ 8, 108) in paper which laid the basis for setting up EOT in terms of Stokes parameters. Note that this paper ignores retardation.

$$\frac{I_0(0,\theta) - I(0,\theta)}{I_0(0,\theta)} = \frac{\beta \cos \theta}{1 + \beta \cos \theta} \left[1 - \frac{1 + \eta_I}{(1 + \eta_I)^2 - \eta_Q^2 - \eta_V^2} \right]$$
$$\frac{V(0,\theta)}{I_0}(0,\theta) = \frac{-\beta \cos \theta}{1 + \beta \cos \theta} \left[\frac{\eta_V}{(1 + \eta_I)^2 - \eta_Q^2 - \eta_V^2} \right]$$

See also Martin & Wickramasinghe 1979, MNRAS 189, 883
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Building a model atmosphere

- Same EOT (or EOTs) are used for building model atmosphere. Usual requirements imposed
 - Hydrostatic equilibrium, one-dimensional (flat)
 - Scattering is ignored, or treated as absorption
 - LTE (source function is Planck function everywhere)
 - Integrated flux is constant through atmosphere (although detailed frequency distribution changes from level to level; this determines temperature structure
 - (At small τ_c where flux not affected much by absorption, local conditions are computed using energy balance)
 - Results depend strongly on good treatment of continuum and line opacity, and on abundance table assumed

Available atmosphere codes & models

- Model atmospheres may be computed using either unpolarised or polarised transfer.
- Until recently, only unpolarised model atmospheres were available
 - Hot stars: ATLAS (cf http://kurucz.harvard.edu/, especially grids and programs)
 - Cool stars: MARCS models (B Gustafsson et al; see also http://www.uwosh.edu/mike/exercises/marcs/marcs.html)
 - Wide range: Phoenix models (P Hauschildt et al)
- D Shulyak and S Khan have recently developed LLModels, a code for hot stars that can compute models including magnetic polarisation effects and arbitrary abundance tables
- It mostly seems to be an acceptable approximation to use unpolarised model atmospheres, or ones with simple line splitting, but the appropriate abundance table has important effects.

Computing an emergent spectrum: what is needed

- Given a suitable model atmosphere (T_{eff}, log g, abundances), what do we have to do to compute the emergent polarised spectrum of a magnetic star (forward computation)?
 - Assume some magnetic field structure, and calculate the vector field at many (50+) grid points on the visible hemisphere
 - For each grid point compute detailed run of polarised opacities and retardances at all relevant depths (60+) at closely spaced frequencies or wavelengths (0.01 Å is barely adequate wavelength grid in visible). A window of 100 Å might be a useful size.
 - Then compute the emergent spectrum along the ray towards observer at each surface grid point, by solving the EOTs outward along the ray, for each wavelength in the window
- A lot of bookkeeping is required!

Computing the emergent spectrum techniques

- Several aspects of this problem require special considerations
 - Need to have a suitable list of spectral lines, with *gf* values and Landé splitting factors. Usual source is VALD database at http://ams.astro.univie.ac.at/vald/ which supplies both data from a variety of sources (some are better than others...)
 - Because one must compute the Voigt and Faraday-Voigt functions millions of times, an efficient algorithm is needed
 - Solving the equations of transfer numerically may be done with standard packages or methods (e.g. Runge-Kutta), but again this has to be done so many times efficiency is essential, and you want a technique that does not require a very dense depth grid
- Descriptions of common codes discuss these points

Example of a spectrum synthesis code: Zeeman

- Zeeman (Landstreet 1988, ApJ 326, 967) is a simple magnetic line synthesis code
 - Contains simple parametrised field structures (colinear dipole, quadrupole, etc), also abundance tables specified on rings colinear with field axis (required for Ap stars)
 - Reads in fundamental parameters of star, assumed magnetic field parameters, spectral window to compute
 - Computes I, Q, U, V spectrum including line splitting and polarised transfer, using precomputed ATLAS atmosphere and VALD line list
 - Compares computed spectrum with an observed spectrum, selects best fit v sin *i*, radial velocity, and χ^2 of fit
 - If desired, can iterate fit to optimise field or abundance parameters

Examples of magnetic line profiles

- Example of line synthesis
 - Cr II 4588 in A0 star
 - Dipole field, polar field strength 1000 G (0.1 T)
 - Star not rotating
 - View from four inclinations from magnetic pole: 0, 30 60, 90 degrees
 - Q, U, V all multiplied by
 10
- Note how much larger V is than Q or U



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Modelling a normal star: example

- Synthesis fits to non-magnetic stars may be very accurate
- Require good choices of T_{eff}, log g, abundances, radial velocity, v sin i, and microturbulence parameter
- T_{eff} and log g often chosen from available Stromgren or Geneva photometry calibrations
- Automated iterative fitting of most remaining parameters works well for such stars





Recent advances in observational capabilities

- Previous example was non-magnetic star, and fit was to simple / spectrum. To model magnetic stars by synthesis, we need more extensive data.
- Major advance during past 10 years has been development of facility instruments capable of high resolution spectropolarimetry in all four Stokes parameters
- Most important: MuSiCoS spectropolarimeter and its successors, ESPaDOnS and Narval, all due to J-F Donati.
- MuSiCoS provided spectra with R = 35000 for window 4600 6600 Å. Main limit was low efficiency, but provided wholly new types of data, provoked several major break-throughs.
- ESPaDOnS and Narval observe region 3700 to 10400 Å with R = 68000 and far higher efficiency.

ESPaDOnS at CFHT



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Least squares deconvolution (LSD)

- Donati et al (1997) have developed a very powerful tool for using spectropolarimetric data to detect weak fields even when polarisation signal is hardly visible: LSD
- Example of weak signal in faint cluster Ap star
 - Even in very strong Fe II line at 4923, V hardly detectable
 - When signals from thousands of lines are averaged, field signature easily visible
- Found that LSD V signal (but not Q,
 U) may be modelled like single line





Measurement of really weak fields

- The bright Ap star ε Uma shows power of LSD
- Although it is extremely bright (m_v = 1.85) it was really difficult to detect with single- or few-line techniques (large error bars on <B_z> curve at right)
- With LSD data from Musicos, the field is very obvious and the uncertainty in <B_z > decreases by about one order of magnitude (small error bars on <B_z > curve)





Modelling a magnetic star: tests of parametrised models

- Using MuSiCoS (or ESPaDOnS) data we can now return to problem of modelling magnetic stars
- We can test models derived from simple field measurements (<B₂> or

other field moments) by observing [I, Q, U, V] spectra as a function of rotational phase and then computing predicted line profiles using the model derived from moments (e.g. 53 Cam)

 Result: poor fits, especially to Q, U.... Simple field models from field average measurements are only first approximations to real structure



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Modelling a magnetic star: detailed models

- Solution is to develop mapping code that can fit spectra of all 4 Stokes parameters at many rotational phases by iterative adjustment of abundance and field maps
- Requires many cycles of forward computation, comparison, backwards feedback to improve maps, then through cycle again (Kochukhov et al 2004, A&A 414, 613)



Further results for 53 Cam



- Above: Fe distribution from 3 lines of multiplet 42 as function of rotational phase
- Right: magnetic field strength and orientation from same three lines



Modelling of cool stars

- Another code for mapping magnetic fields from spectropolarimetry of cool stars has been developed by Donati
- This has been used to map / and V LSD spectra of active cool stars, as in the map of HR 1099 to right (Petit et al 2004, MNRAS 348, 1175), where the observations at the bottom are the thin lines and the fitted model gives the bold lines



Figure 6. Same as Fig. 2 for the 2001 96 data set

Summary

- The point of this lecture is that computation of spectra of magnetic stars using a good underlying physical model is quite practical, and with sophisticated mapping techniques is beginning to yield detailed maps of both hot (well, tepid) and cool magnetic stars
- The first such maps reinforce the impression that magnetic Ap stars have fields that are really quite different from those of cool, solar-like stars.