Impact of magnetic fields on stellar structure and evolution

Jean-Paul Zahn

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Impact on stellar structure

Are magnetic fields strong enough to play a role in the structure of stars ?

main forces that rule hydrostatic equilibrium in stars:
pressure and gravity

$$\frac{P}{H_P} = \rho \frac{GM}{r^2}$$

sound speed in solar interior

$$c_s^2 \approx \frac{P}{\rho} \approx \frac{GM}{R} \quad \longrightarrow \quad c_s \approx 5\,10^7\ cm/s$$

magnetic force

$$\frac{\vec{j} \times \vec{B}}{c}$$

Lorentz, or rather Laplace ?

magnetic pressure is of same order as gas pressure
when Alfvén velocity equals sound speed :

$$c_A = \frac{B}{\sqrt{4\pi\rho}} \approx c_s \quad \longrightarrow \quad B \approx 2\,10^8 G = 2\,10^4 T$$

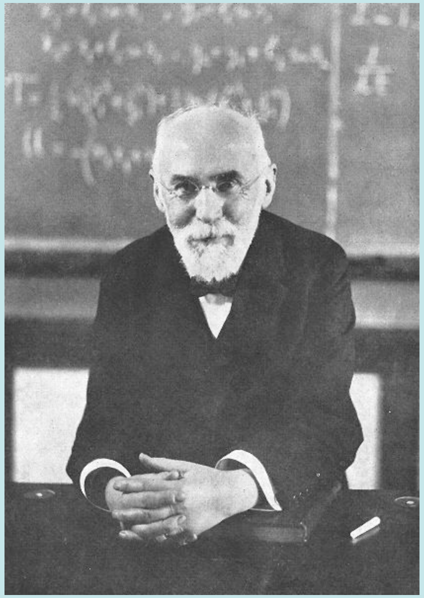
Laplace or Lorentz ?

Pierre-Simon Laplace
1749 - 1827

$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

force exerted by a magnetic field on
an element of electric courant

Hendrik Lorentz
1853 - 1928



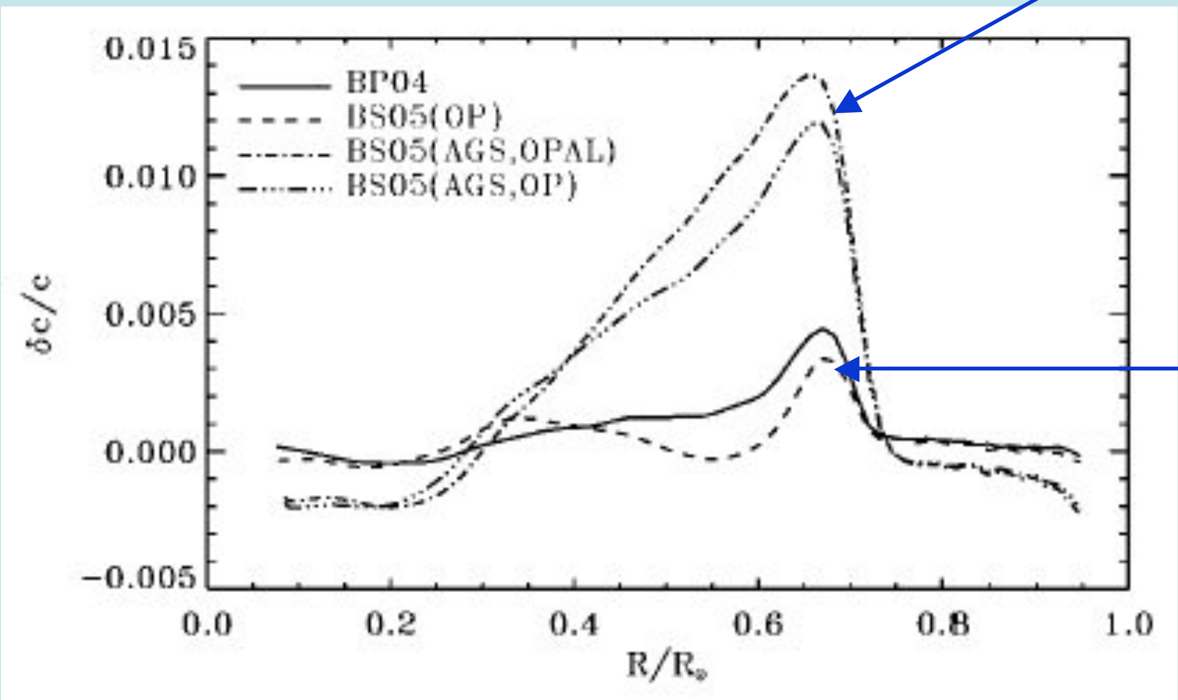
$$\vec{F} = q\vec{V} \times \vec{B}$$

force exerted by an magnetic field on
a moving charged particle

Can a field of MegaGauss strength exist in the Sun?

Models built without magnetic field
compared with helioseismic data

observed - model sound speed



new composition
(Asplund et al.)

due to a field of ≈ 3 MG ?

old composition
(Grevesse et al.)

Bahcall & coll. 2005

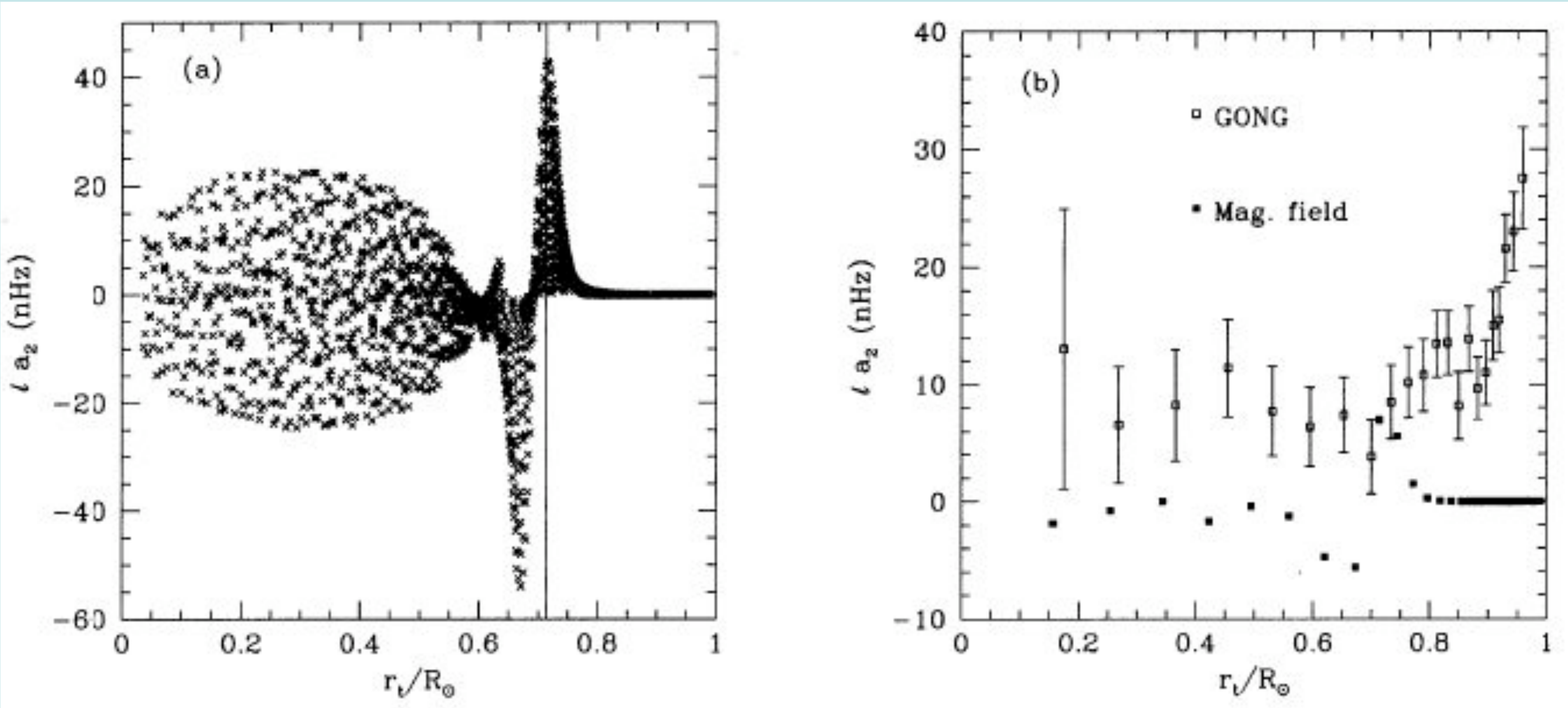
Can a field of MegaGauss strength exist in the Sun?

A closer look at helioseismology:
frequencies are split by mag. field

→ upper limit ≈ 300 kG

Splitting coefficients expected from a 4 MG toroidal field located in the tachocline (Δ=0.04 $R_\odot$)

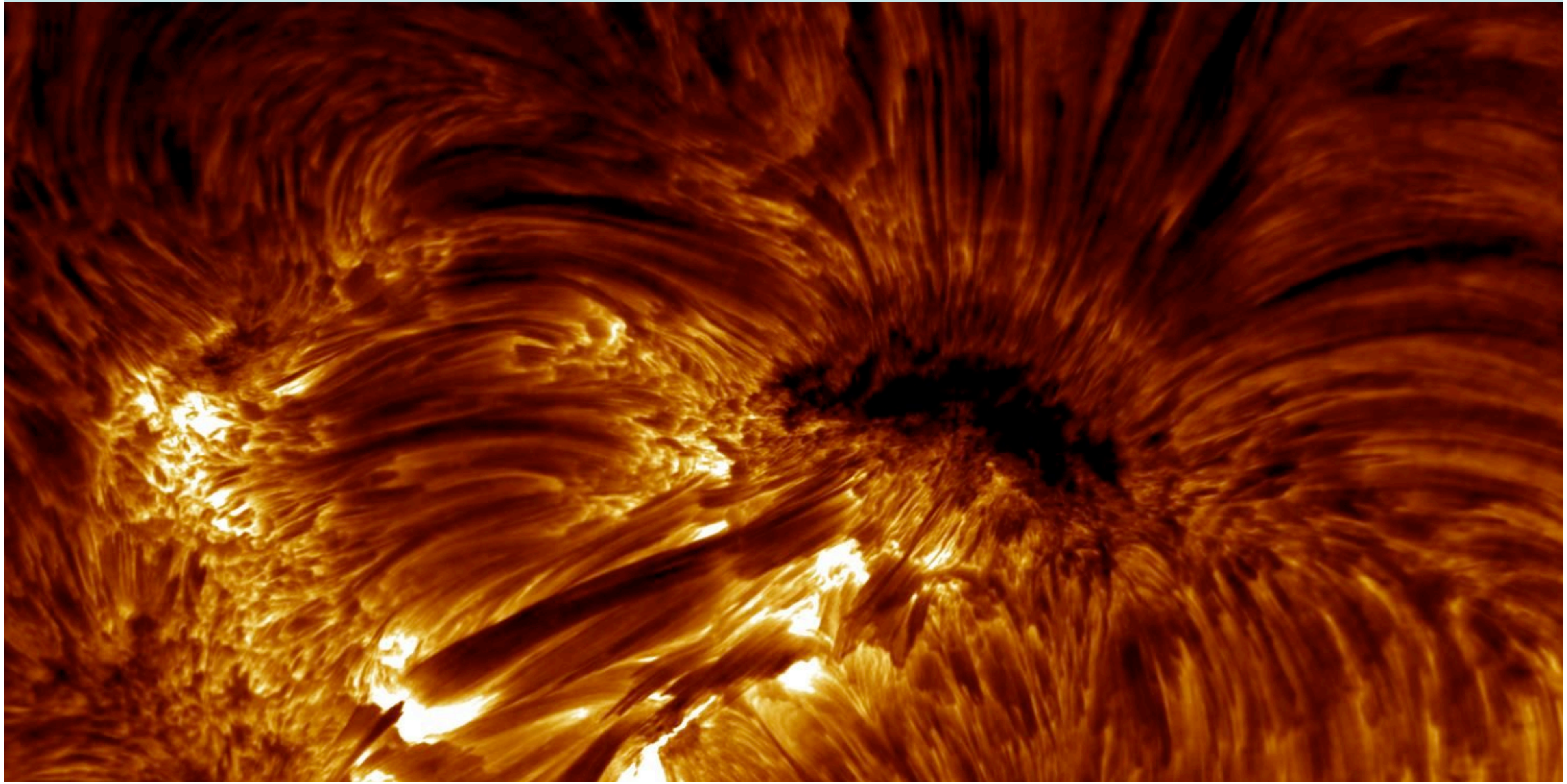
same, averaged over 30 neighbouring modes



Anita, Chitre & Thompson 2000

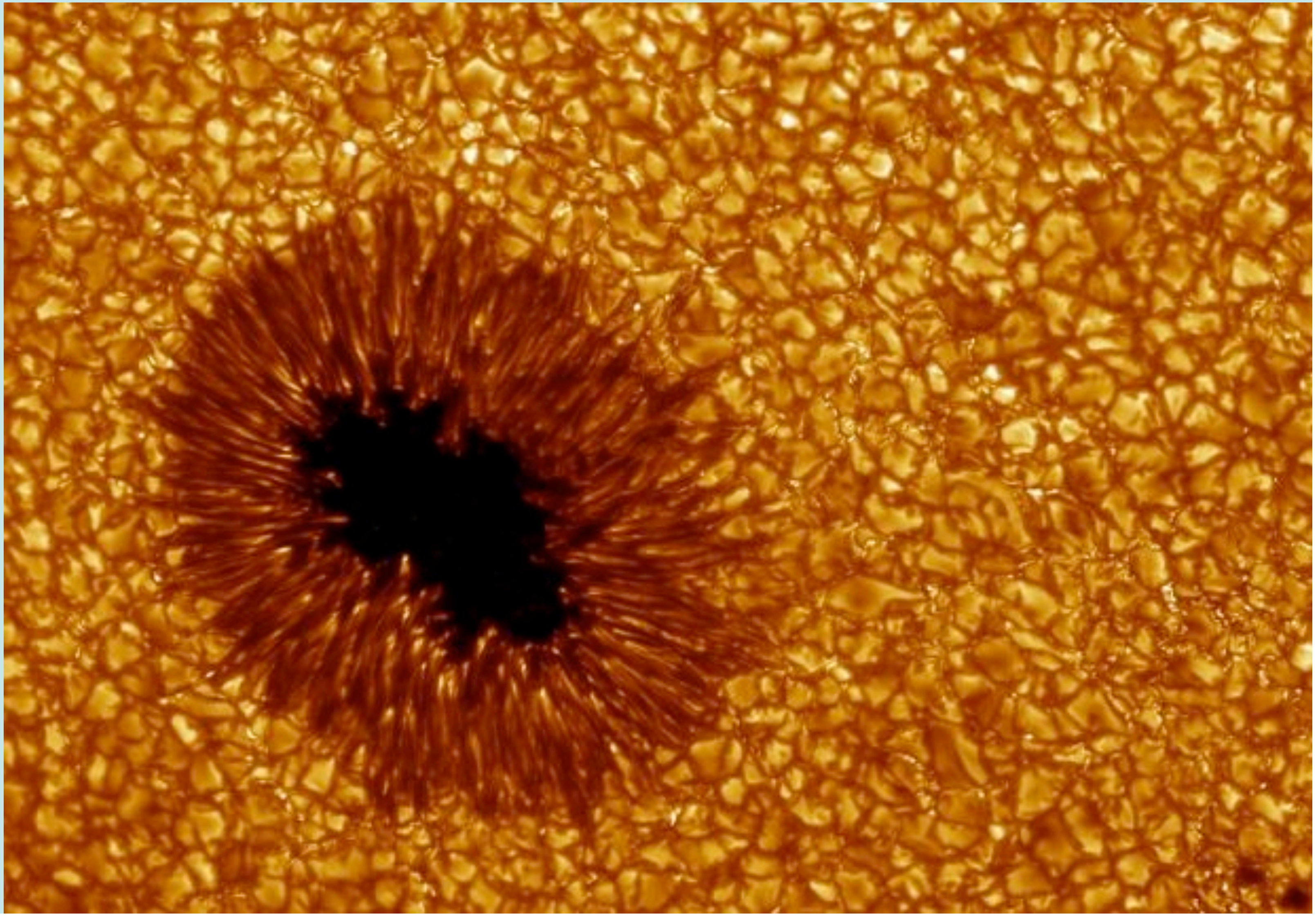
Magnetic pressure dominates gas presure in the surface layers

magnetic pressure: $P_m = \frac{B^2}{8\pi}$ gas pressure: $P_g = \frac{\mathrm{R}\rho T}{\mu}$



Magnetic fields interfer
with thermal convection

Strong magnetic fields block convective heat transport in sunspots



SST
La Palma

Strong magnetic fields suppress the convective instability

Linear instability in a unstably stratified magnetized medium

perturb by $\xi \propto \exp\left[s\,t + i\,\vec{k}\cdot\vec{x}\right]$

dispersion relation, neglecting thermal and Ohmic diffusion :

$$s^2 = \left(\frac{k_h}{k}\right)^2 \frac{g}{H_P}\left[\nabla - \nabla_{ad}\right] \quad - \quad \left(\vec{k}\cdot\vec{V}_A\right)^2$$

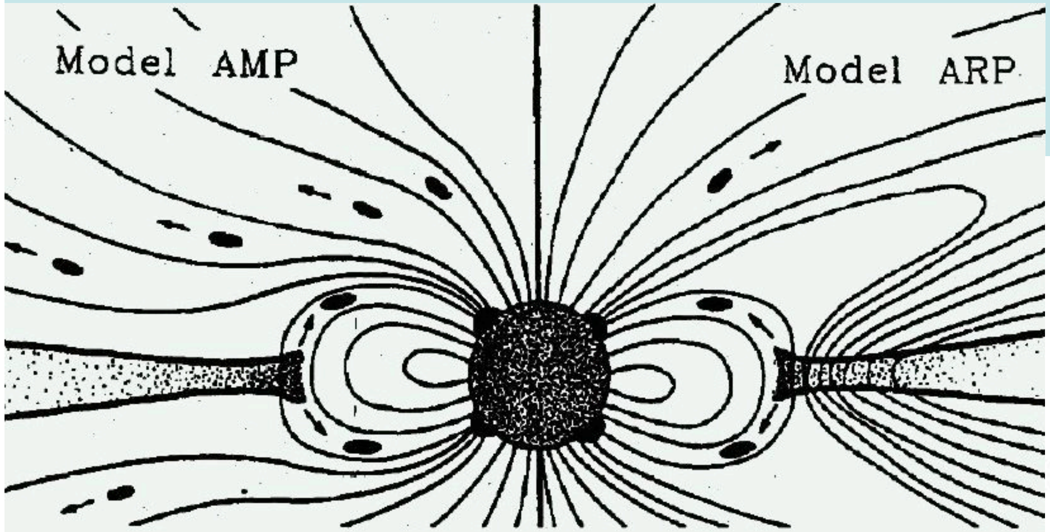
Chandrasekhar; Weiss 1960's

→ most unstable for horizontal wave-vector,
may be stabilized by sufficiently strong horizontal field
~ 10^7G below the solar CZ
~ 10^3G at surface

- explains why inhomogeneities in surface composition of Ap stars are not smooted out by convection
- displaces somewhat the boundary of CZ; effect on Li burning during PMS ?

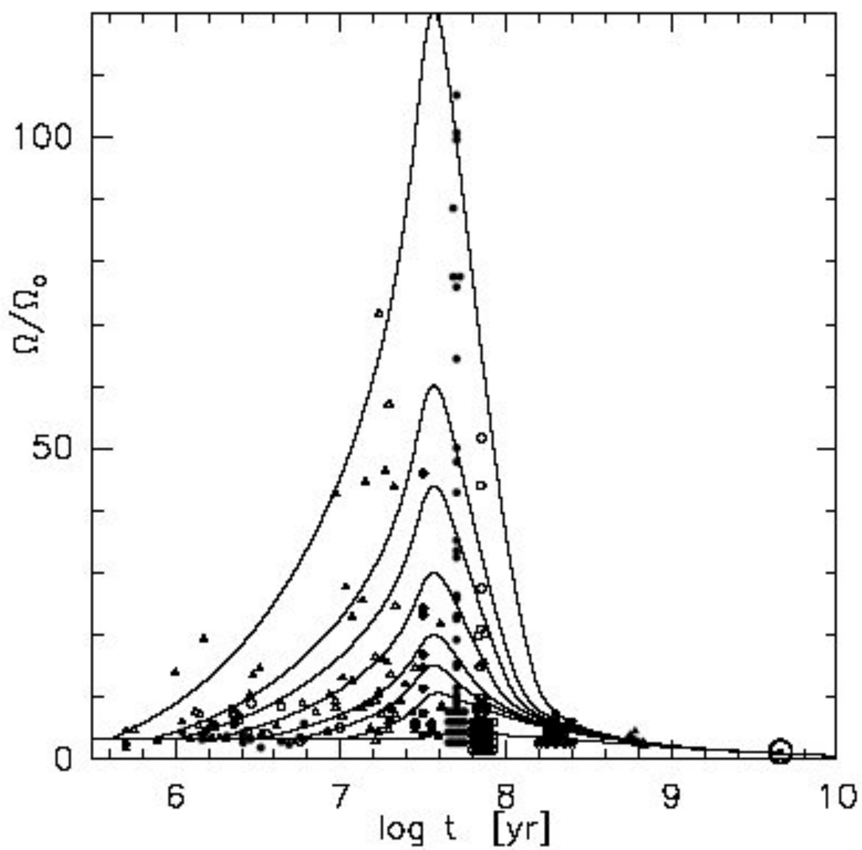
Magnetic fields couple stars to their environment

Magnetized star coupled to accretion disk



Fendt 1994

Bouvier et al. 1997



→ explains the relatively slow rotation of TT stars

Magnetized winds → strong angular momentum loss

If Sun loses matter at equator :

$$\frac{d}{dt} I\Omega = R^2\Omega \frac{d}{dt} M$$

$$\frac{d}{dt} k^2 M R^2 \Omega = R^2\Omega \frac{d}{dt} M$$

$$\frac{(R^2\Omega)_f}{(R^2\Omega)_i} = \left[\frac{M_f}{M_i}\right]^p$$

$$p = k^{-2} - 1 = 16$$

$$\left[\frac{M_f}{M_i}\right] = 0.99 \qquad \frac{(R^2\Omega)_f}{(R^2\Omega)_i} = 0.85$$

Fessenkov 1949 Schatzman 1954

but Sun loses matter at distance D (Alfvén radius) :

$$\frac{d}{dt} I\Omega = D^2\Omega \frac{d}{dt} M$$

$$\frac{d}{dt} k^2 M R^2 \Omega = D^2\Omega \frac{d}{dt} M$$

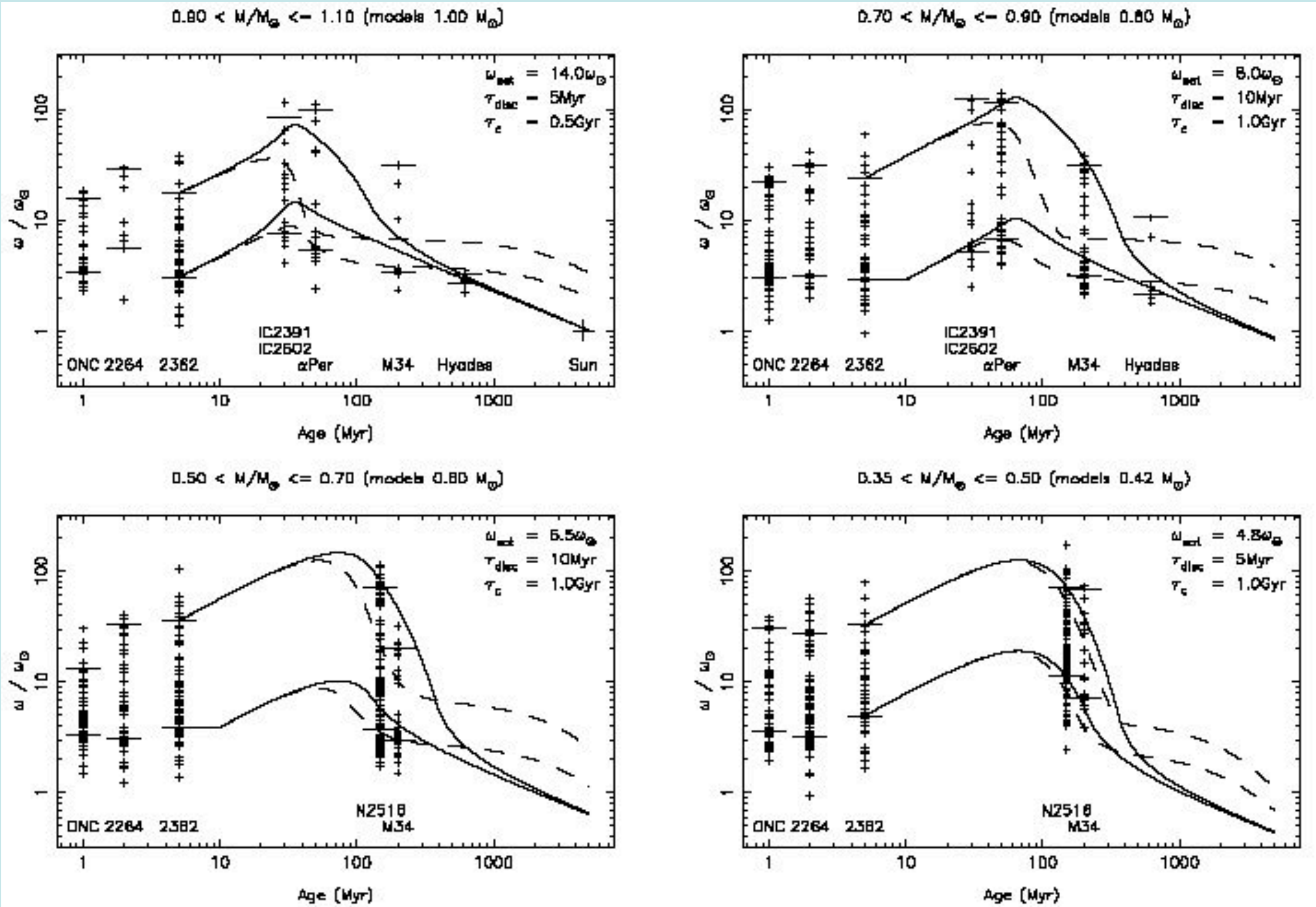
$$\frac{(R^2\Omega)_f}{(R^2\Omega)_i} = \left[\frac{M_f}{M_i}\right]^p \qquad D/R = 5$$

$$p = (D/R)^2 k^{-2} - 1 = 425$$

$$\left[\frac{M_f}{M_i}\right] = 0.99 \qquad \frac{(R^2\Omega)_f}{(R^2\Omega)_i} = 0.014$$

Schatzman 1962

Disc-coupling and mass loss by magnetized wind determine the rotation of stars



Monitor Project
(Irwin et al. 2006, 2007)

→ the young Sun was a fast rotator

Rotational mixing in radiation zones

Meridional circulation

Classical picture: circulation is due to thermal imbalance
caused by perturbing force (centrifugal, magn. field, etc.)
Eddington (1925), Vogt (1925), Sweet (1950), etc

Eddington-Sweet time $t_{ES} = t_{KH} \frac{GM}{\Omega^2 R^3}$ with $t_{KH} = \frac{GM^2}{RL}$

Revised picture: after a transient phase of about t_{ES},
circulation is driven by the loss (or gain) of angular momentum
and structural changes due to evolution
Busse (1981), JPZ (1992), Maeder & JPZ (1998)

- AM loss by wind: need to transport AM to surface → strong circulation
- no AM loss: no need to transport AM → weak circulation

shear-induced turbulence and internal gravity waves
contributes to AM transport

Rotational mixing in magnetized radiation zones

Transport of angular momentum

$$\rho \frac{d}{dt}\left(r^2 \sin^2\theta\,\Omega\right) = -\nabla\cdot\left(\rho r^2 \sin^2\theta\,\Omega\,\vec{U}\right) + \frac{\sin^2\theta}{r^2}\partial_r\left(\rho\nu_v r^4 \partial_r\Omega\right) - \nabla\cdot\left(\rho r^2 F_{IGW}\right) \quad + r\sin\theta\,\vec{e}_\phi\cdot\vec{L}$$

advection thru MC — turbulent diffusion — internal gravity waves — Laplace torque

Even a weak field can inhibit the transport of AM

$$B^2 > 4\pi\bar{\rho}\,\frac{R^2\Omega}{t_{AML}}$$

t_{AML}: characteristic time for AM loss

For $\bar{\rho} = 1\,g/cm^3 \quad R = 7\;10^{10}\,cm \quad R\Omega = 10^7\,cm/s \quad t_{AML} = 10^9\,yr$

$$\rightarrow \quad B_{crit} \approx 20\,G$$

But the exact figure depends sensitively on the topology of magnetic field

Rotational mixing in magnetized radiation zones

Evolution of an axisymmetric field

poloidal (meridian) field $\vec{B}_P = \nabla \times \vec{A}, \quad \vec{A} = A\vec{e}_\phi$

toroidal (azimuthal) field $\vec{B}_T = B_T \vec{e}_\phi$

$$\partial_t A + \frac{1}{s}\vec{U}\cdot\nabla(sA) = \eta\left(\nabla^2 A - \frac{A}{s^2}\right) \qquad s = r\sin\theta$$

advection diffusion

induction equations

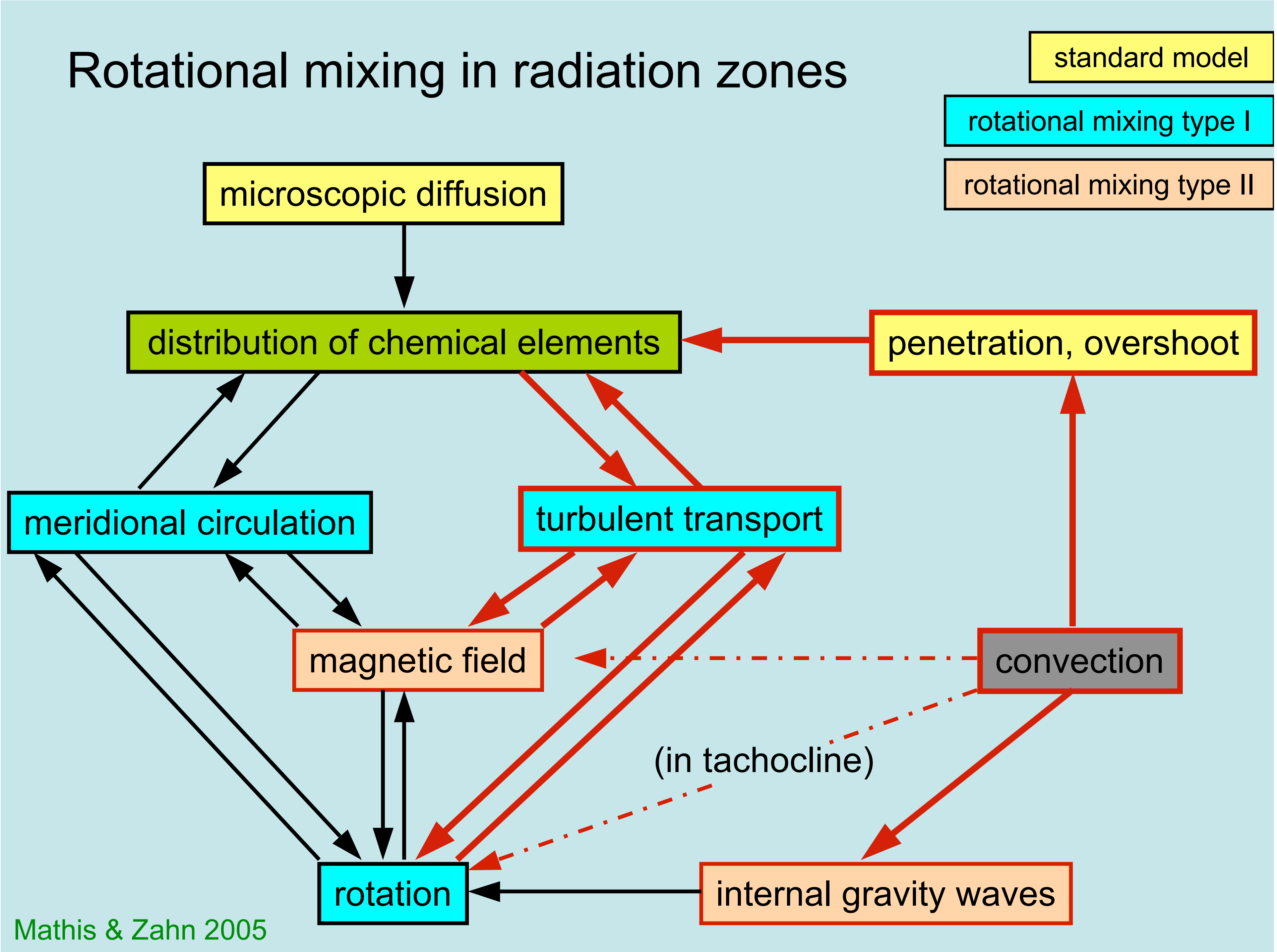
$$\partial_t B_T + s\vec{U}\cdot\nabla\left(\frac{B_T}{s}\right) = -B_T\nabla\cdot\vec{U} + s\vec{B}_P\cdot\nabla\Omega + \eta\left(\nabla^2 B_T - \frac{B_T}{s^2}\right)$$

advection stretching diffusion

Ω-effect → $s\vec{B}_P\cdot\nabla\Omega$

suppressed when Ω cst on field lines of B_P (Ferraro law)

2D equations are projected on spherical harmonics
to be implemented in stellar evolution codes (thesis S. Mathis)



Rotational mixing in radiation zones
standard model
rotational mixing type I
rotational mixing type II
microscopic diffusion
distribution of chemical elements
penetration, overshoot
meridional circulation
turbulent transport
magnetic field
convection
(in tachocline)
rotation
internal gravity waves
Mathis & Zahn 2005

The solar tachocline problem (Spiegel & Z 1992)

Differential rotation $\Omega(\theta)$ applied at top of RZ

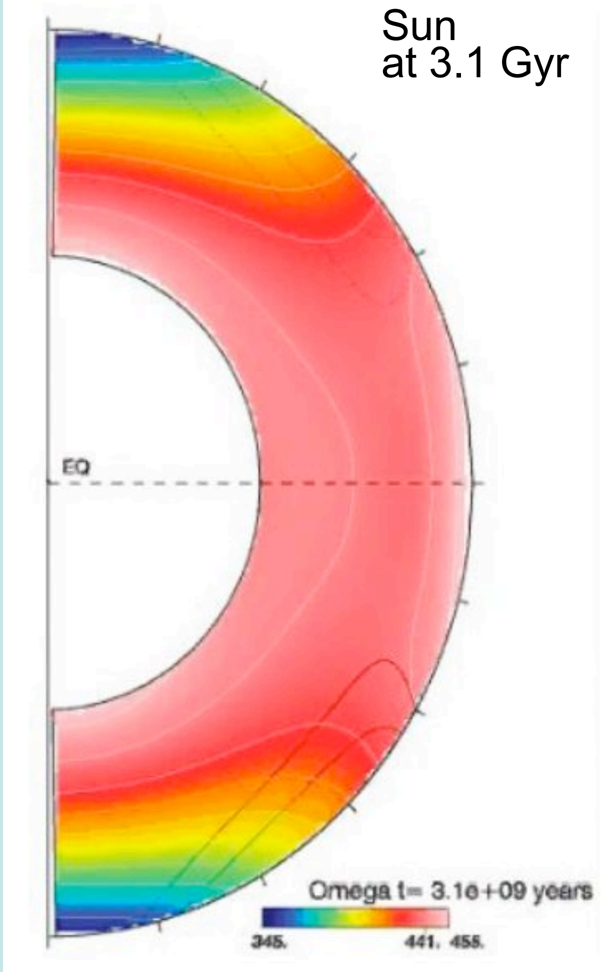
Hydrostatic and geostrophic equilibrium
conservation of angular momentum
conservation of thermal energy
Boussinesq approximation

solutions are separable : $\Omega(r,\theta) = \Omega(r) + \sum_i \widetilde{\Omega}_i(r) f_i(\theta)$

In thin layer approximation, for $t \gg r_0^2/K$

$$\frac{\partial \widetilde{\Omega}}{\partial t} = -K \left(\frac{2\Omega}{N}\right)^2 \left(\frac{r_0}{\lambda}\right)^2 \frac{\partial^4 \widetilde{\Omega}}{\partial r^4} + \nu_v \frac{\partial^2 \widetilde{\Omega}}{\partial r^2}$$

In present Sun, differential rotation would have spread down to $r = 0.3\, R_\odot$ - - - - - - - - - - - - - - - ►



Brun 2006

→ not observed - why is the tachocline so thin ?

Another physical process must confine the tacholine

Anisotropic turbulence ? Spiegel & Z 1992

Fossil magnetic field ? Gough & McIntyre 1998

Can the tachocline be confined by a fossil field ?

no

Numerical simulations by Sacha Brun

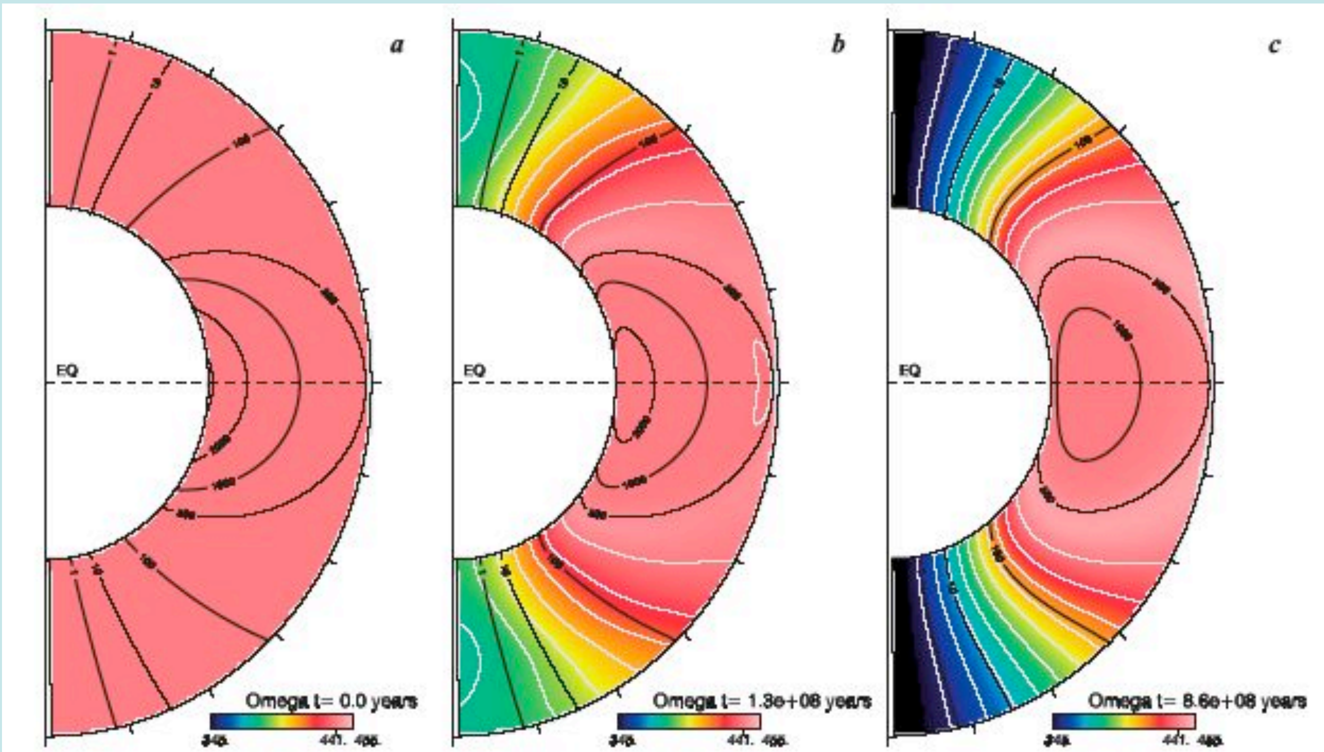
$\Rightarrow$ Ferraro
Ω ~ cst on field lines of B_{pol}

diff. rotation imposed at top of RZ

initial dipolar penetrates in CZ

ASH code
tuned for RZ
optimized for massively parallel machines

193x128x256



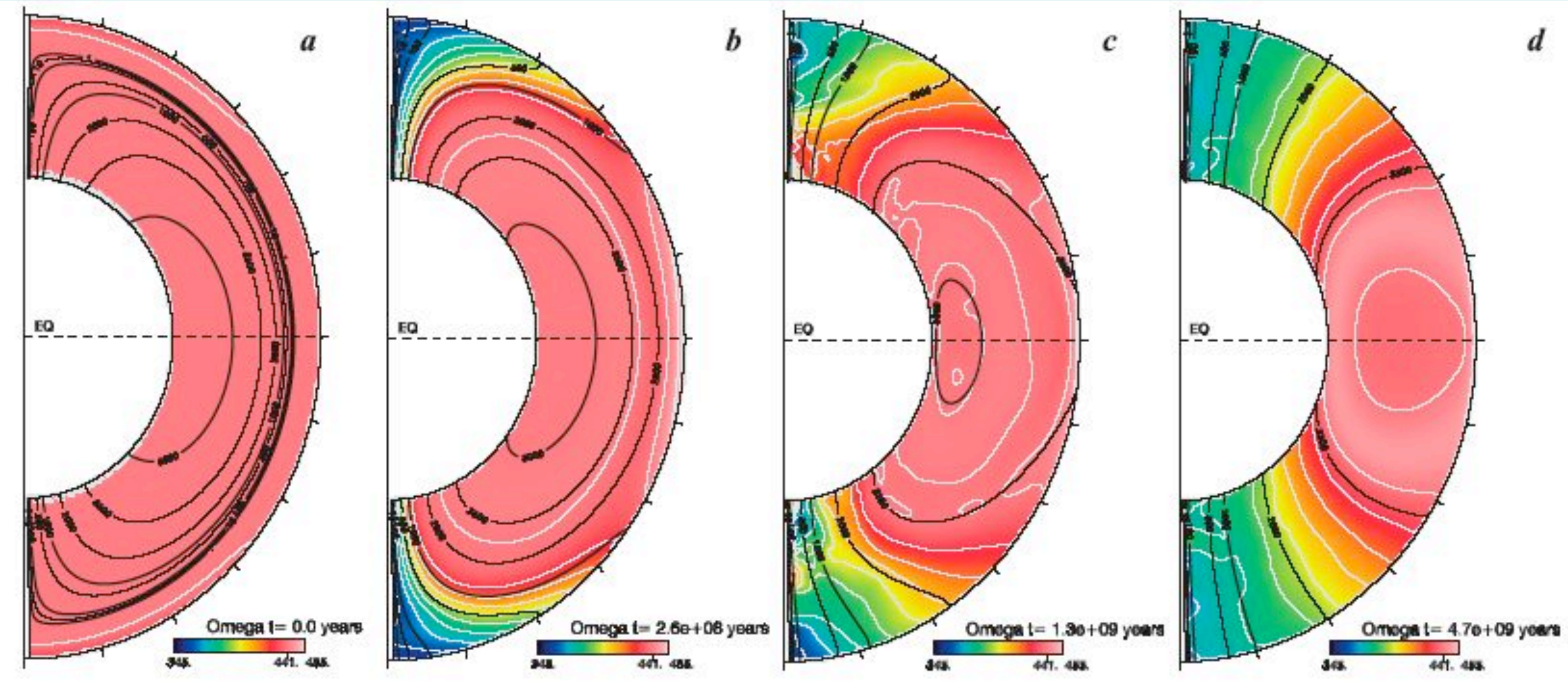
Can the tachocline be confined by a fossil field ?

No : such a field eventually connects with the CZ
and imprints its differential rotation on the RZ

Brun & Z 2006

initial dipolar field burried in RZ

⇒ Ferraro



Magnetic fields generate instabilities

MHD instabilities

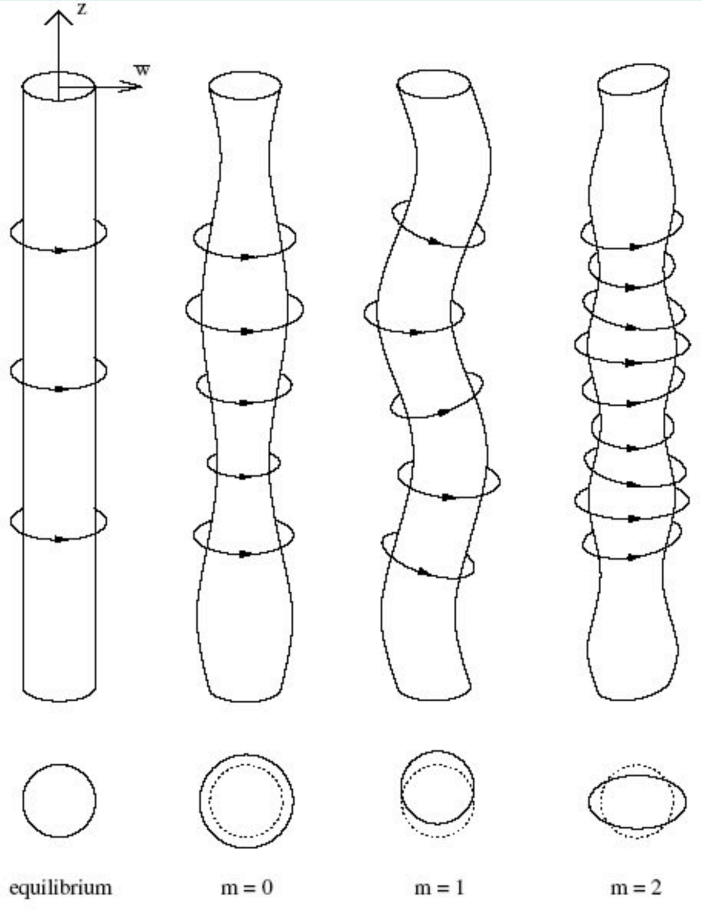
Theoretical results, mostly by Tayler & collaborators

- A purely poloidal field is unstable
  to non-axisymmetric perturbations
  (Markey & Tayler 1973)

- A purely toroidal field is unstable
  to non-axisymmetric perturbations
  (Tayler 1973; Wright 1973; Goossens et al. 1981)

- Stable fields are probably
  a mix of poloidal and toroidal fields
  of comparable strength

- Rotation stabilizes somewhat a purely toroidal field,
  but it cannot suppress entirely the instability
  (Pitts & Tayler 1973)



Results obtained in the ideal case (no thermal and Ohmic diffusions)

MHD instabilities

Linear analysis, adding diffusion (Acheson 1978; Spruit 1999, 2002)

Radiation zone, stable stratification
buoyancy frequency :

$$N^2 = N_t^2 + N_\mu^2 = \frac{g}{H_P}\left(\nabla_{ad} - \nabla\right) + \frac{g}{H_P}\left(\frac{d\ln\mu}{d\ln P}\right)$$

Purely toroidal field
Alfvén frequency :

$$\omega_A^2 = \frac{B_\varphi^2}{4\pi\rho s^2} \qquad s = r\sin\theta$$

Diffusivities - thermal: $\kappa \approx 10^7\, cm^2/s$ Ohmic: $\eta \approx 10^3\, cm^2/s$

Perturbation, near axis: $\xi \propto \exp i\left(l s + m\varphi + n z - \sigma t\right)$

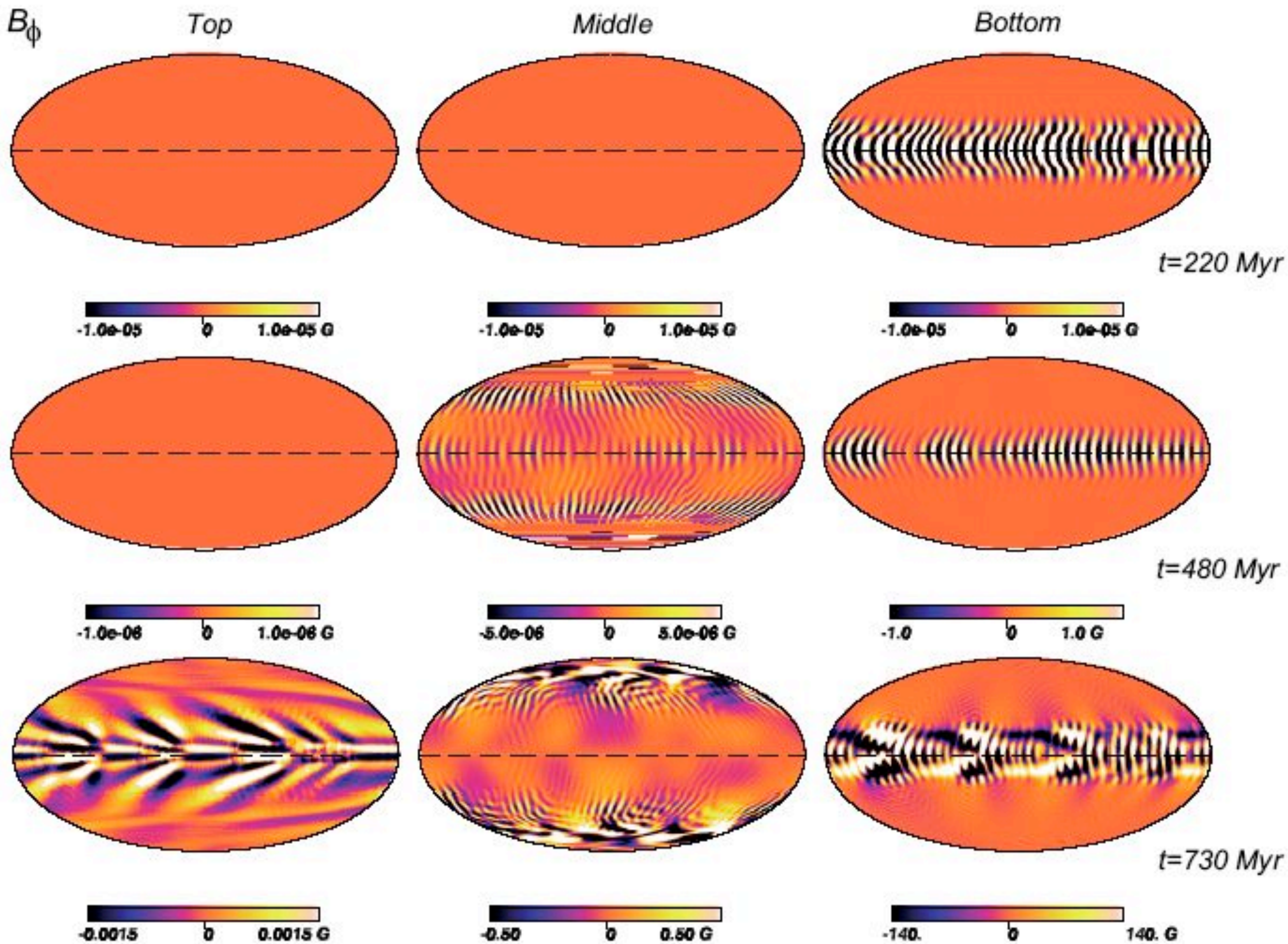
$$\mathrm{Im}(\sigma) = 0$$

Instability for $\omega_A^4 > C\,\Omega\,\eta\, l^2 \left[\frac{\eta}{\kappa} N_t^2 + N_\mu^2\right]$ $\quad C = \mathrm{O}(1)$

Spruit's conjectures :

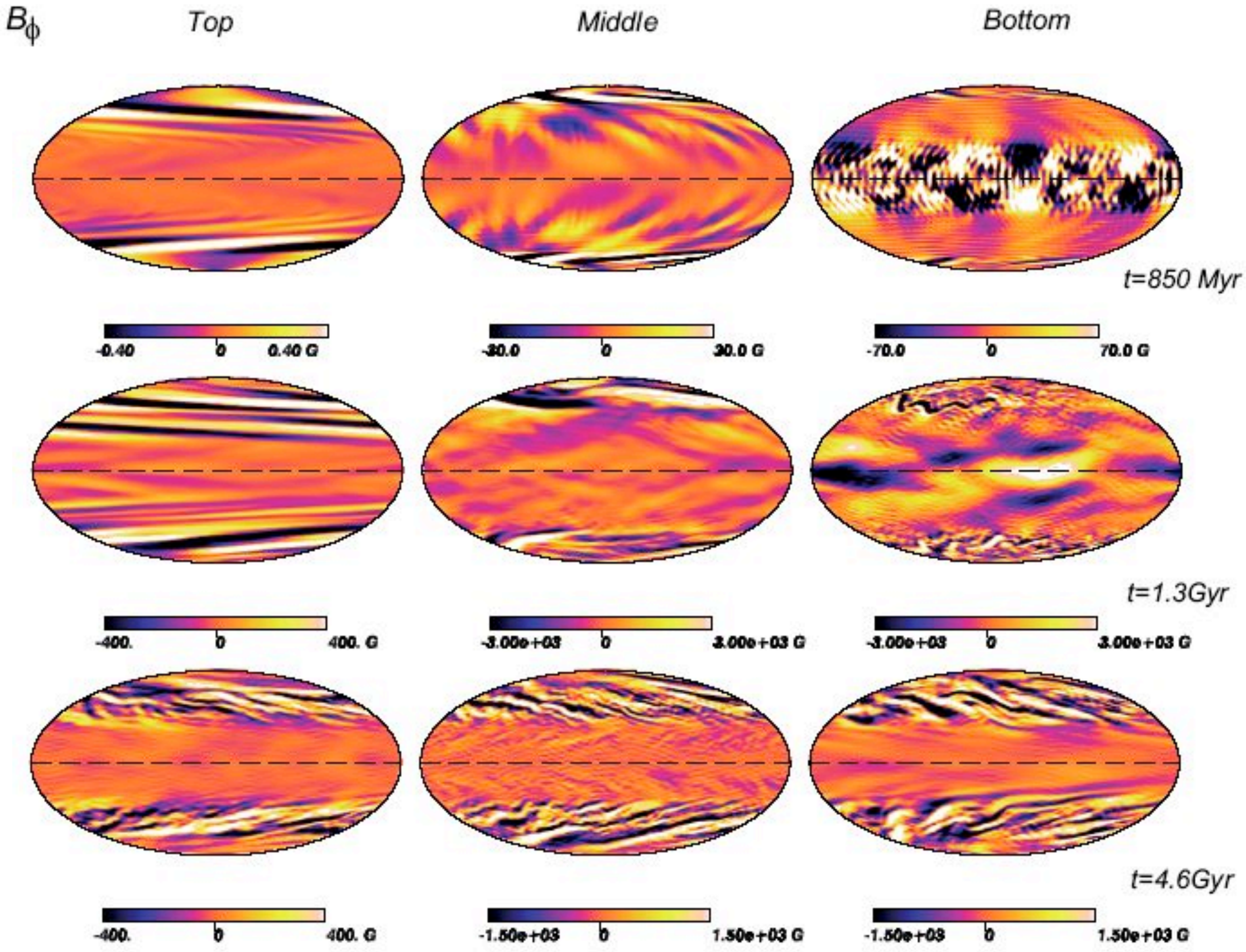
- instability saturates when turbulent η ensures marginal stability
- turbulence operates a dynamo in radiation zone

Tayler instabilities in the solar radiation zone
(magnetic tachocline simulation)



Tayler instabilities in the solar radiation zone

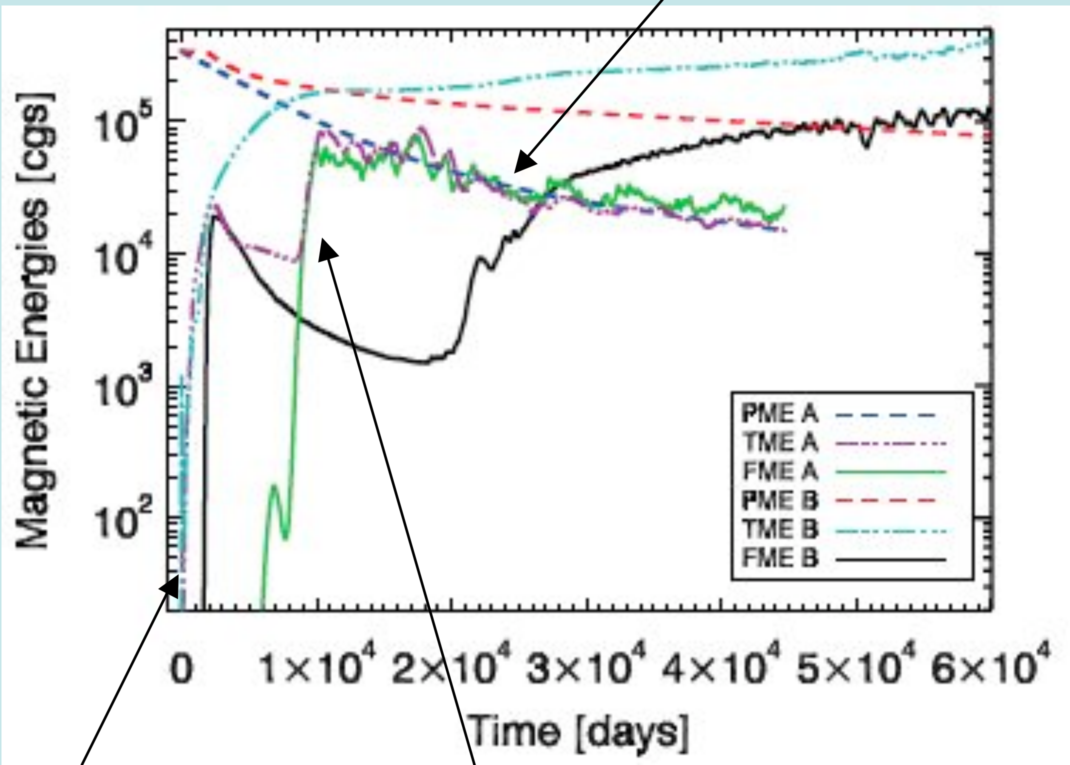
(magnetic tachocline simulation, cont.)



Tayler instabilities in the solar radiation zone
(magnetic tachocline simulation)

Brun & JPZ 2006
JPZ, Brun & Mathis 2007

poloidal field decays steadily



instability of poloidal field

instability of toroidal field

Poloidal field
is not
regenerated

→ no dynamo

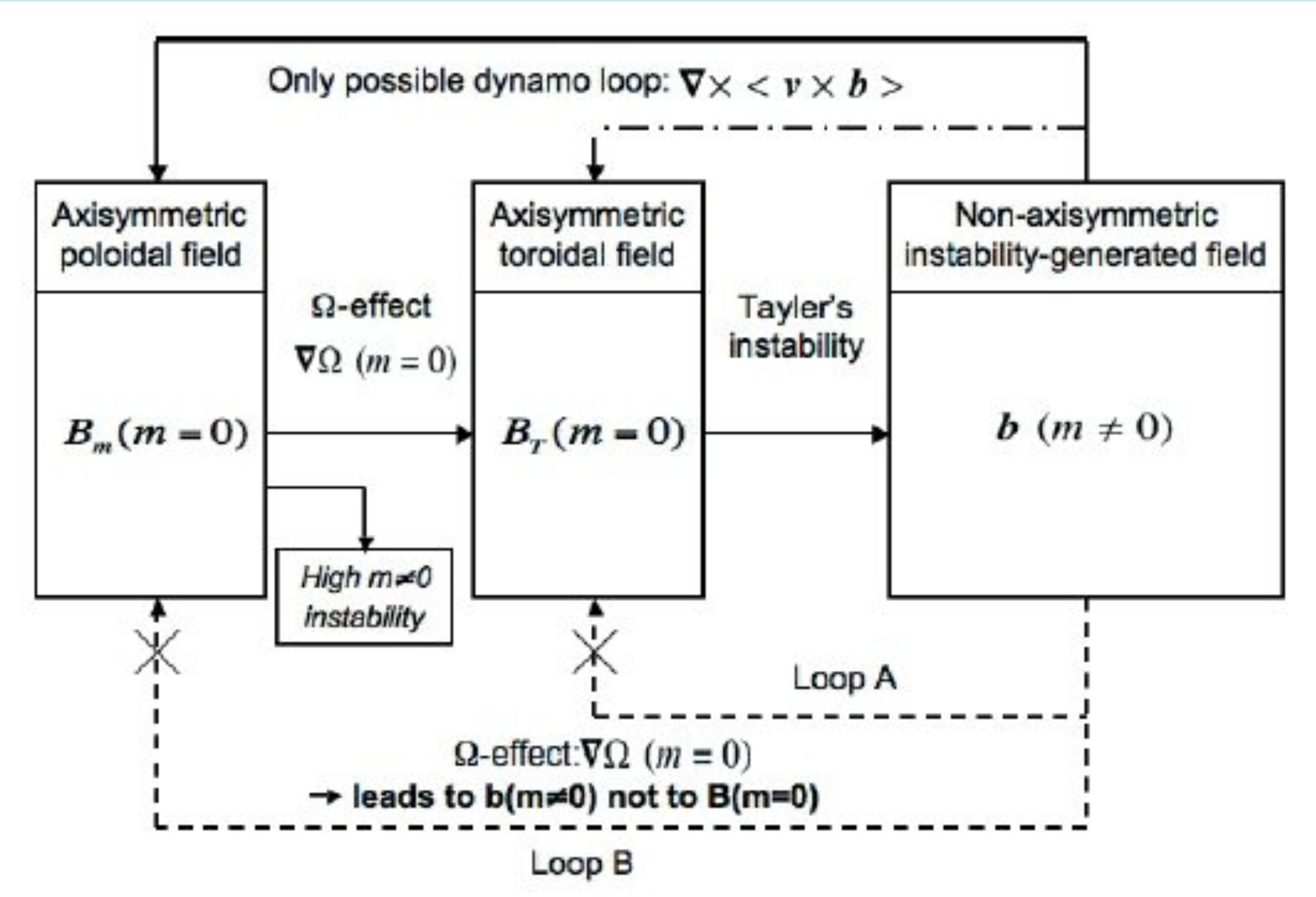
Decay of
poloidal field
not enhanced
by instability

→ no eddy diff.

→ no mixing

The dynamo loop

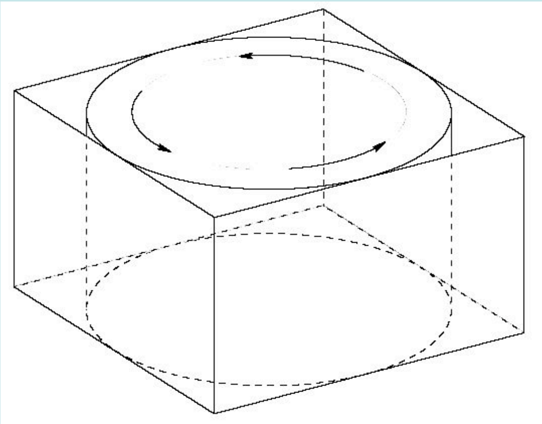
→ It cannot work as explained by Spruit and Braithwaite



Z, Brun & Mathis 2007

Why does Braithwaite find a dynamo ?

How Braithwaite's 2006 simulation (in black) differs from ours (in red)



geometry

boundary conditions :

field normal to surface

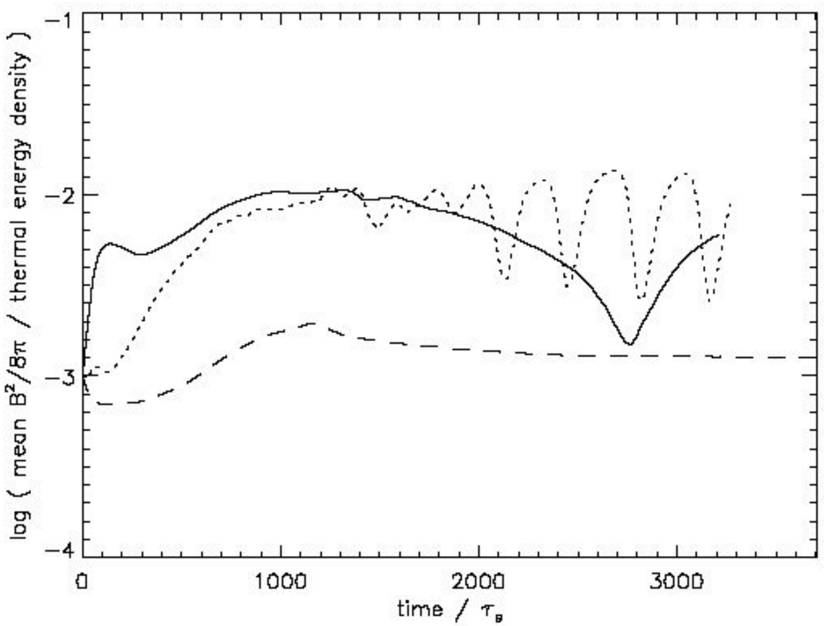
potential field

resolution

64x64x63 128x256x192

Euler, numerical dissipation : Rm=?

DNS, enhanced diffusion : $Rm=10^5$



was the simulation pursued long enough ?

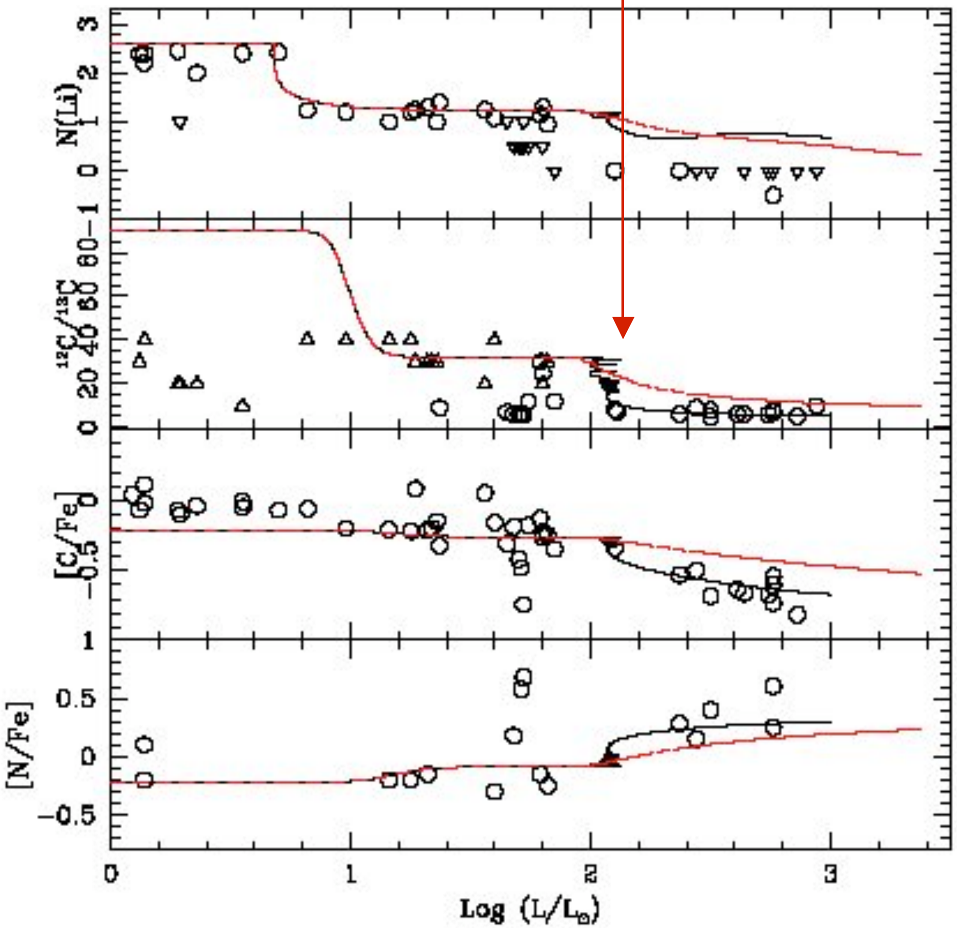
Magnetic fields may inhibit instabilities

Another example: thermohaline instability in RG

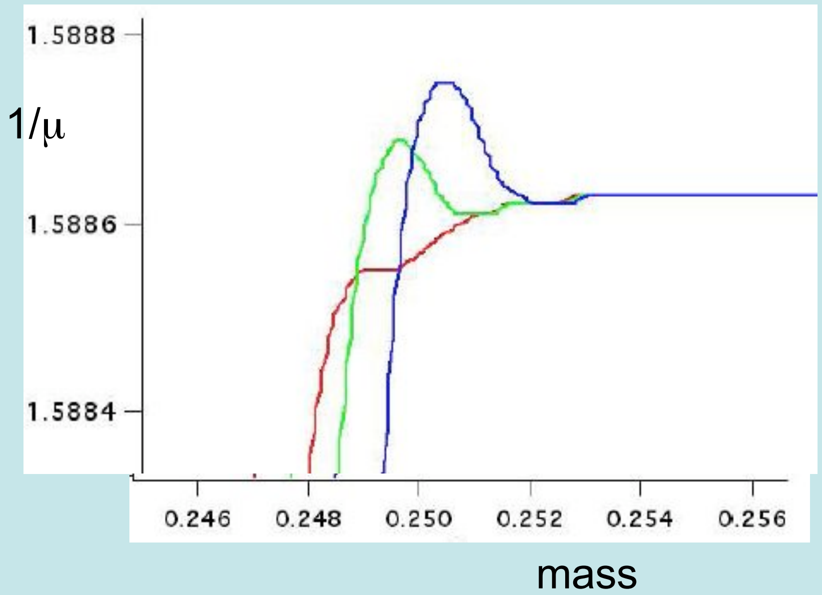
Thermohaline mixing in red giant stars

first dredge-up

extra mixing



due to inversion of μ-gradient produced by $^3He(^3He,2p)^4He$

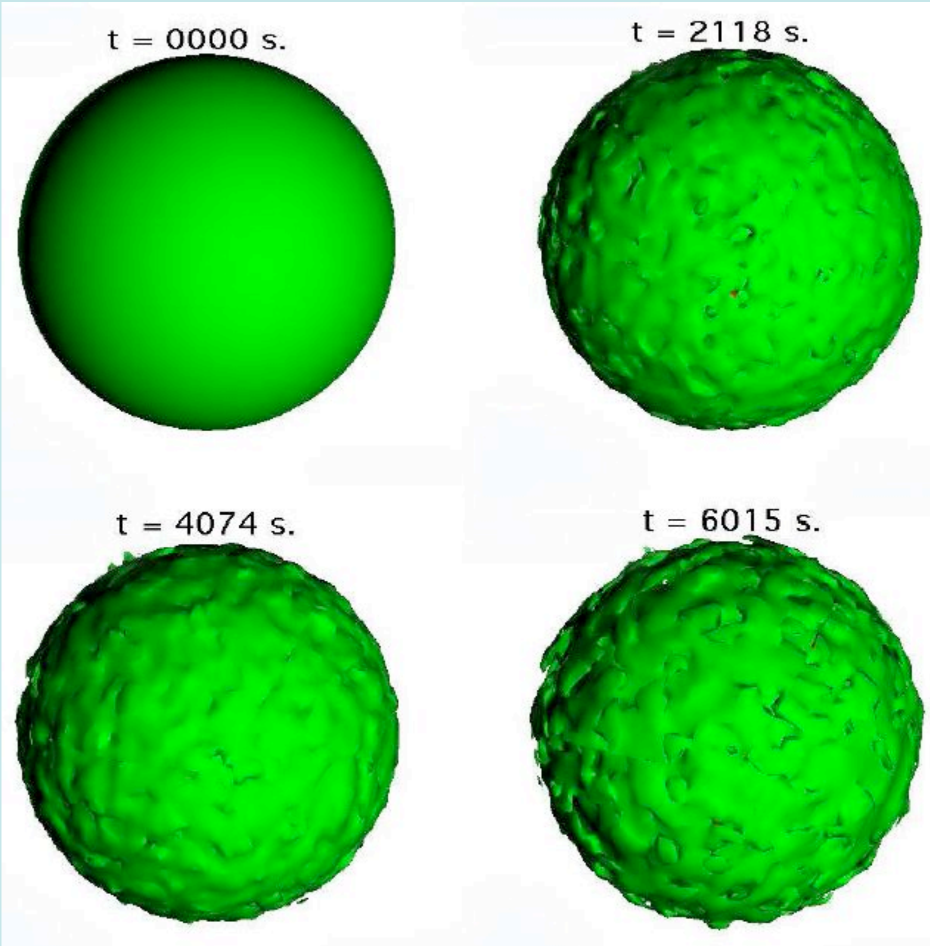


Eggleton, Dearborn & Lattanzio et al. 2006

In fact, Eggleton et al. observed convective instability, which occurs when

$$\nabla > \nabla_{ad} + \frac{d\ln\mu}{d\ln P}$$

Ledoux criterion



In reality, as the μ-gradient builds up, the first instability to arise as soon as

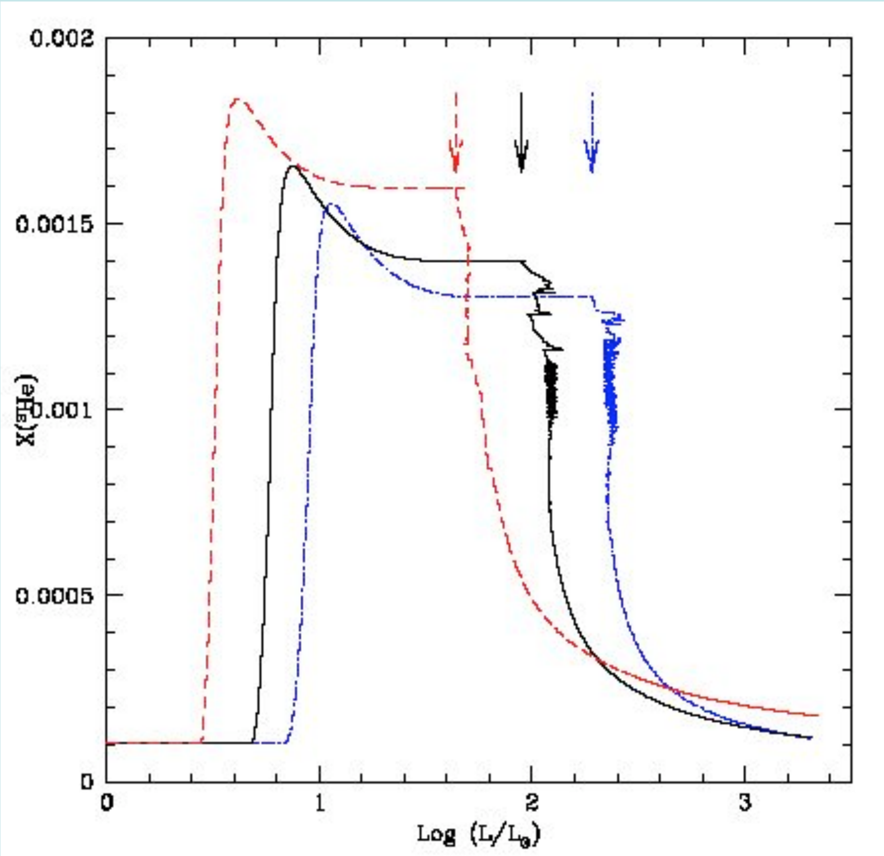
$$\frac{d\ln\mu}{d\ln P} < 0$$

is the thermohaline instability.

Charbonnel & Z 2007

who use Ulrich's 1972 prescription

Such extra-mixing destroys ^{3}He ;
which explains its Galactic abundance



However, observations show that a small fraction of stars (~4%) avoid this extra-mixing
(Charbonnel & do Nascimento 1998)

Moreover,
2 PNe have been observed with high ^{3}He abundance ~10^{-3}
(NGC 3242, J320)
(Balser et al. 2006)

Our explanation:
the thermohaline instability is suppressed by magn. field ~10^5G in those RGB stars that are the descendants of Ap stars

(Charbonnel & Z, submitted to A&A)

Conclusions

Magnetic fields play little rôle in the structure of stars

but they have an impact on their evolution

- by determining their rotation state
- by suppressing instabilities
- by interfering with mixing processes operating in RZ:
  rotational mixing, thermohaline mixing
- possibly by triggering MHD instabilities

Obviously, the effect depends on field strength

→ observational constraints are highly needed