Stellar Magnetospheres

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“Magnetosphere”

- Space around magnetized planet or astron.body
- coined in late ‘50’s
  - satellites discovery of Earth’s “radiation belts”
  - trapped, high-energy plasma
- also detected in other planets with magnetic field
  - e.g. Jupiter (biggest), Saturn etc. (but not Mars, Venus)
- concept since extended to Sun and other stars
  - sun has “magnetized corona”
  - stretched out by solar wind
- but study has some key differences...

Planetary vs. Stellar Magnetospheres

- outside-in compression
- Space physics
- local in-situ meas.
- plasma parameters
- plasma physics models

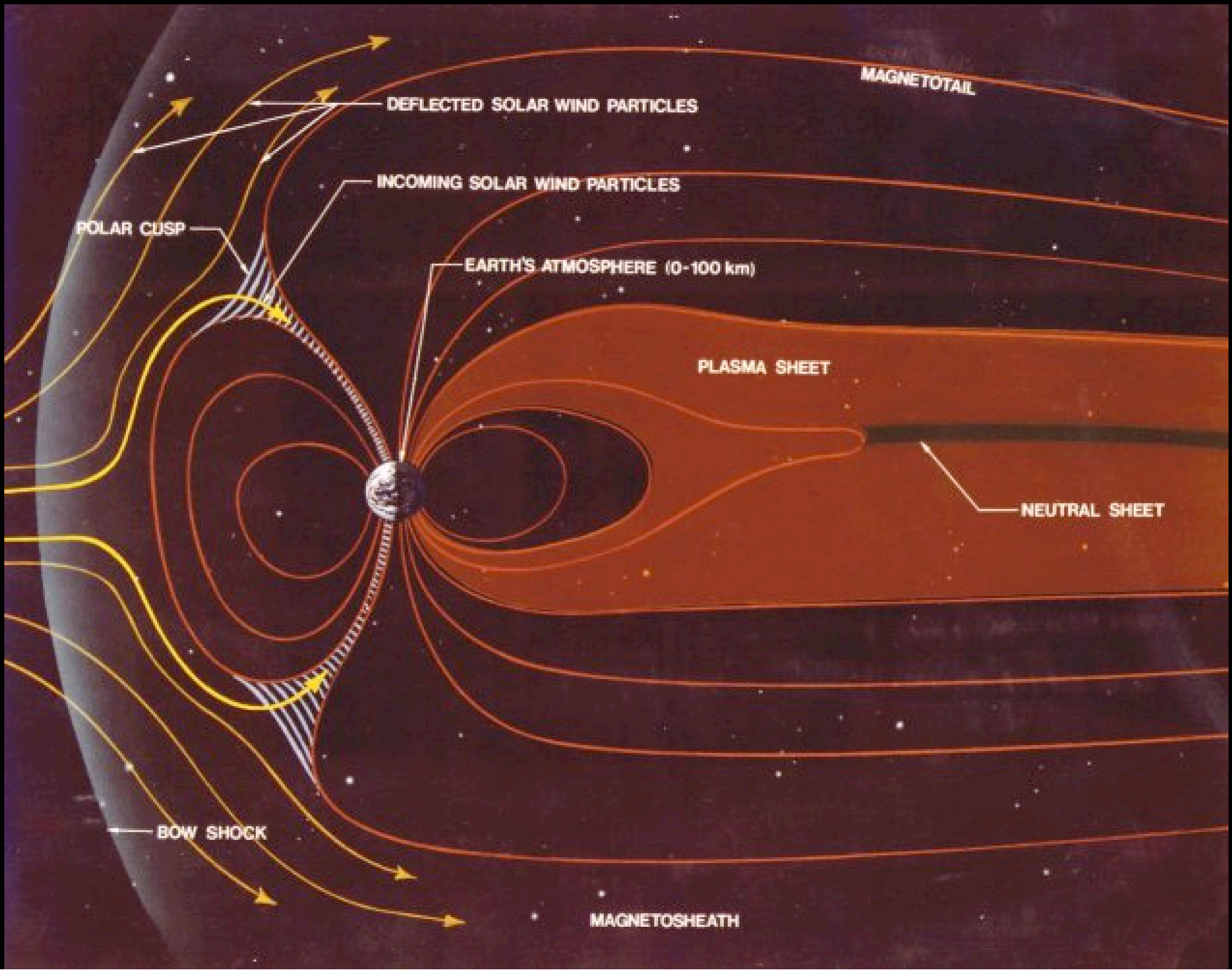
- inside-out expansion
- Astrophysics
- global, remote obs.
- gas & stellar parameters
- hydro or MHD models

Interplanetary medium

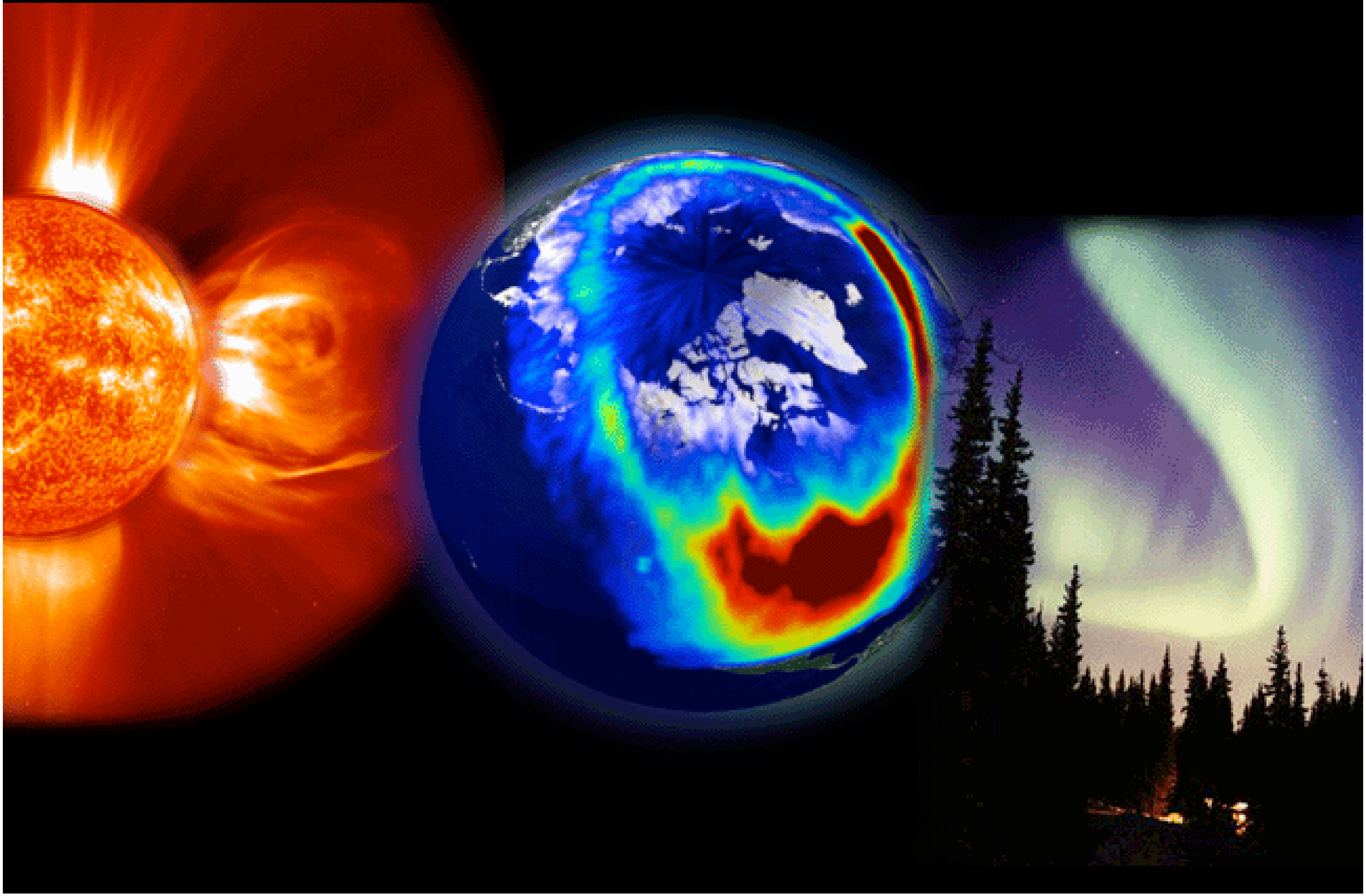
Solar coronal expansion

Solar astrophysics includes both approaches

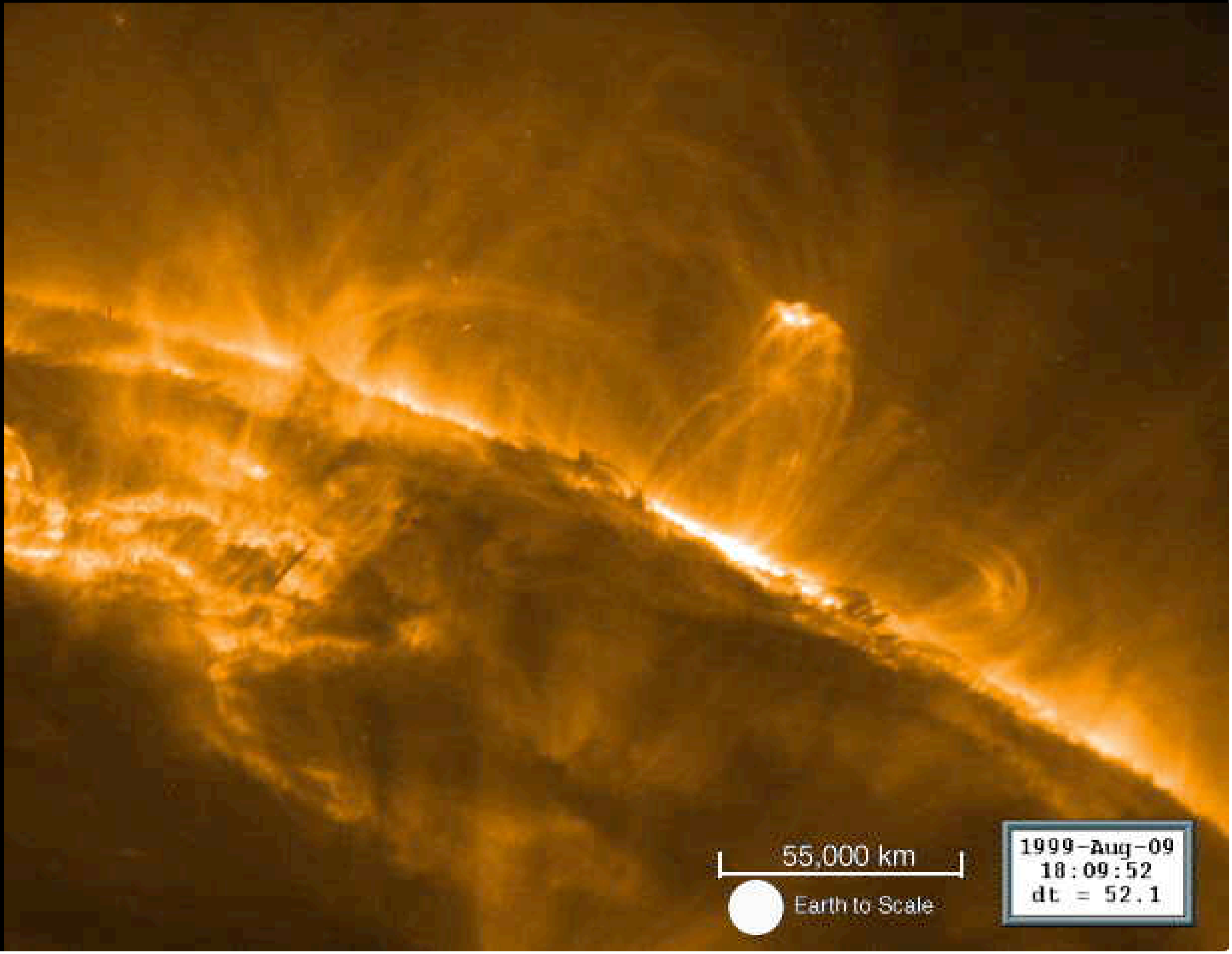
Earth’s Magnetosphere



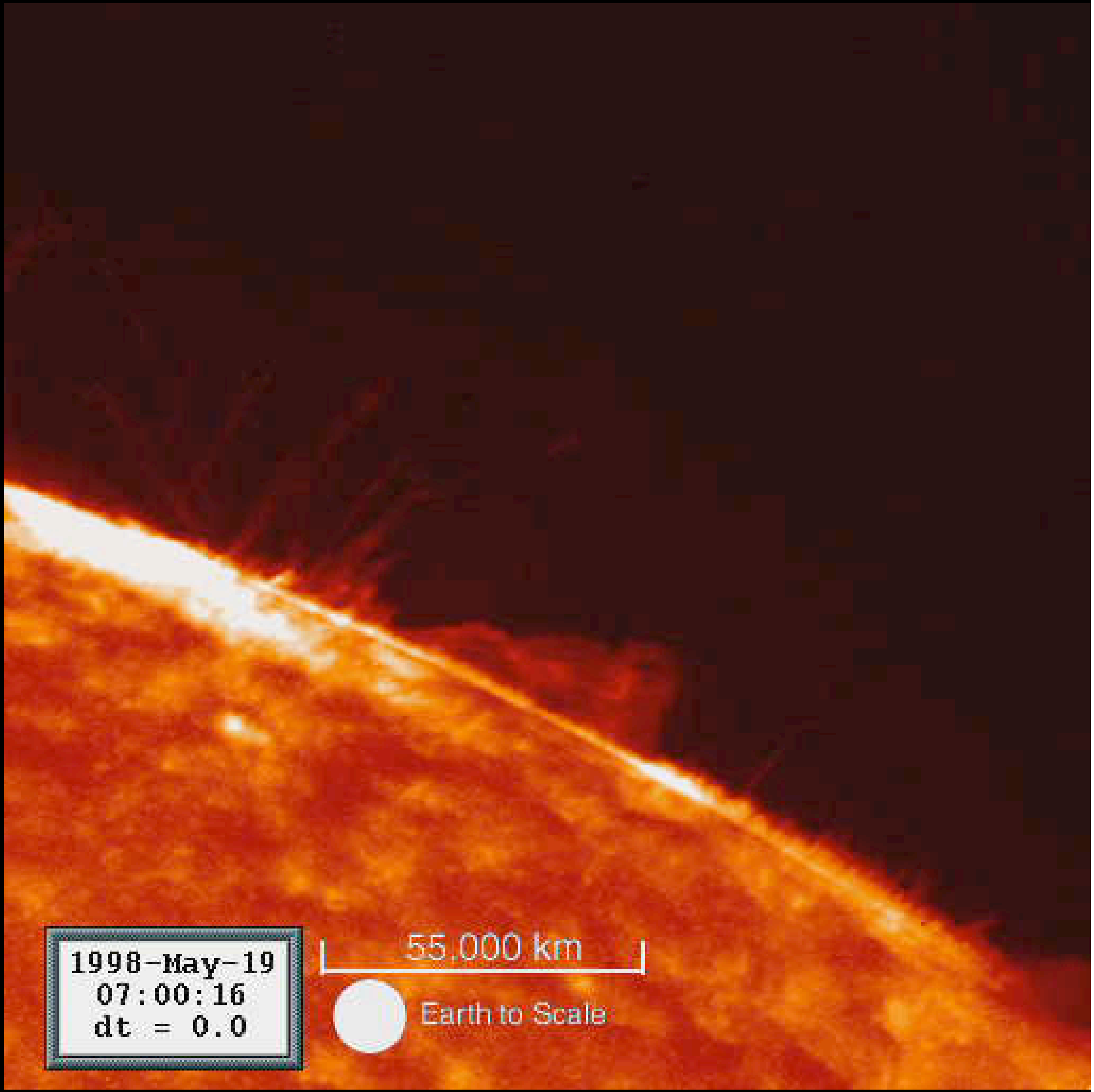
Sun-Earth Connection



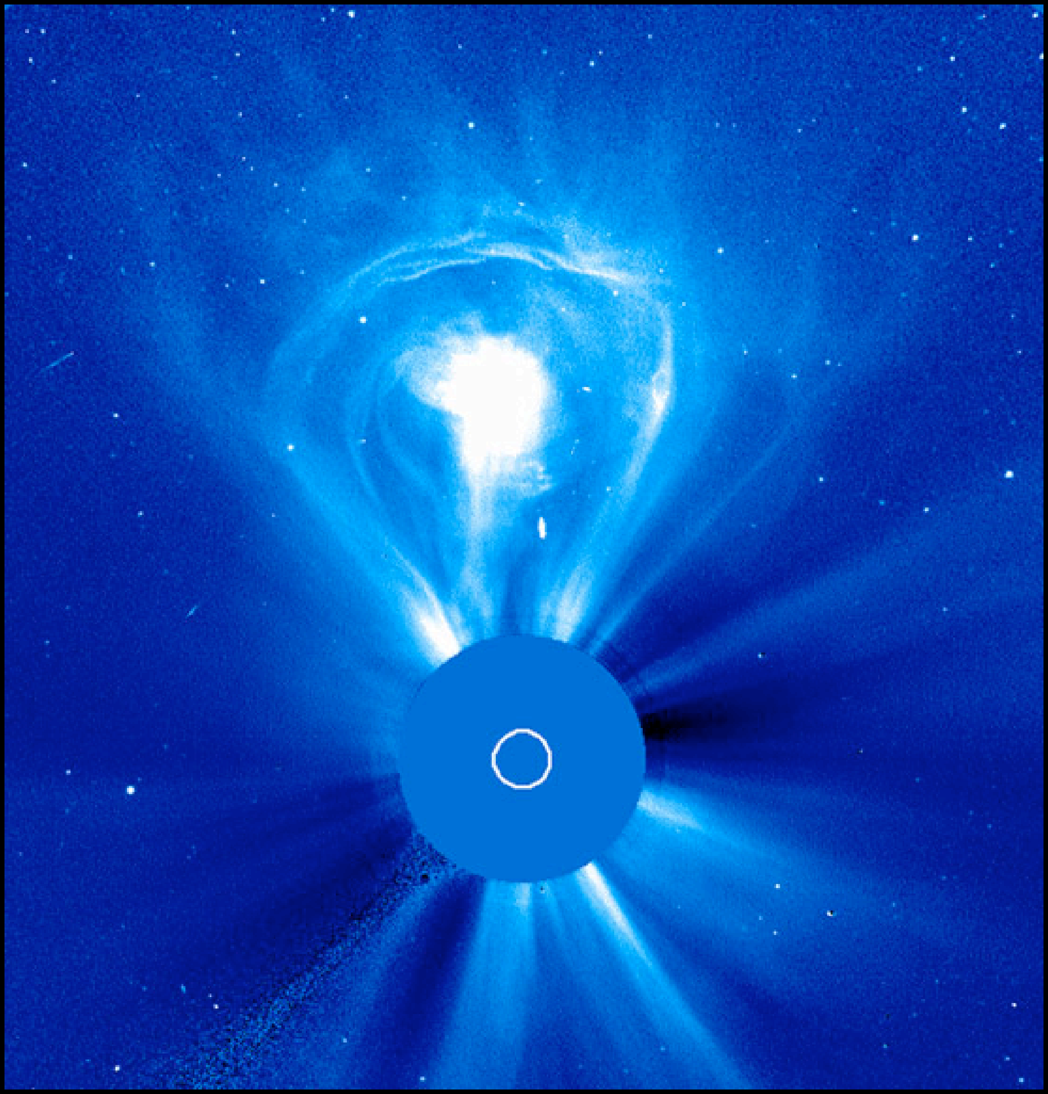
Magnetic loops on Solar limb



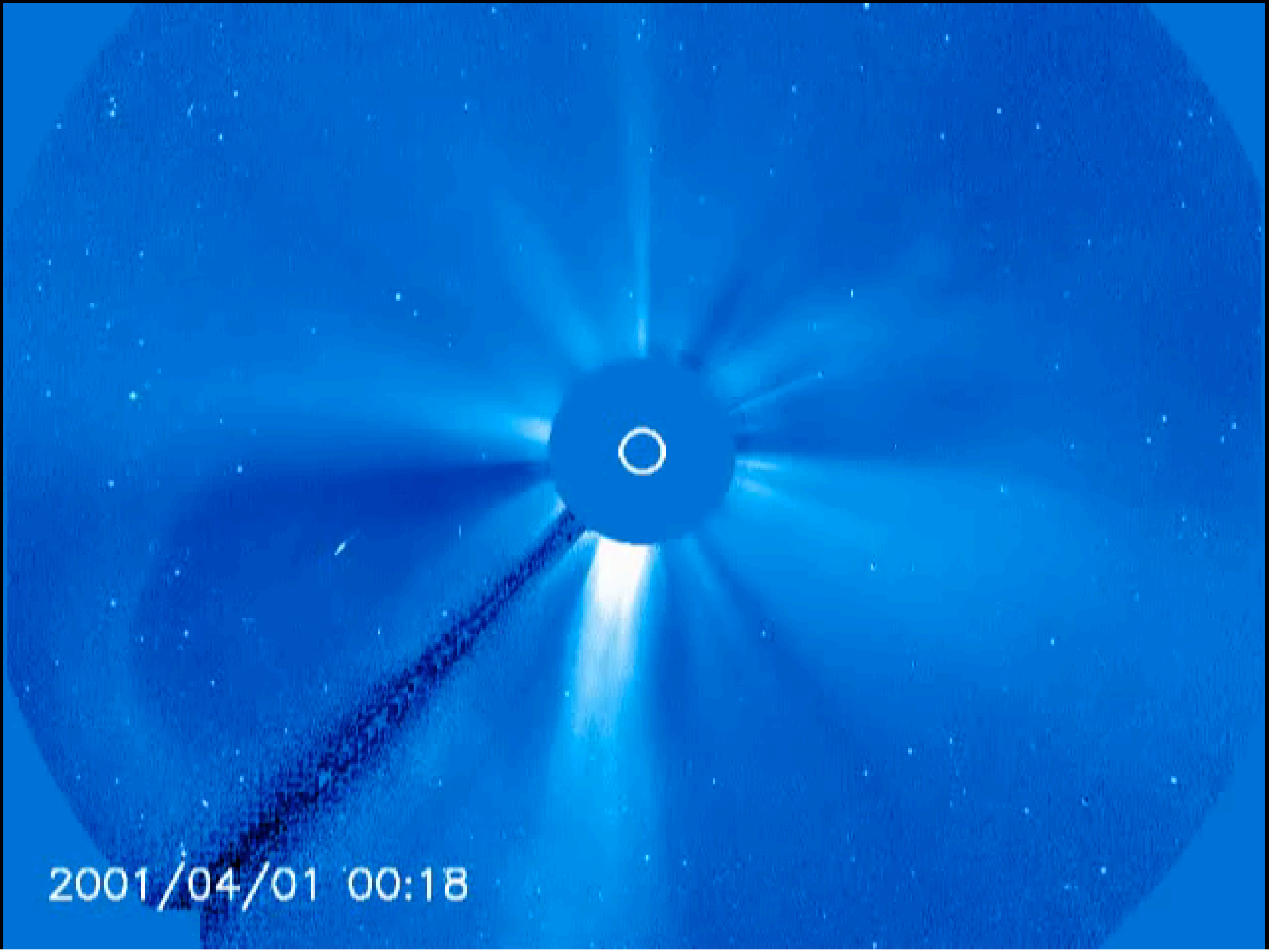
Solar Flare



Sun’s Active “Magnetosphere”



Solar Activity: Coronal Mass Ejections

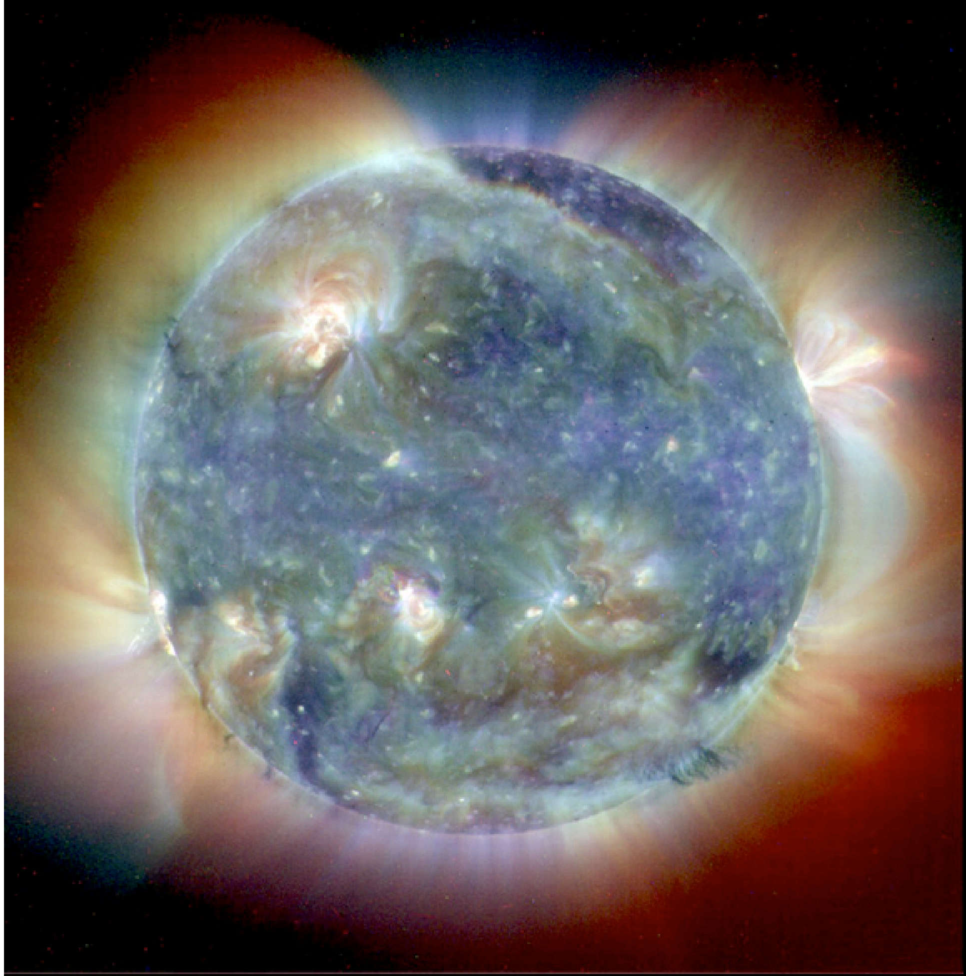


Processes for Solar Activity

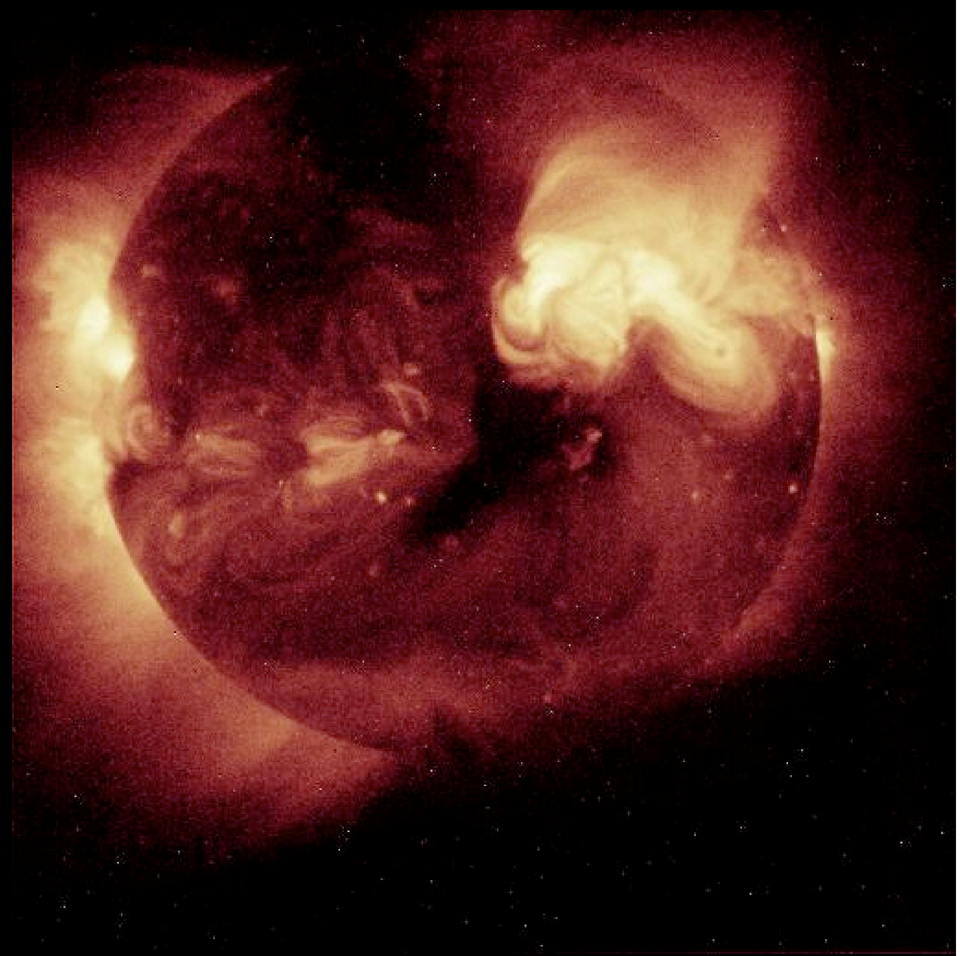
- Convection + Rotation Dynamo
  - Complex Magnetic Structure
  - Magnetic Reconnection
  - Coronal Heating
  - Solar Wind Expansion + CME

Solar Corona in EUV & X-rays

Composite EUV image from EIT/SOHO

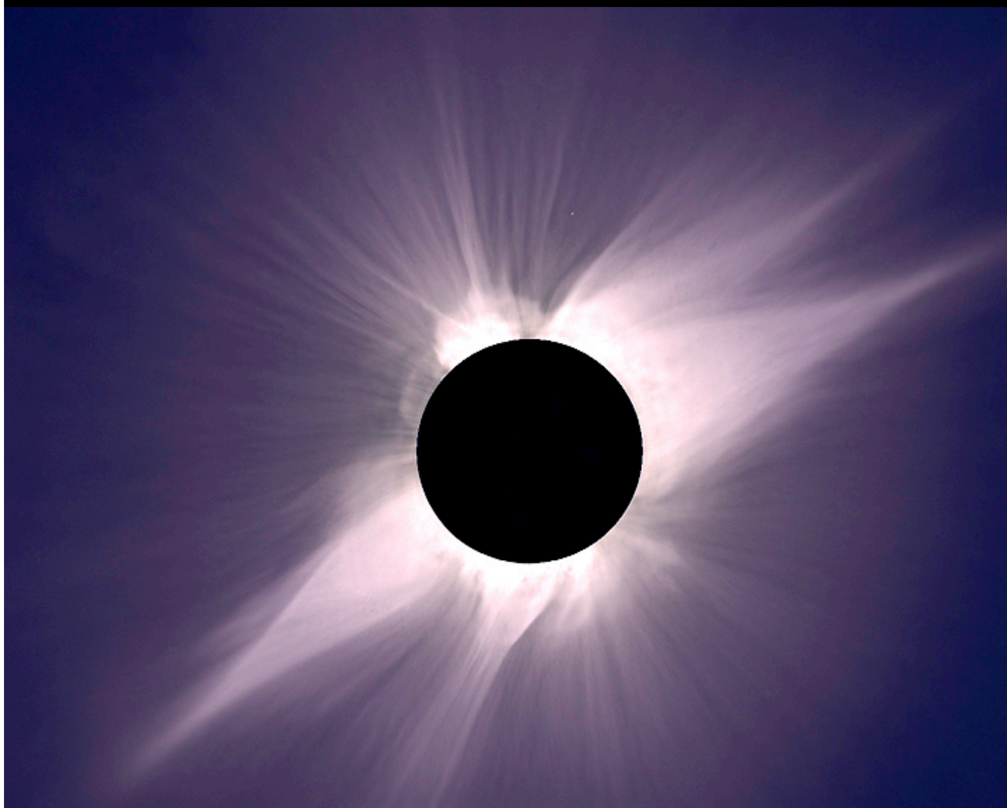


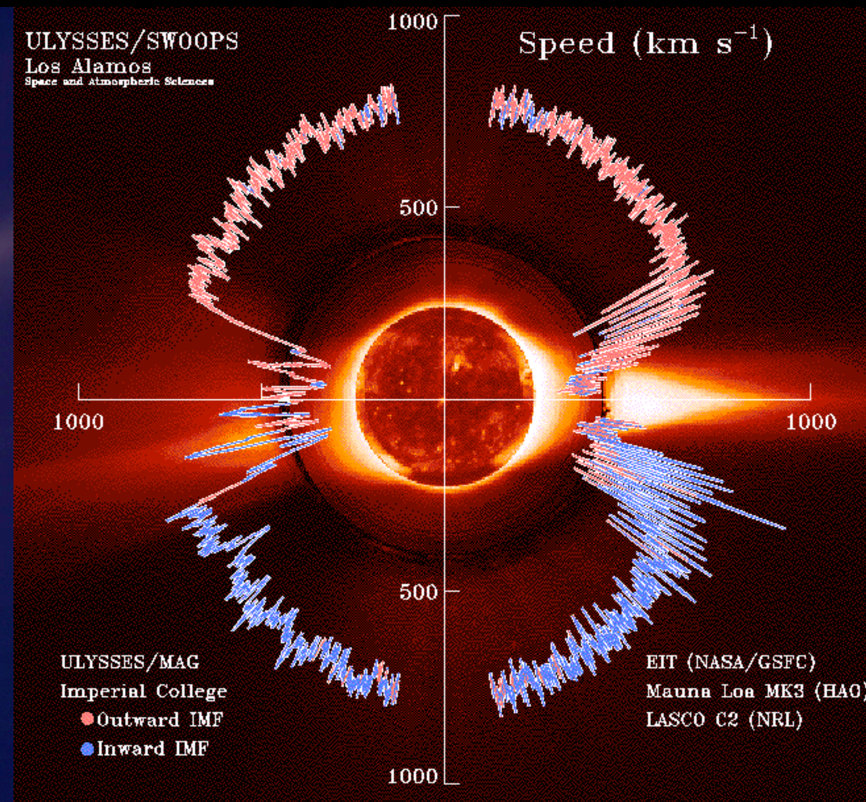
X-ray Corona from SOHO



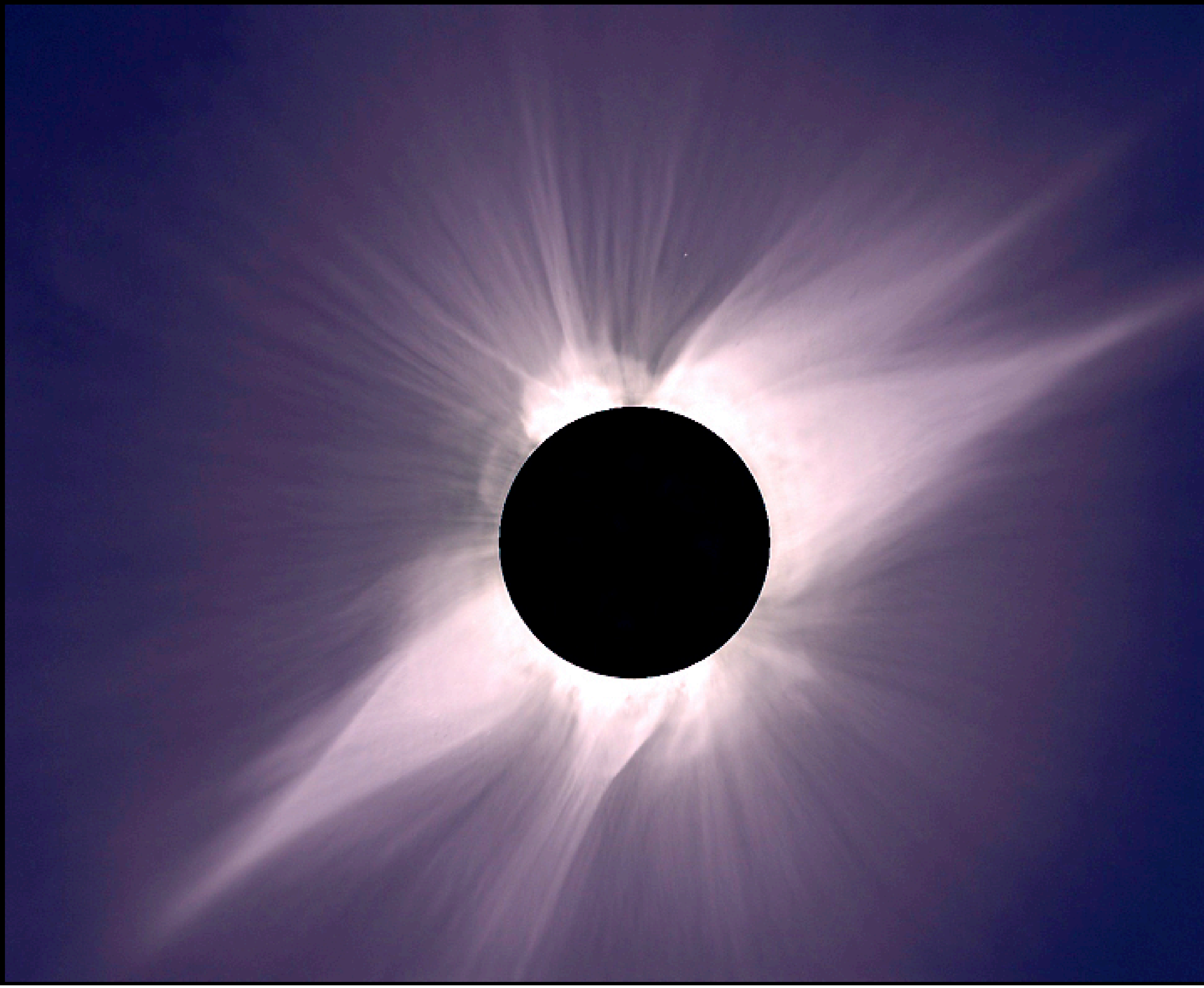
Magnetic Effects on Solar Coronal Expansion

Coronal streamers

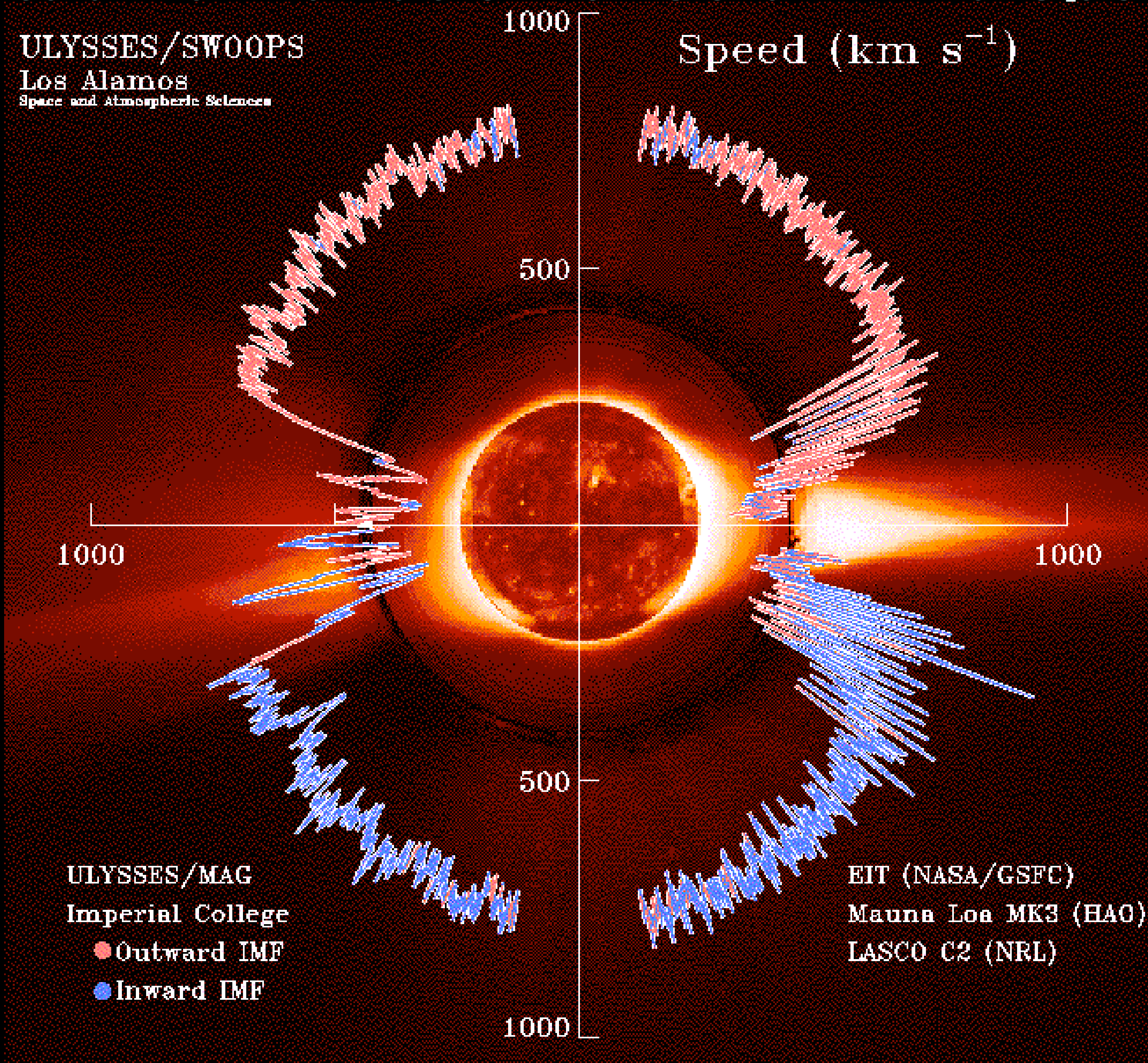




Corona during Solar Eclipse



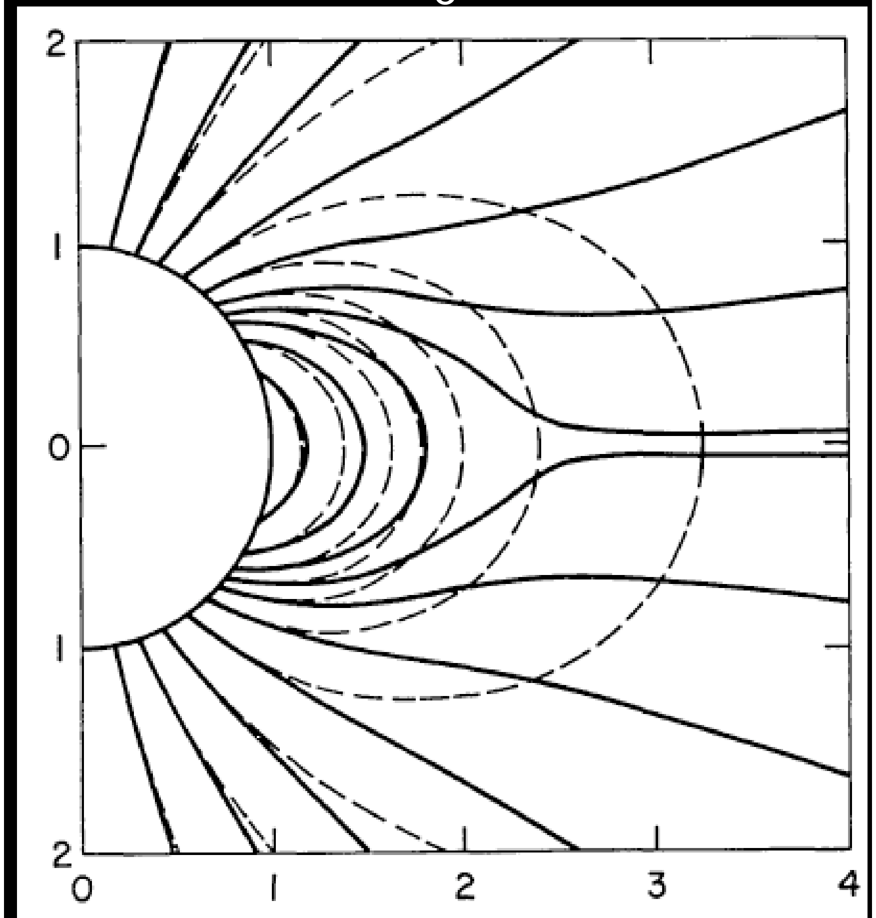
Latitudinal variation of solar wind speed



Pneuman and Kopp (1971)

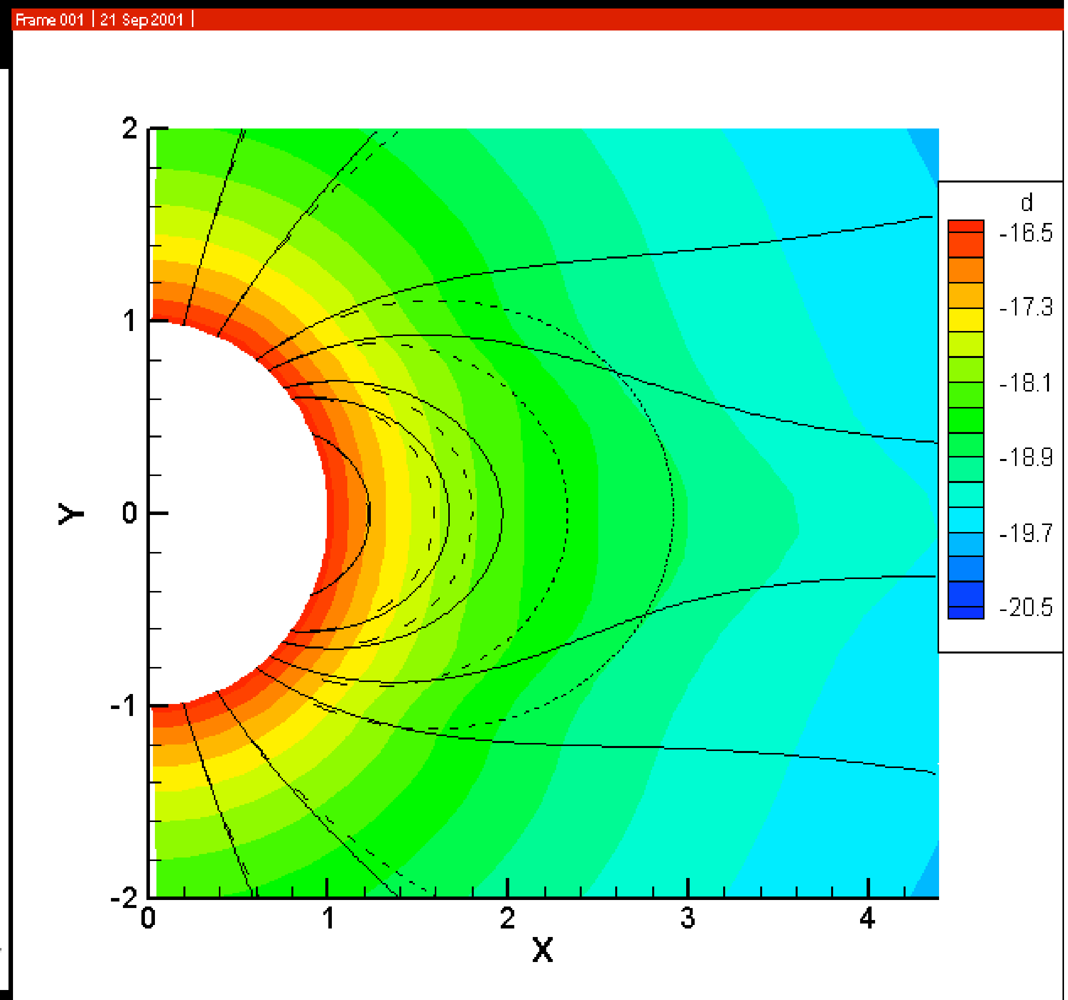
Iterative scheme

MHD model for base dipole with B_o=1 G



Fully dynamic, time dependent

MHD Simulation



MHD model for magnetized plasmas

- ions & electrons **coupled** in "single fluid"
  - "effective" collisions =>
  - Maxwellian distributions
- large scale in length & time
  - >> Larmour radius, mfp, Debye, skin
  - >> ion gyration period
- ideal MHD => infinite conductivity

Magnetohydrodynamic (MHD) Equations

$$\frac{\partial B}{\partial t} = \nabla \times \left(v \times B \right)$$

mag Induction

$$\nabla \bullet B = 0$$

Divergence free B

$$\frac{D\rho}{Dt} + \rho \nabla \bullet v = 0$$

Mass

$$\rho \frac{Dv}{Dt} = -\nabla p + \frac{1}{4\pi} \left(\nabla \times B \right) \times B + \rho (g_{ext} - g_{grav})$$

Momentum

$$\frac{\partial e}{\partial t} + \nabla \cdot e\mathbf{v} = -P\nabla \cdot \mathbf{v} + H - C$$

Energy

$$P = \rho a^2 = (\gamma - 1)e$$

Ideal Gas E.O.S.

Maxwell's equations

$$\nabla \bullet E = 4\pi\rho_e$$

Coulomb

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

Induction

$$\nabla \bullet B = 0$$

no mag. monopoles

$$\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c}\cancel{\frac{\partial E}{\partial t}}$$

Ampere

$$O\left(\frac{v}{c}\right)^2 << 1$$

Magnetic Induction & Diffusion

Combine Maxwell's eqns. with **Ohm's law:**

$$J = \sigma(E + v \times B/c)$$

Eliminate E and J to obtain:

$$\frac{\partial B}{\partial t} = \nabla \times \left(v \times B\right) + \frac{c^2}{4\sigma}\nabla^2 B$$

magnetic induction

magnetic diffusion

Magnetic Reynold's number

$$\mathrm{Re} \cong \frac{4\pi\sigma L v}{c^2} \cong \frac{induction}{diffusion} >> 1 \quad \text{e.g., } 10^{10}!$$

induction **diffusion**

$$\frac{\partial B}{\partial t} = \nabla \times \left(v \times B \right) + \cancel{\frac{c^2}{4\sigma} \nabla^2} B$$

"**Ideal**" MHD

$$O\left(\frac{1}{\mathrm{Re}}\right) << 1$$

Frozen Flux theorum

Ideal MHD induction eqn.: $$\frac{\partial B}{\partial t} = \nabla \times \left(v \times B \right)$$

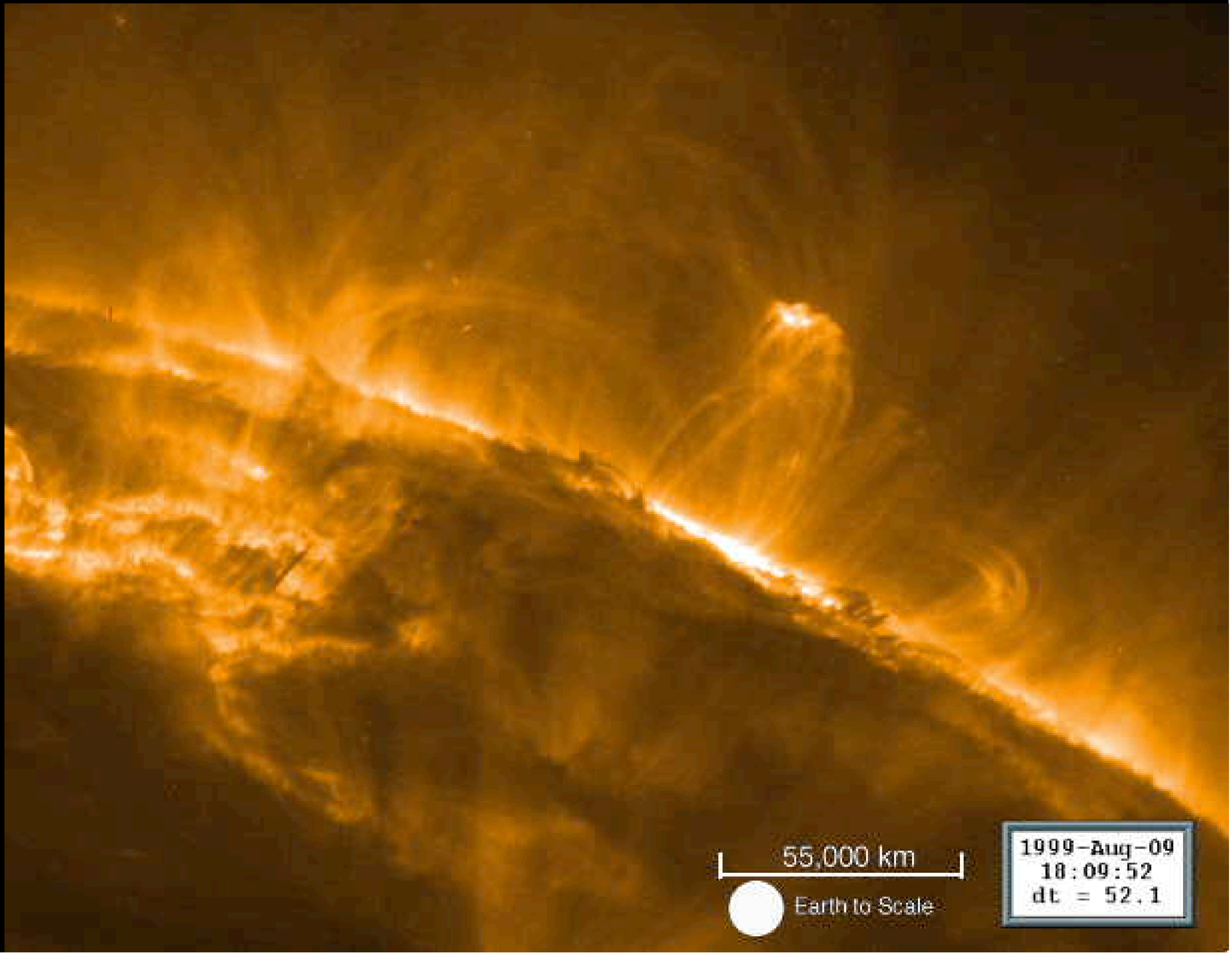
implies **flux** F through any **material surface** σ,

$$F \equiv \int_{\sigma} B \bullet dA$$

does not change in time, i.e. is “frozen”:

$$\frac{dF}{dt} = 0$$

Magnetic loops on Solar limb



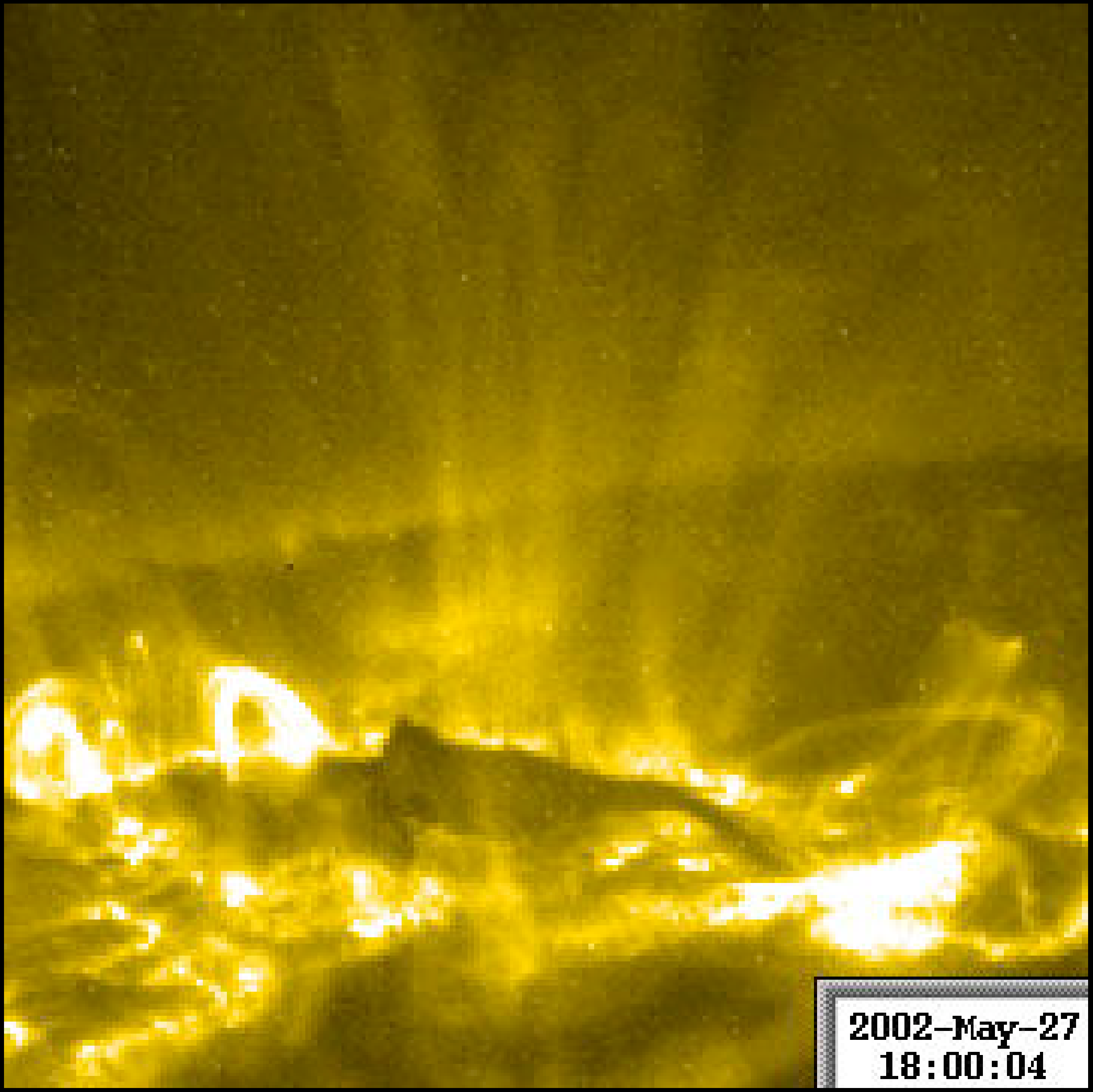
Magnetic Reconnection

If/when/where B field varies over a small enough scale, L << R, i.e. such that local Re~1, then frozen flux breaks down,

$$\frac{dF}{dt} \neq 0$$

The associated magnetic reconnection can occur suddenly, leading to dramatic plasma heating through dissipation of magnetic energy $B^2/8\pi$.

Solar Flare



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Magnetic Lorentz force: magnetic tension and pressure

Lorentz force

$$f_{Lor} = \frac{J \times B}{c} = \frac{1}{4\pi}\left(\nabla \times B\right) \times B$$

Use BAC-CAB + no. div. to rewrite:

$$f_{Lor} = \frac{B \bullet \nabla B}{4\pi} - \nabla\left(\frac{B^2}{8\pi}\right)$$

magnetic tension **magnetic pressure**

Alfven speed

Alfven speed

$$V_A \equiv \frac{B}{\sqrt{4\pi\rho}}$$

Note:

$$\frac{\rho V_A^2}{2} = \frac{B^2}{8\pi} = P_{mag} = E_{mag}$$

Compare sound speed

$$a \equiv \sqrt{\frac{P_{gas}}{\rho}}$$

$$P_{gas} = \rho a^2$$

Plasma "beta"

$$\beta \equiv \frac{P_{gas}}{P_{mag}} = \frac{P_{gas}}{B^2/8\pi} = 2\left(\frac{a}{V_A}\right)^2$$

low"beta" plasma strongly magnetic: $V_A > a$

high "beta" plasma weakly magnetic: $V_A < a$

Key MHD concepts

- Reynolds no. Re >> 1
- Re->inf => “ideal” => frozen flux
- breaks down at small scales: reconnection
- Lorentz force ~ mag. pressure + tension
- Alfven speed V_A~B/sqrt(ρ)
- plasma beta ~ P_{gas}/P_{mag} ~ $(a/V_A)^2$

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Apply to solar corona & wind

- Solar corona
  - high T => high P_{gas}
  - scale height H ~< R
  - breakdown of hydrostatic equilibrium
  - pressure-driven solar wind expansion
- How does magnetic field alter this?
  - closed loops => magnetic confinement
  - open field => coronal holes
  - source of high speed solar wind

Hydrostatic Scale Height

Hydrostatic equilibrium:

$$-\frac{GM}{r^2} = \frac{1}{\rho}\frac{dP}{dr} \equiv \frac{a^2}{H} \qquad P = \rho a^2$$

Scale Height:

$$\frac{H}{R} = \frac{a^2 R}{GM} \approx \frac{T_6}{14}$$

solar photosphere: $T_6 = 0.006$ $\qquad \dfrac{H}{R} \approx \dfrac{1}{2000}$

solar corona: $T_6 = 2$ $\qquad \dfrac{H}{R} \approx \dfrac{1}{7}$

Failure of hydrostatic equilibrium for hot, isothermal corona

hydrostatic equilibrium:

$$0 = -\frac{GM}{r^2} - \frac{a^2}{P}\frac{dP}{dr}$$

$$\frac{P(r)}{P_o} = \exp\left[-\frac{R}{H}\left(1-\frac{R}{r}\right)\right] \rightarrow \exp\left[-\frac{R}{H}\right] \; for\ r \rightarrow \infty$$

 # decades of pressure decline:

$$\log\left(\frac{P_o}{P_\infty}\right) = \frac{R}{H}\log e \approx \frac{6}{T_6}$$

observations for TR vs. ISM:

$$\log\left(\frac{P_{TR}}{P_{ISM}}\right) = 12$$

Solar corona $T_6 \sim 2$ => must expand!

Spherical Expansion of Isothermal Solar Wind

Momentum and Mass Conservation:

$$\mathrm{v}\frac{d\mathrm{v}}{\mathrm{dr}} = -\frac{GM}{r^2} - \frac{a^2}{\rho}\frac{d\rho}{dr} \qquad \frac{d\left(\rho \mathrm{v} r^2\right)}{dr} = 0$$

Combine to **eliminate density**:

$$\left(1 - \frac{\mathrm{a}^2}{\mathrm{v}^2}\right)\mathrm{v}\frac{d\mathrm{v}}{\mathrm{dr}} = \frac{2\mathrm{a}^2}{r} - \frac{GM}{r^2}$$

RHS=0 at “critical” radius:

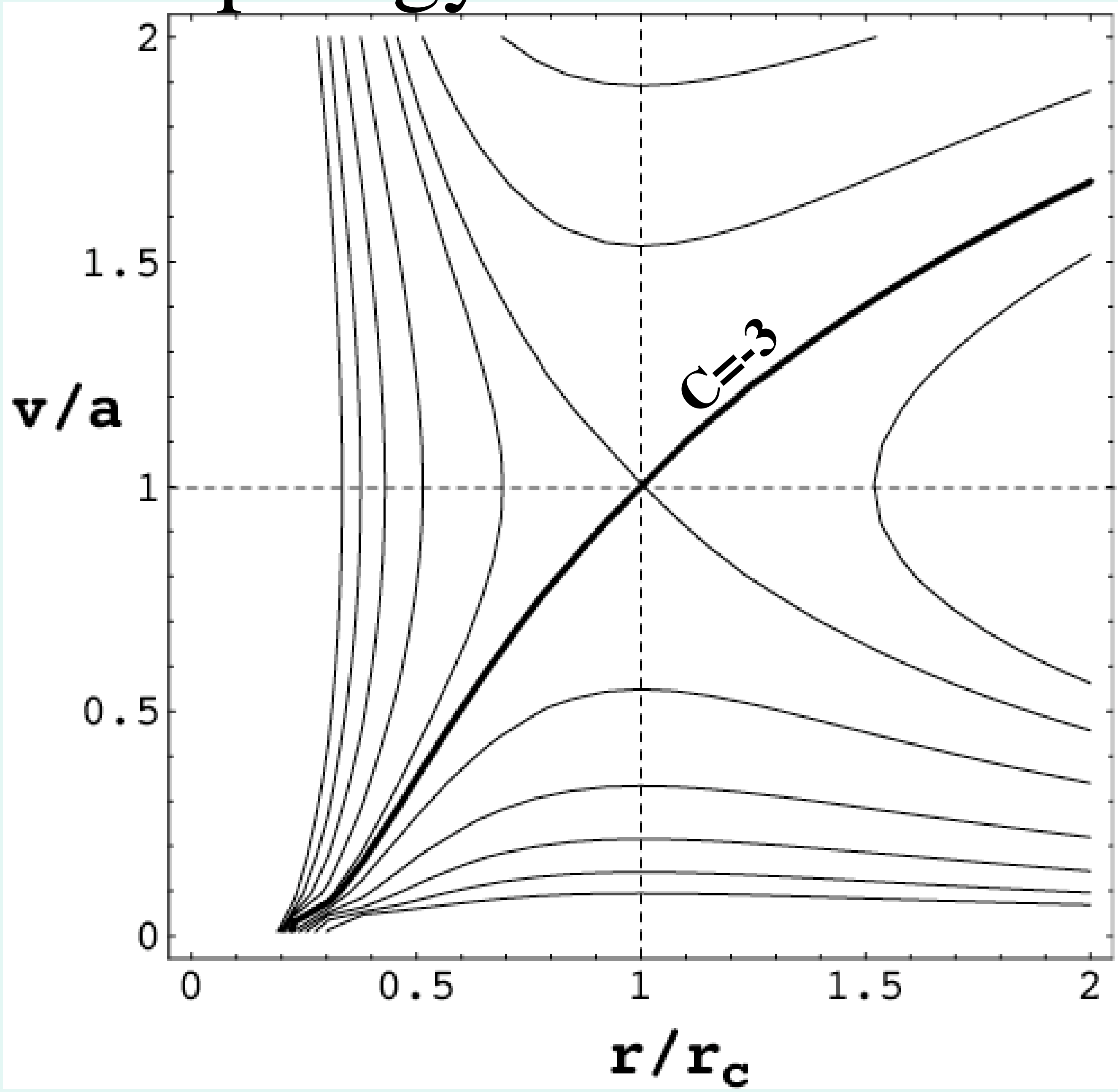
$$r_c = \frac{GM}{2a^2}$$

Integrate for transcendental soln:

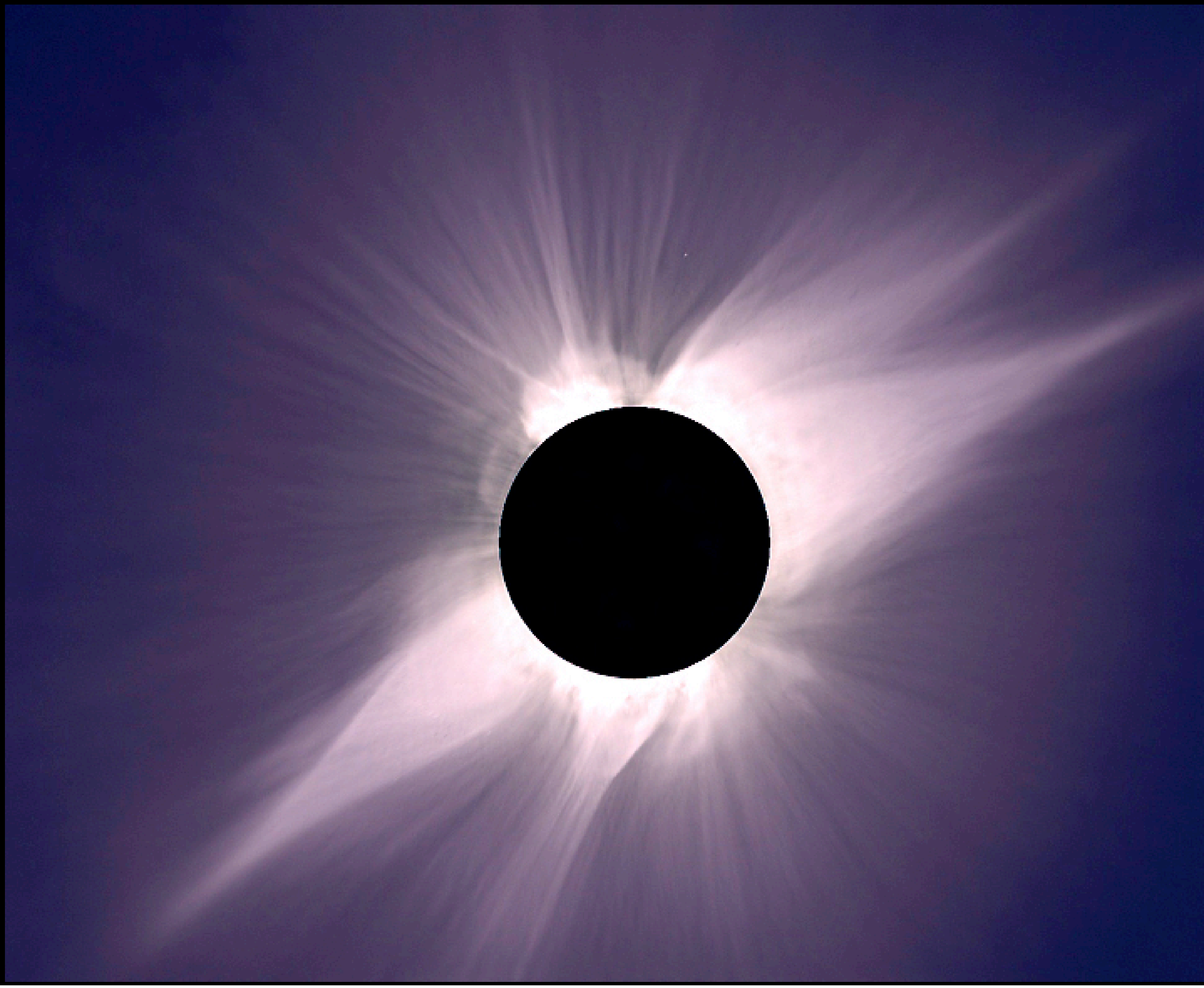
$$\frac{\mathrm{v}^2}{a^2} - \ln\frac{\mathrm{v}^2}{a^2} = 4\ln\frac{r}{r_c} + \frac{4r_c}{r} + C$$

$C = -3$ => Transonic soln: $\mathrm{v}\left(r_c\right) \equiv a \qquad r_c = r_s$ **sonic** radius

Solution topology for isothermal wind



Corona during Solar Eclipse



Kopp-Holzer Non-Radial Expansion

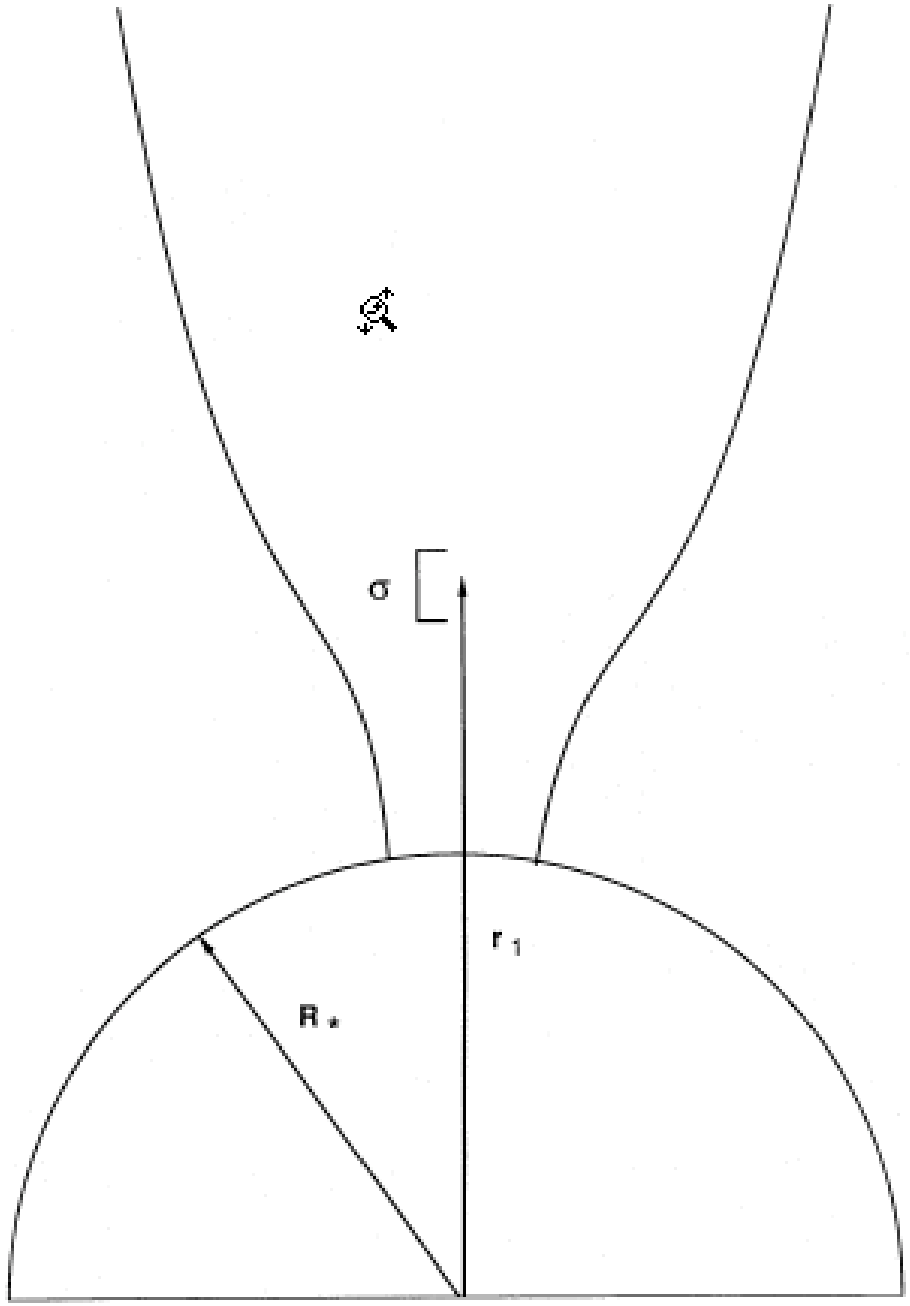


FIG. 1.—Flow tube boundaries derived from eqs. (6) and (7) with $f_{max} = 2.0$, $r_1 = 1.5\ R_*$, and $\sigma = 0.1\ R_*$. The angular width of the region at its base was taken to be 20°.

Mach number topology

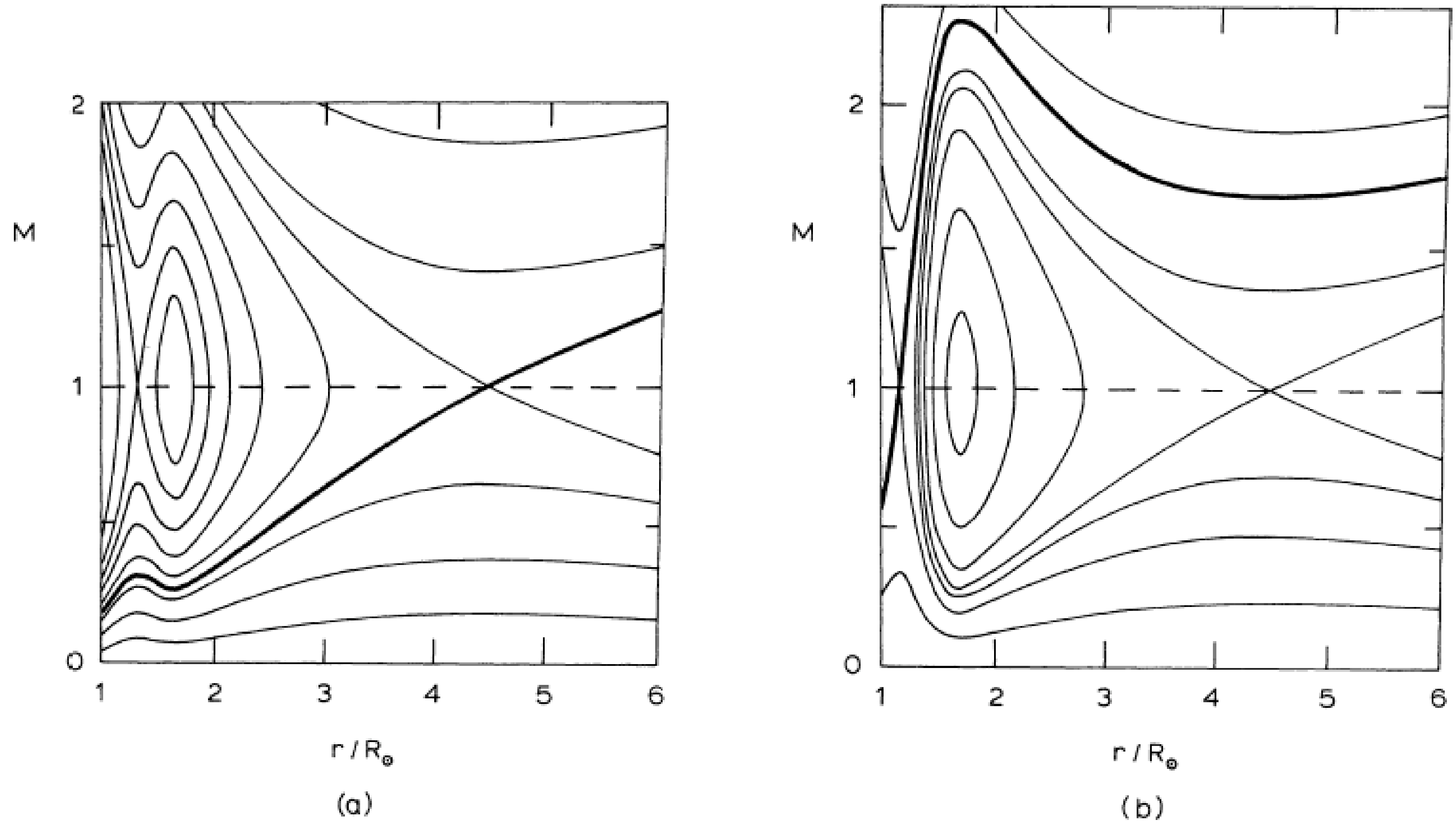
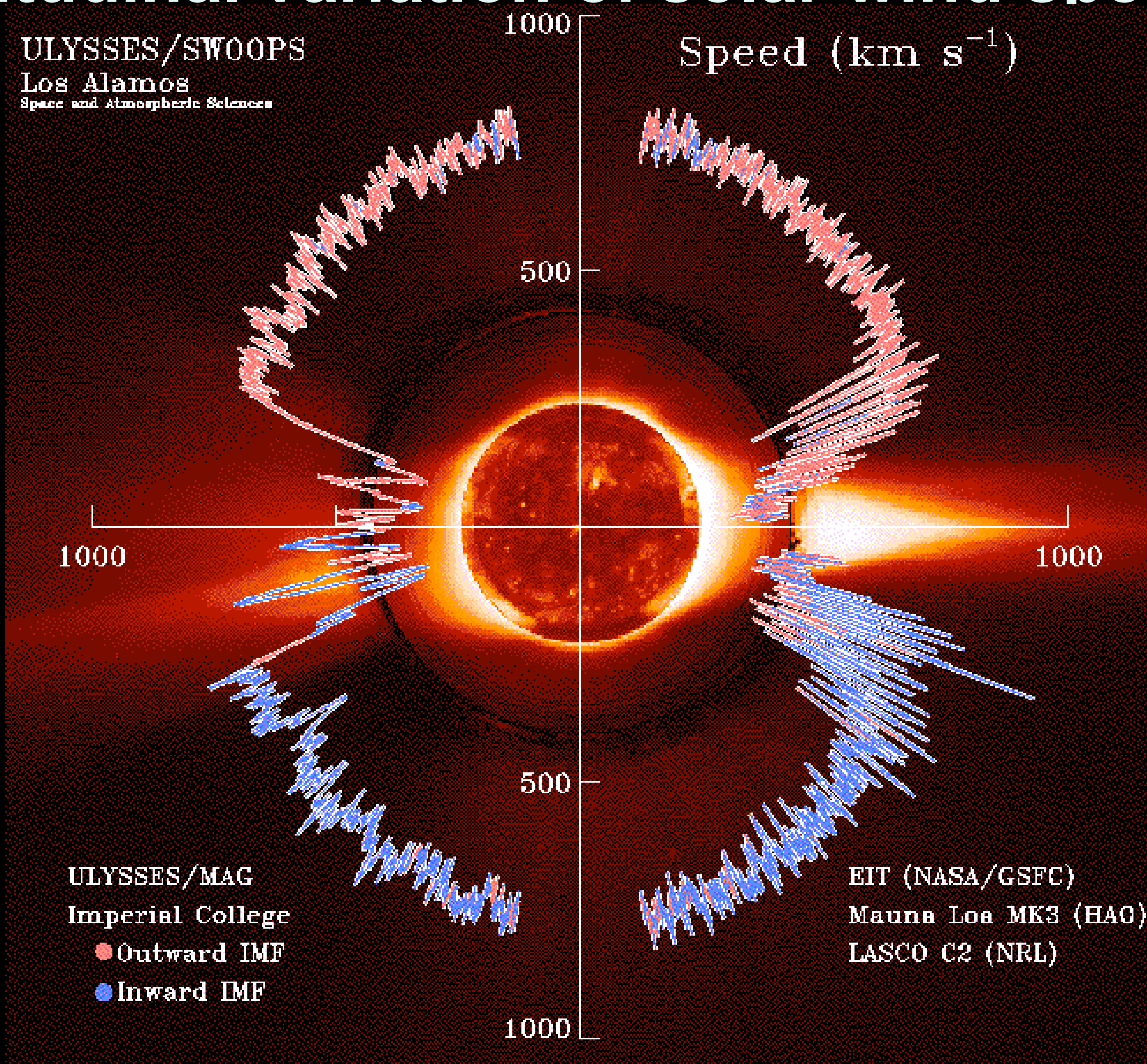


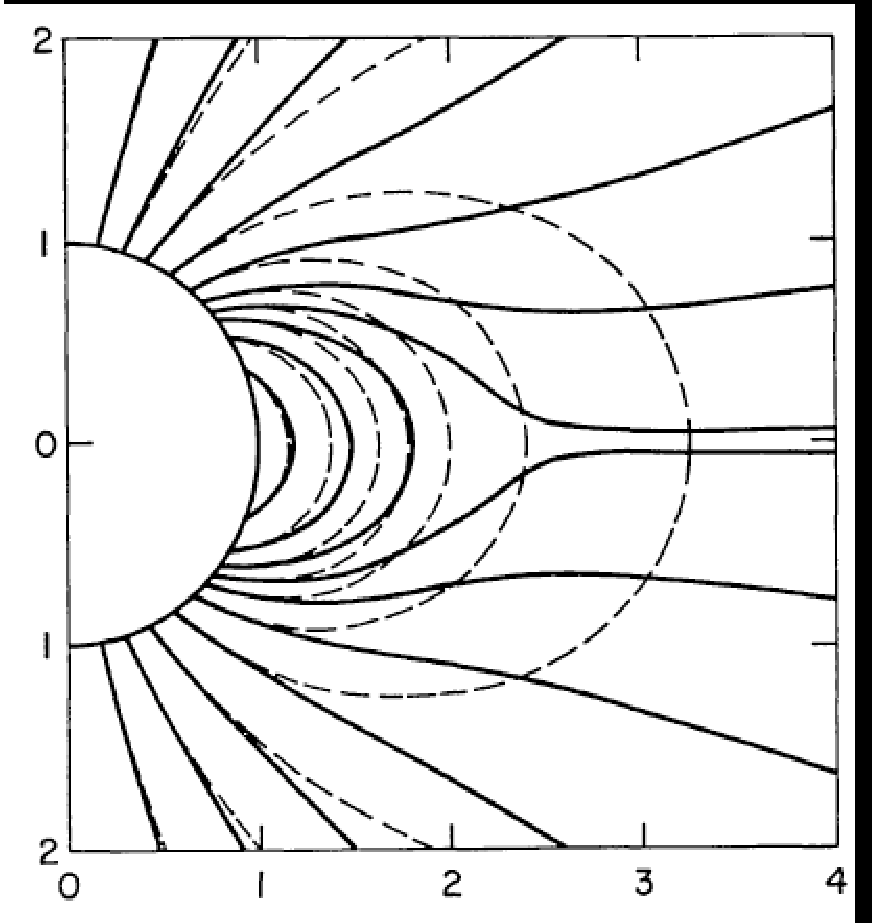
Fig. 2. Solution topologies for polytropic flow ($\alpha = 1.1$) in the divergent geometries described in the text. The energy per gram, E, is the same (1.8×10^{15} ergs/g) for all the curves. (*a*) $f_{max} = 3$, for which the physically realistic solution crosses the outermost critical point at 4.5 $R_{\odot}$. (*b*) $f_{max} = 12$, for which the solution starting at the base and extending to infinity becomes supersonic at the innermost critical point at 1.15 $R_{\odot}$.

Latitudinal variation of solar wind speed

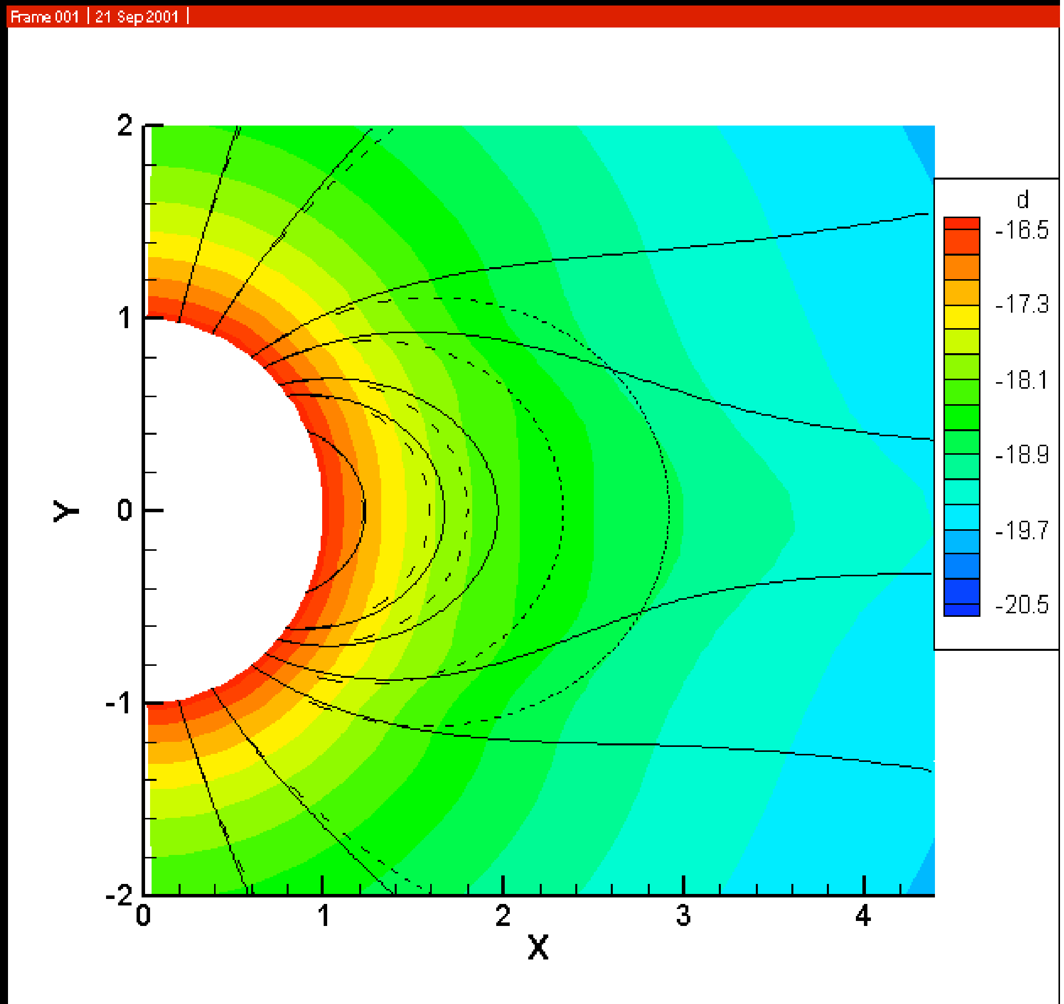


MHD model for coronal expansion vs. solar dipole

Pneumann & Kopp 1971
Iterative solution



MHD Simulation



Wind Magnetic Confinement

Ratio of magnetic to kinetic energy density:

$$\eta(r) \equiv \frac{B^2/8\pi}{\rho v^2/2}$$

$$= \frac{B^2 r^2}{\dot{M} v} = \boxed{\frac{B_*^{\,2} R_*^{\,2}}{\dot{M} v_\infty}} \frac{(r/R_*)^{2-2\boxed{q}}}{(1-R_*/r)^\beta}$$

η_*

e.g, for dipole field,
q=3; $\eta \sim 1/r^4$

$\eta_* >> 1$ => strong magnetic confinement of wind

for solar wind with $B_{dipole} \sim 1$ G: $\eta_* \sim 40$