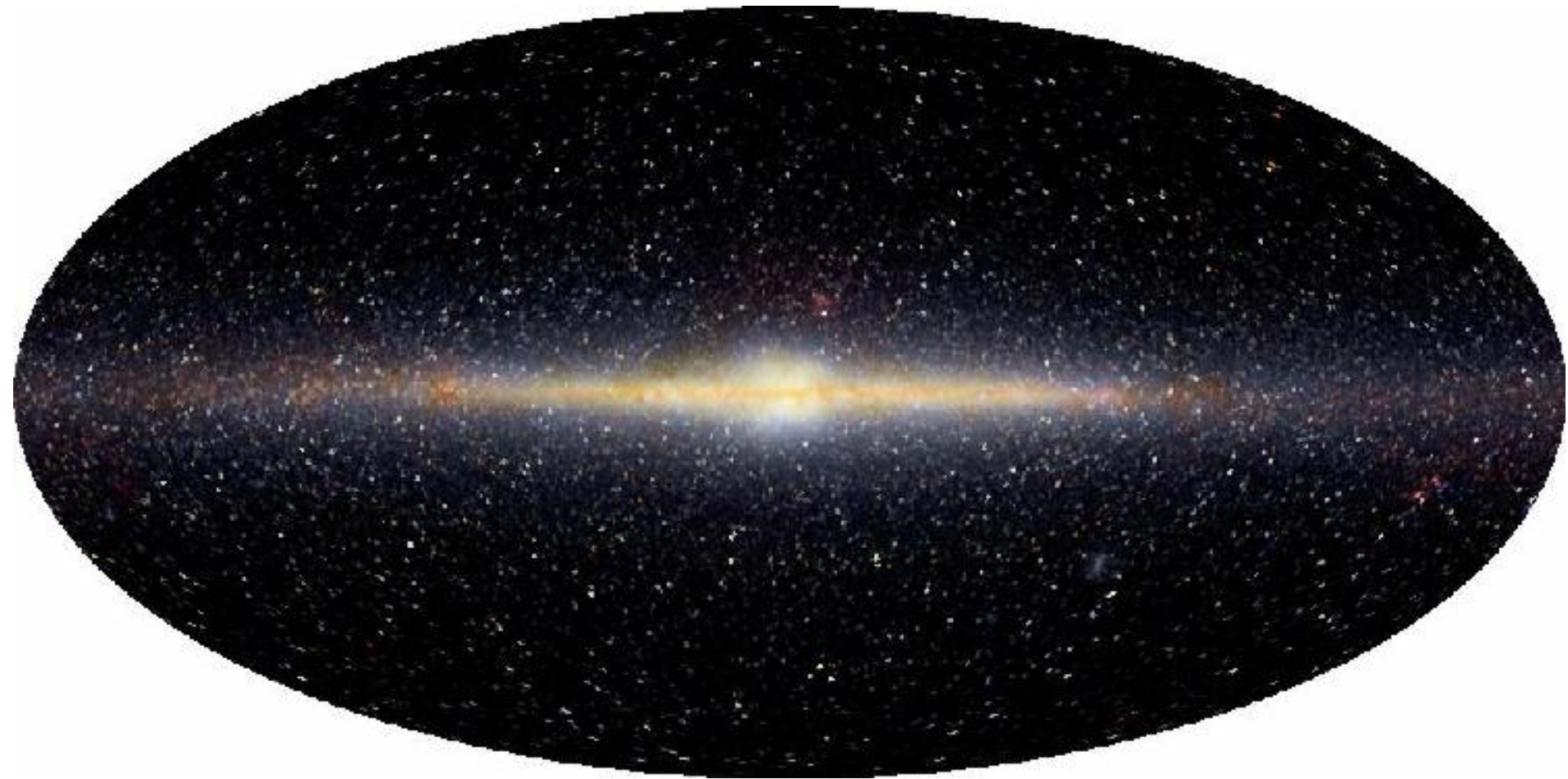
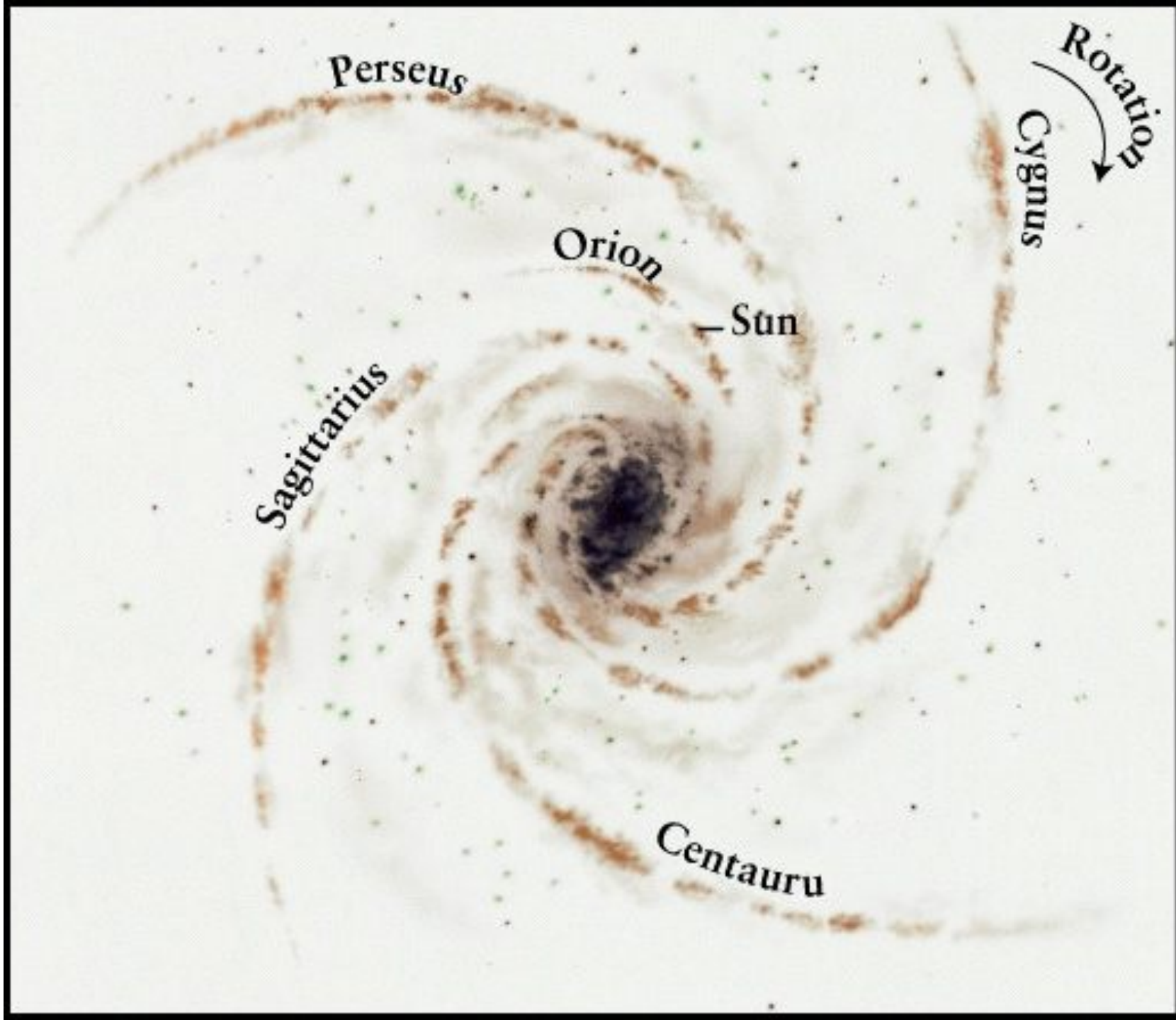


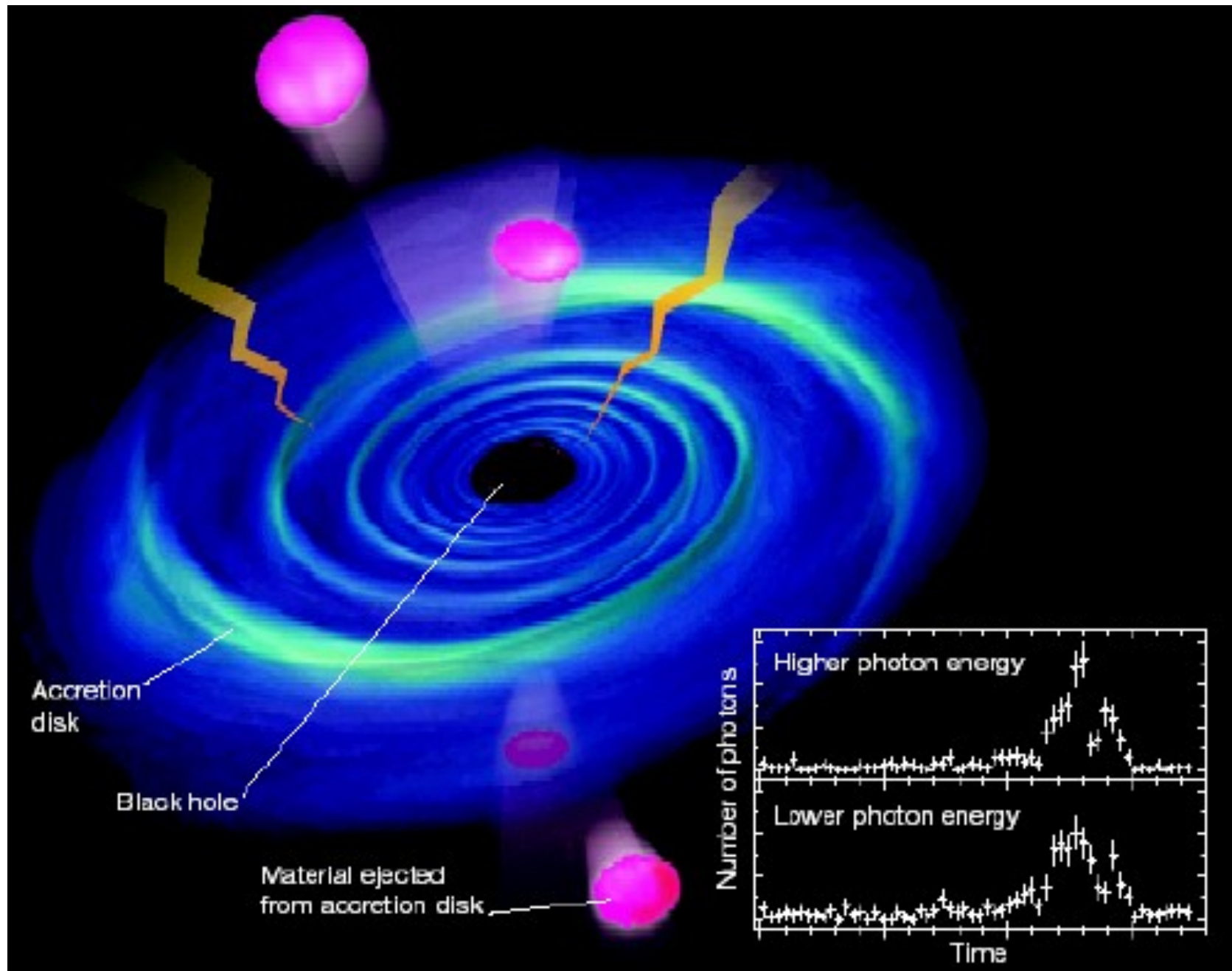
résonances dans les disques

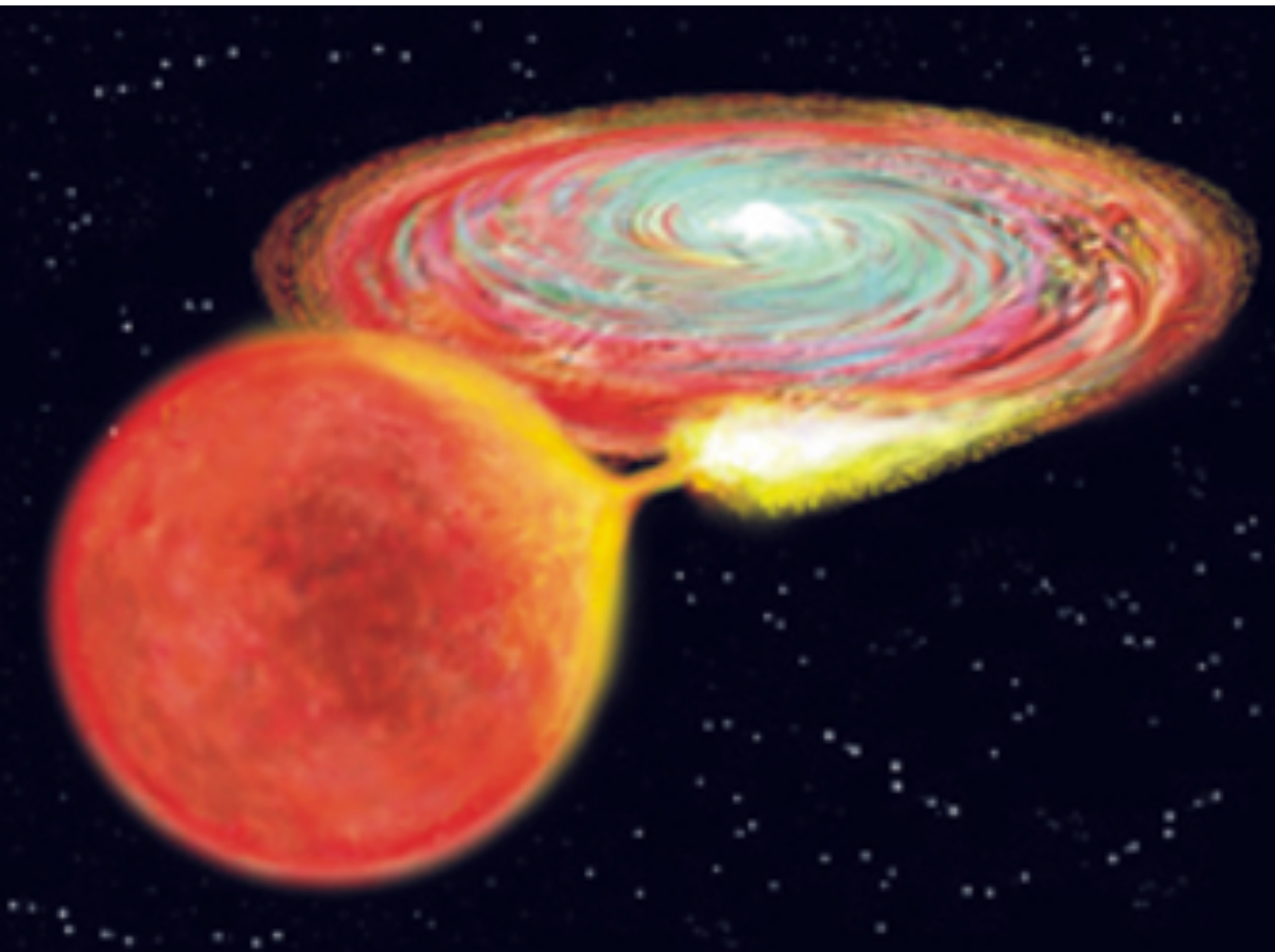


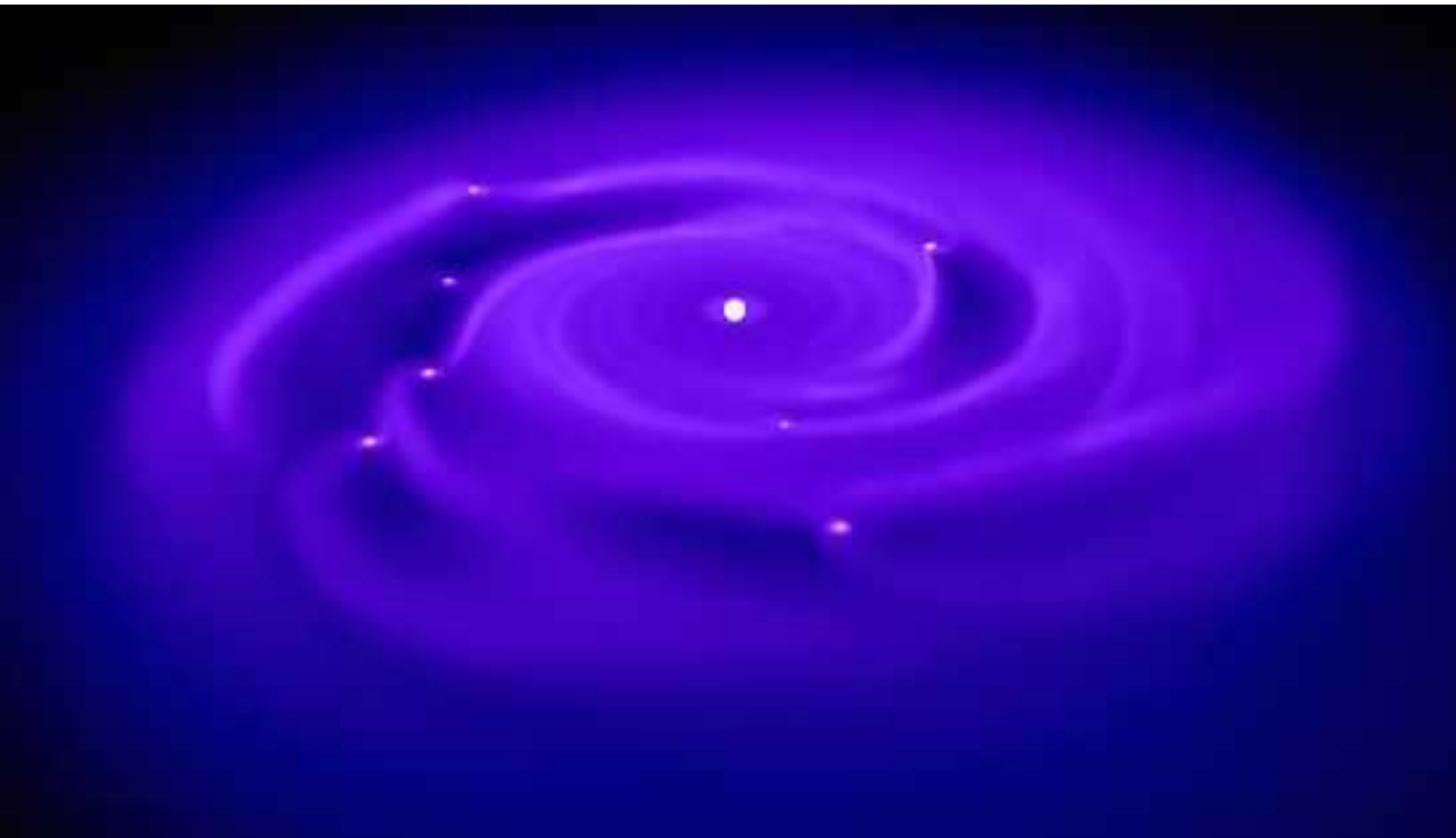


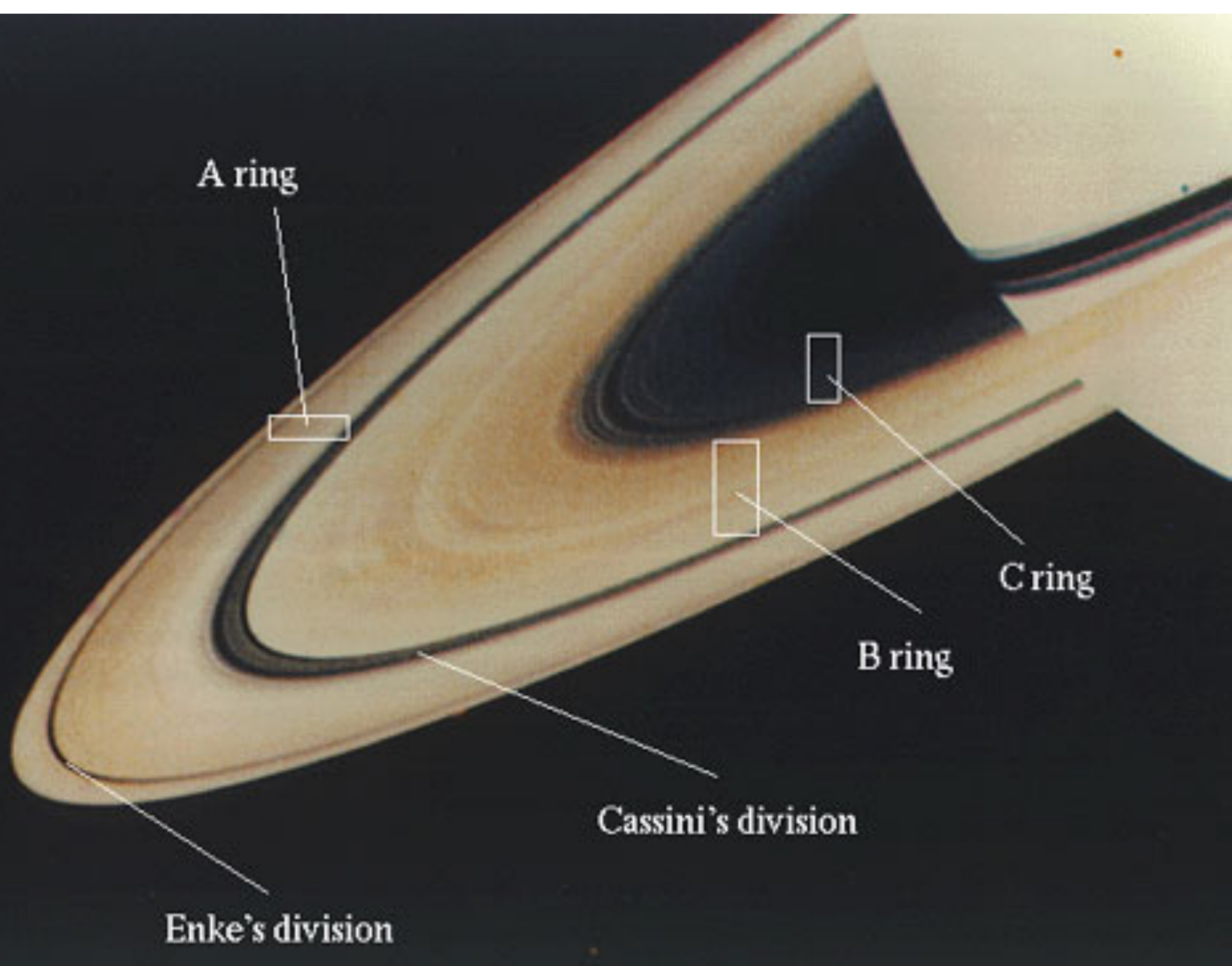














difficultés: problème à N-corps où  $N \gg 1$  ! →

phénomènes **collectifs** complexes (pression, auto-gravité, collisions, viscosité)

simplifications:

si perturbations assez faibles, on peut rester dans un cadre **linéaire**

approche:

on oublie le comportement individuel des particules → description **fluide** (hydrodynamique)

## idées générales sur la dynamique des disques

une fois la linéarisation faite, étudier d'abord la partie **libre**, puis la partie **forcée**. Plus précisément:

(1) étudier d'abord les modes **propres** du disque, i.e. son comportement sous l'effet du corps central, de sa propre gravité ("auto-gravitant"), et d'effets hydrodynamiques locaux (pression, viscosité)

(2) étudier ensuite la réponse **forcée** par un perturbateur extérieur (e.g. un satellite), en particulier près d'une **résonance** avec ce satellite

**donc:**

**[équation linéaire décrivant la solution libre] =**

**[les termes de forçage dus à un perturbateur extérieur]**

**équations fondamentales de la physique:**

- p.f.d.
- conservation de la masse
- équation d'état (e.g. gaz parfait)
- équation de champ (e.g. Poisson)

un disque dissipatif orbitant autour d'un corps central massif conserve son moment cinétique et diminue son énergie → il **s'aplatit**

moment cinétique:

$$\mathbf{H} = \sum_i m_i \mathbf{r}_i \times \mathbf{v}_i = \sum_i m_i \mathbf{r}_i \times \mathbf{v}_{iz} + \sum_i m_i \mathbf{r}_i \times \mathbf{v}_{ih}$$

module du moment cinétique:

$$H = \mathbf{H} \cdot \hat{\mathbf{z}} = \sum_i m_i (\mathbf{r}_i \times \mathbf{v}_{iz}) \cdot \hat{\mathbf{z}} + \sum_i m_i (\mathbf{r}_i \times \mathbf{v}_{ih}) \cdot \hat{\mathbf{z}} = \sum_i m_i r_i v_{i\theta}$$

énergie:

$$E = \sum_i m_i \Phi_P(r_i) + \sum_i m_i v_{i\theta}^2 / 2 + \sum_i m_i v_{ir}^2 / 2 + \sum_i m_i v_{iz}^2 / 2$$

$E$  minimale à  $H$  constant →  $v_\theta = 0$  et  $v_z = 0$  → disque plat et circulaire



**un disque dissipatif orbitant autour d'un corps central massif conserve son moment cinétique et diminue son énergie → il **s'aplatit****

**question:**

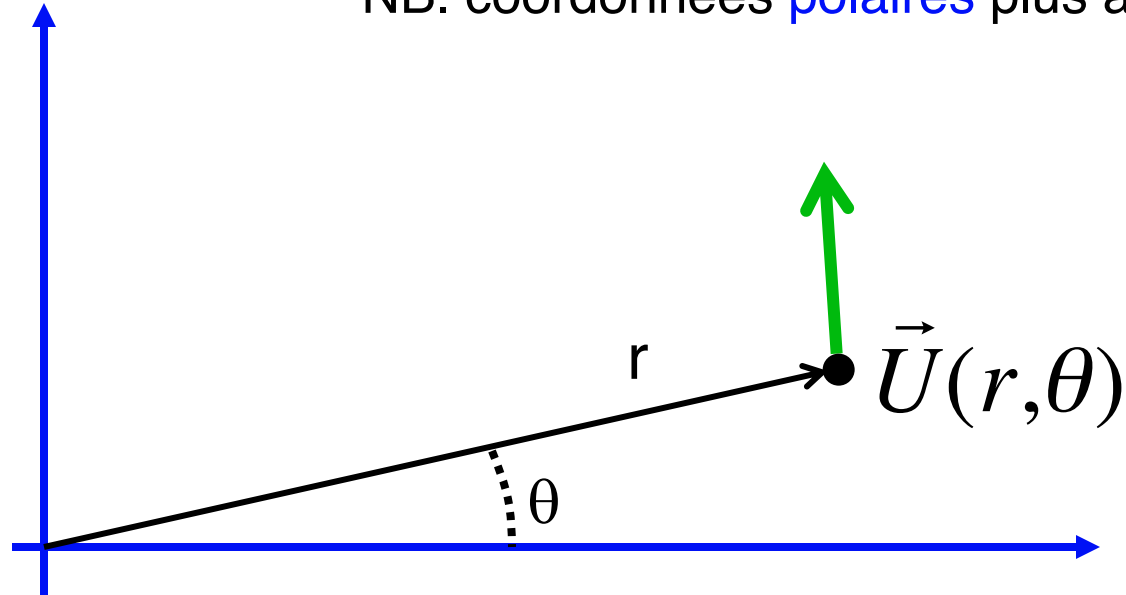
un disque devient-il infiniment mince?

**réponse:**

non, l'autogravité et la viscosité maintiennent une épaisseur faible mais *non nulle*. Par exemple, les anneaux de Saturne ont une épaisseur de ~ qqs mètres, pour une taille de ~ 100000 km.

L'autogravité et la viscosité expliquent également la présence d'ondes résonantes, excitées par des satellites.

NB. coordonnées **polaires** plus appropriées



description **eulérienne**: on considère les quantités physiques en un point fixe  $(r, \theta)$ :

$\vec{U}(r, \theta)$

vitesse en  $(r, \theta)$

$\rho(r, \theta)$  ou  $\Sigma(r, \theta)$

masse volumique ou mieux, surfacique en  $(r, \theta)$

$P(r, \theta)$

pression en  $(r, \theta)$

...

(NB. description **eulérienne**  $\neq$  **lagrangienne**: les quantités physiques sont alors décrites en *suivant* l'élément de fluide en déplacement)

on considère d'abord un disque infiniment mince *non* perturbé  
le plus simple possible:

$\vec{U}_0(r)$  vitesse képlérienne

$\Sigma_0 = \text{cste}$  (le plus simple pour l'instant...)

$P_0 = \text{cste}$

...

puis le disque **perturbé**:

$\vec{U}_0(r, \theta) = \vec{U}_0(r) + \vec{u}(r, \theta)$   $u \ll \vec{U}_0$

$\Sigma = \Sigma_0 + \sigma(r, \theta)$  avec:  $\sigma \ll \Sigma_0$

$P = P_0 + p(r, \theta)$   $p \ll P_0$

...

...

équation du mouvement:

$$\delta m \cdot \frac{D\vec{U}}{Dt} = \vec{F}$$

où:  $\frac{D\vec{U}}{Dt}$  est la dérivé *totale* la vitesse (i.e. en suivant le mouvement, c.a.d. **lagrangienne**)

$$\frac{D\vec{U}}{Dt} = \frac{\vec{U}(\vec{r} + \delta\vec{r}, t + \delta t) - \vec{u}(\vec{r}, t)}{\delta t} =$$

$$\frac{\partial\vec{U}}{\partial x} \cdot \dot{x} + \frac{\partial\vec{U}}{\partial y} \cdot \dot{y} + \frac{\partial\vec{U}}{\partial t} = \frac{\partial\vec{U}}{\partial r} \cdot \dot{r} + \frac{\partial\vec{U}}{\partial \theta} \cdot \dot{\theta} + \frac{\partial\vec{U}}{\partial t}$$

soit: 
$$\boxed{\frac{D\vec{U}}{Dt} = (\vec{U} \cdot \vec{\nabla})\vec{U} + \frac{\partial\vec{U}}{\partial t}}$$

(relation entre dérivées  
**lagrangienne** et **eulérienne**)



$$\left\{ \begin{array}{l} \frac{D\vec{U}}{Dt} = \frac{\partial\vec{U}}{\partial t} + (\vec{U} \cdot \vec{\nabla})\vec{U} = \vec{F} \quad \text{équation d'Euler (p.f.d.)} \\ \frac{\partial\Sigma}{\partial t} + \vec{\nabla} \cdot (\Sigma\vec{U}) = 0 \quad \text{équation de continuité (conservation de la masse)} \\ \nabla^2\Phi_D = 4\pi G\Sigma\delta(z) \quad \text{équation de Poisson (th. de Gauss)} \\ P = \Sigma c^2 \quad \text{équation d'état (ici, par exemple gaz parfait)} \end{array} \right.$$

où:

$$\vec{F} = \vec{F}_P + \vec{F}_D + \vec{F}_S - \frac{\vec{\nabla} \cdot \vec{P}}{\Sigma} = -\vec{\nabla}(\underbrace{\Phi_P}_{\text{planète}} + \underbrace{\Phi_D}_{\text{disque}} + \underbrace{\Phi_S}_{\text{satellite}}) - \frac{\vec{\nabla} \cdot \vec{P}}{\Sigma}$$

tenseur de pression  
(pression et viscosité)

$$\vec{U}_0(r, \theta) = \vec{U}_0(r) + \vec{u}_1(r, \theta)$$

$$\Sigma = \Sigma_0 + \Sigma_1(r, \theta)$$

$$P = P_0 + p_1(r, \theta)$$

$$\Phi_D = \Phi_{0,D} + \Phi_{1,D}$$

avec:

$$u_1 \ll \vec{U}_0$$

$$\Sigma_1 \ll \Sigma_0$$

$$p_1 \ll P_0$$

$$\Phi_{01,D} \ll \Phi_{0,D}$$

pour commencer: solution **libre** (pas de satellite) **sans viscosité**

$$\left\{ \begin{array}{l} \frac{\partial \vec{u}_1}{\partial t} + (\vec{u}_1 \cdot \vec{\nabla}) \vec{u}_0 + (\vec{u}_0 \cdot \vec{\nabla}) \vec{u}_1 = -\vec{\nabla}(\Phi_P + \Phi_D) - \frac{\vec{\nabla} p_1}{\Sigma_0} \\ \frac{\partial \Sigma_1}{\partial t} + \Sigma_0 \vec{\nabla} \cdot \vec{u}_1 = 0 \\ \nabla^2 \Phi_{1,D} = 4\pi G \Sigma_1 \delta(z) \\ p_1 = \Sigma_1 c^2 \end{array} \right.$$

**systeme equations lineaires decrivant l'evolution d'un disque soumis au potentiel central de la planete, a son propre potentiel, et a sa pression interne (sans viscosite)**

voir les **detaills** dans le dossier **“7- Resonances dans les disques”** sur ma page:

[http://www.lesia.obspm.fr/perso/bruno-sicardy/ensei/m2\\_obs/index.html](http://www.lesia.obspm.fr/perso/bruno-sicardy/ensei/m2_obs/index.html)

on "lance" dans le disque des modes propres de la forme:

module (amplitude)  
de la perturbation

$$\Sigma_1 = \Sigma_a \exp[i(\mathbf{k} \cdot \mathbf{x} - \omega t)] \quad \text{etc...}$$

avec (par exemple)  $\mathbf{k} = k\hat{\mathbf{e}}_x$

valeur complexe  
de la perturbation

$$\text{Equation de Poisson} \longrightarrow |\kappa| \Phi_a = -2\pi G \Sigma_a$$

alors les opérateurs  $\partial/\partial t$  et  $\nabla$  deviennent **algébriques**:

$$\partial/\partial t \Leftrightarrow -i\omega \quad \text{où } \omega = \text{vitesse angulaire du mode}$$

$$\nabla \cdot \Leftrightarrow i\mathbf{k} \cdot \quad \text{où } \mathbf{k} = \text{nombre d'onde du mode, } |\mathbf{k}| = 2\pi/\lambda$$



$$ik\Sigma_0 v_{ax} - i\omega\Sigma_a = 0$$

$$i\omega v_{ax} + 2\kappa v_{ay} + \left( \frac{2\pi G}{|k|} - \frac{c_s^2}{\Sigma_0} \right) ik\Sigma_a = 0$$

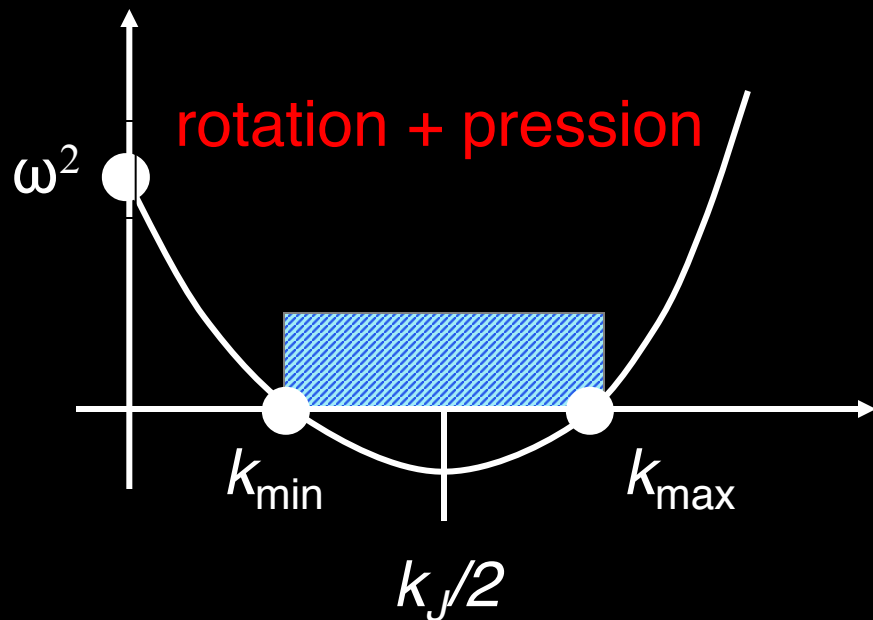
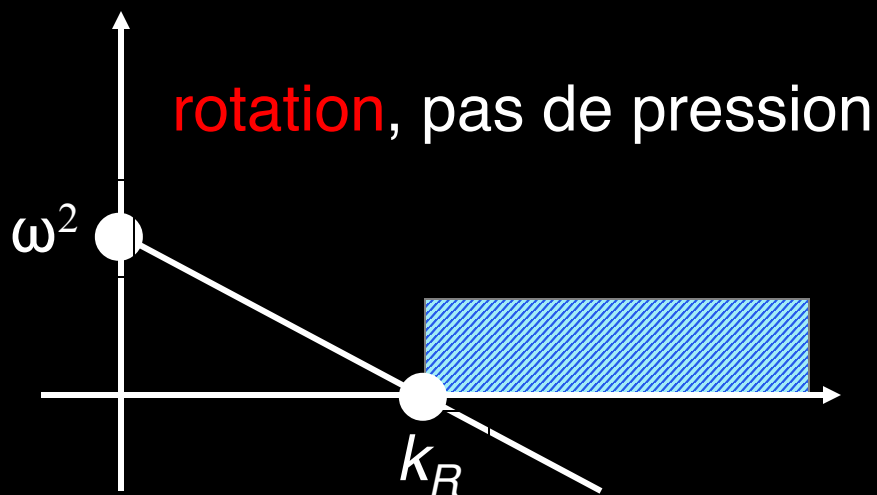
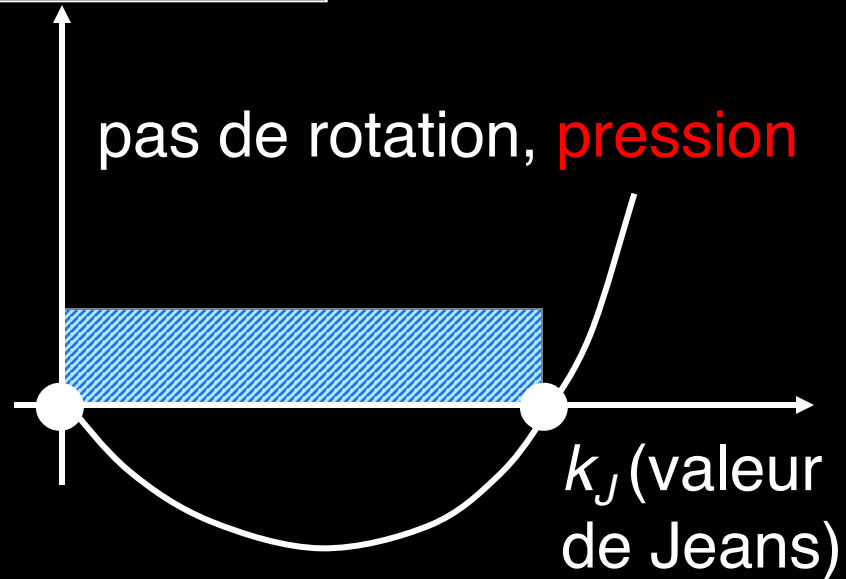
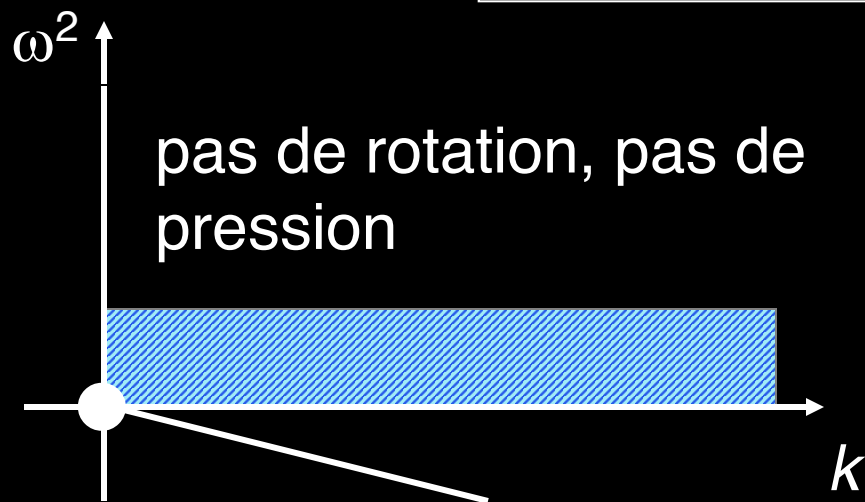
$$\kappa/2 v_{ax} - i\omega v_{ay} = 0$$

solution non triviale pour:

$$k^2 c_s^2 - 2\pi G \Sigma_0 |k| + \kappa^2 = \omega^2$$

→ c'est **relation de dispersion** des modes propres dans un disque **auto-gravitant** ( $\Sigma_0$ ) en **rotation** ( $\kappa$ ) et avec **pression** ( $c_s$ )

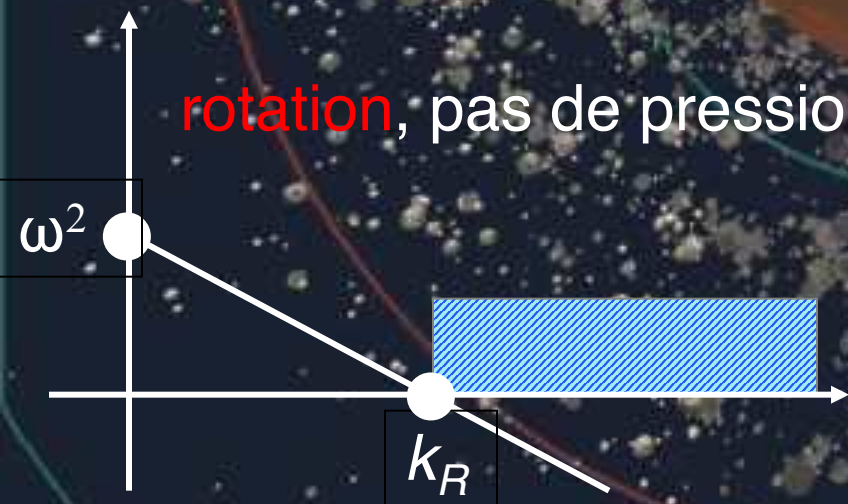
$$k^2 c_s^2 - 2\pi G \Sigma_0 |k| + \kappa^2 = \omega^2$$



**grandes** structures stabilisées  
par la rotation

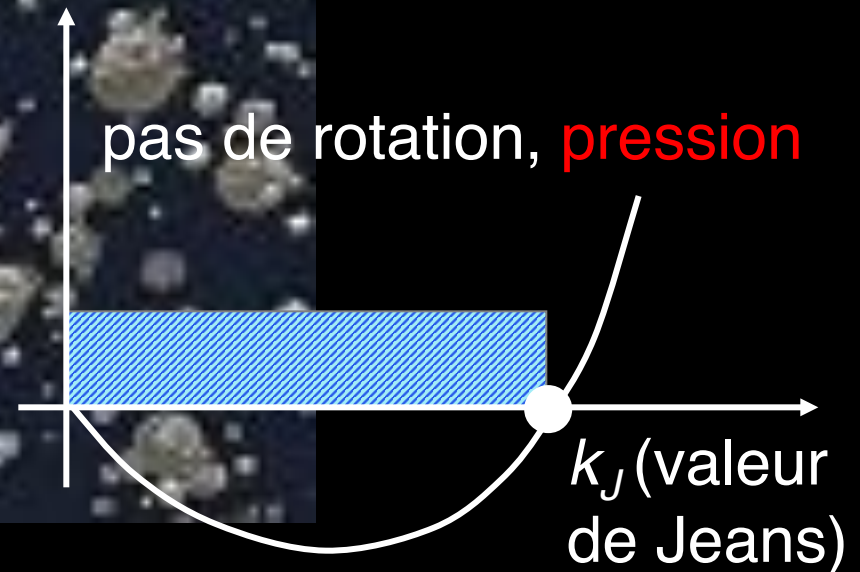
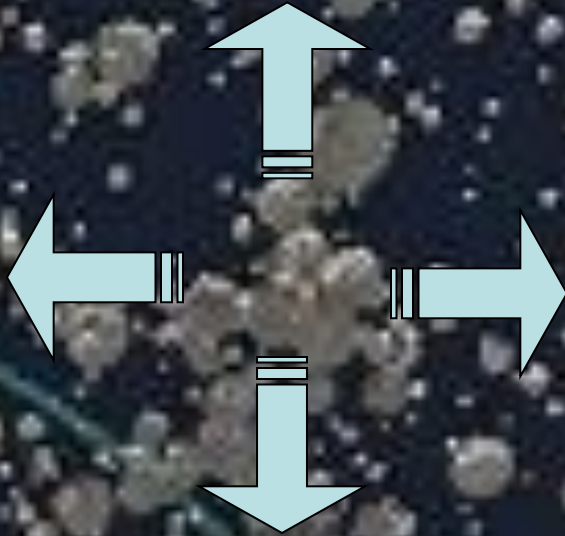
( $R^2 \Omega^2$  grand  
comparé à l'énergie  
potentielle de la  
structure)

**rotation**, pas de pression



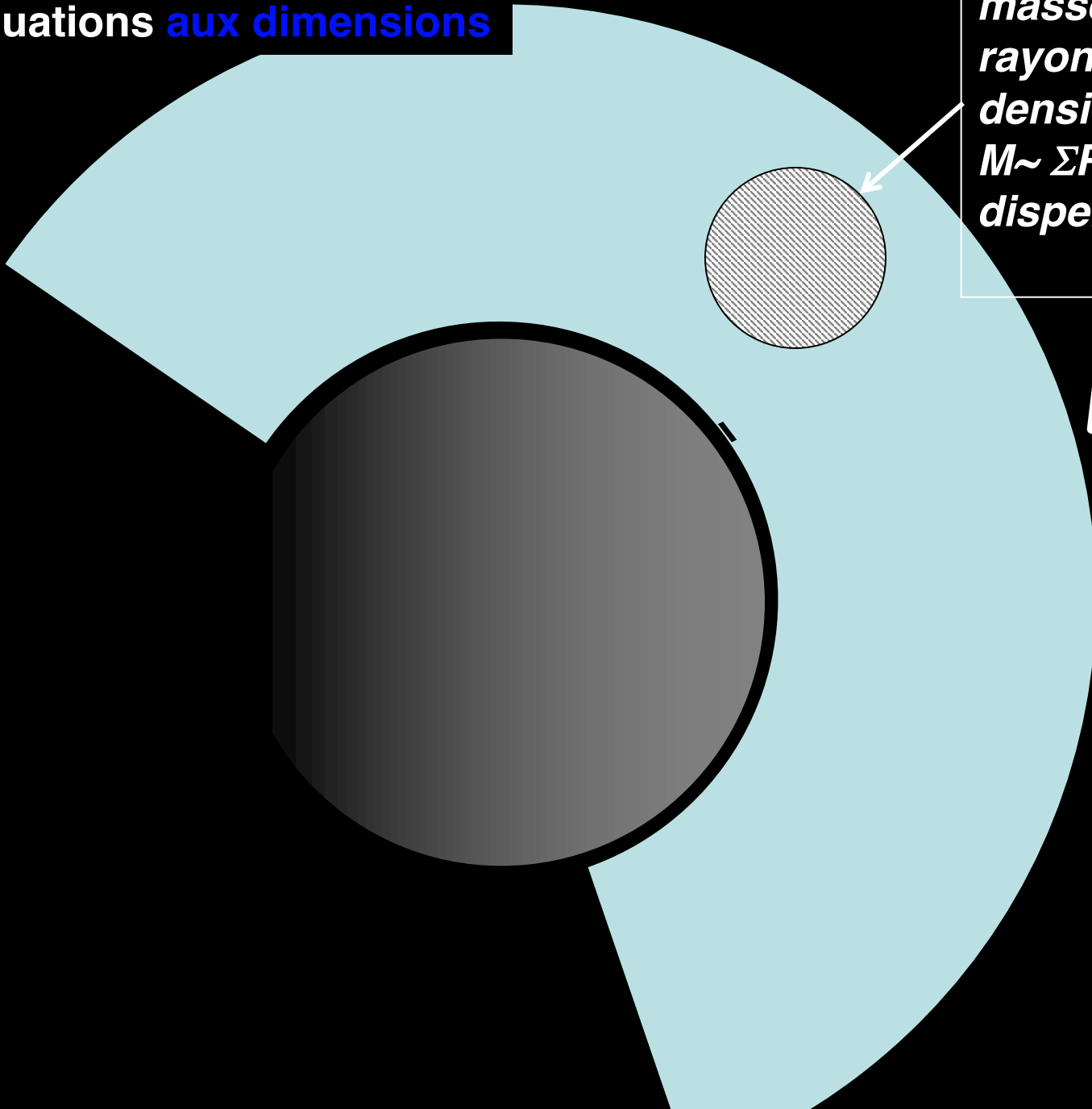
**petites** structures stabilisées  
par la **pression** (critère de Jeans)

( $mc_s^2$  grand comparé  
à l'énergie potentielle  
de la structure)



équations aux dimensions

*masse  $M$*   
*rayon  $R$*   
*densité surfacique  $\Sigma$*   
 *$M \sim \Sigma R^2$*   
*dispersion vitesse  $c_s$*



$\Omega$

énergie gravitationnelle:

$$U_G \sim -\frac{GM^2}{R} \sim -G\Sigma^2 R^3$$

énergie rotationnelle:

$$U_R \sim I\Omega^2 \sim MR^2\Omega^2 \sim \Sigma R^4\Omega^2$$

énergie thermique:

$$U_T \sim Mc_s^2 \sim \Sigma R^2 c_s^2$$

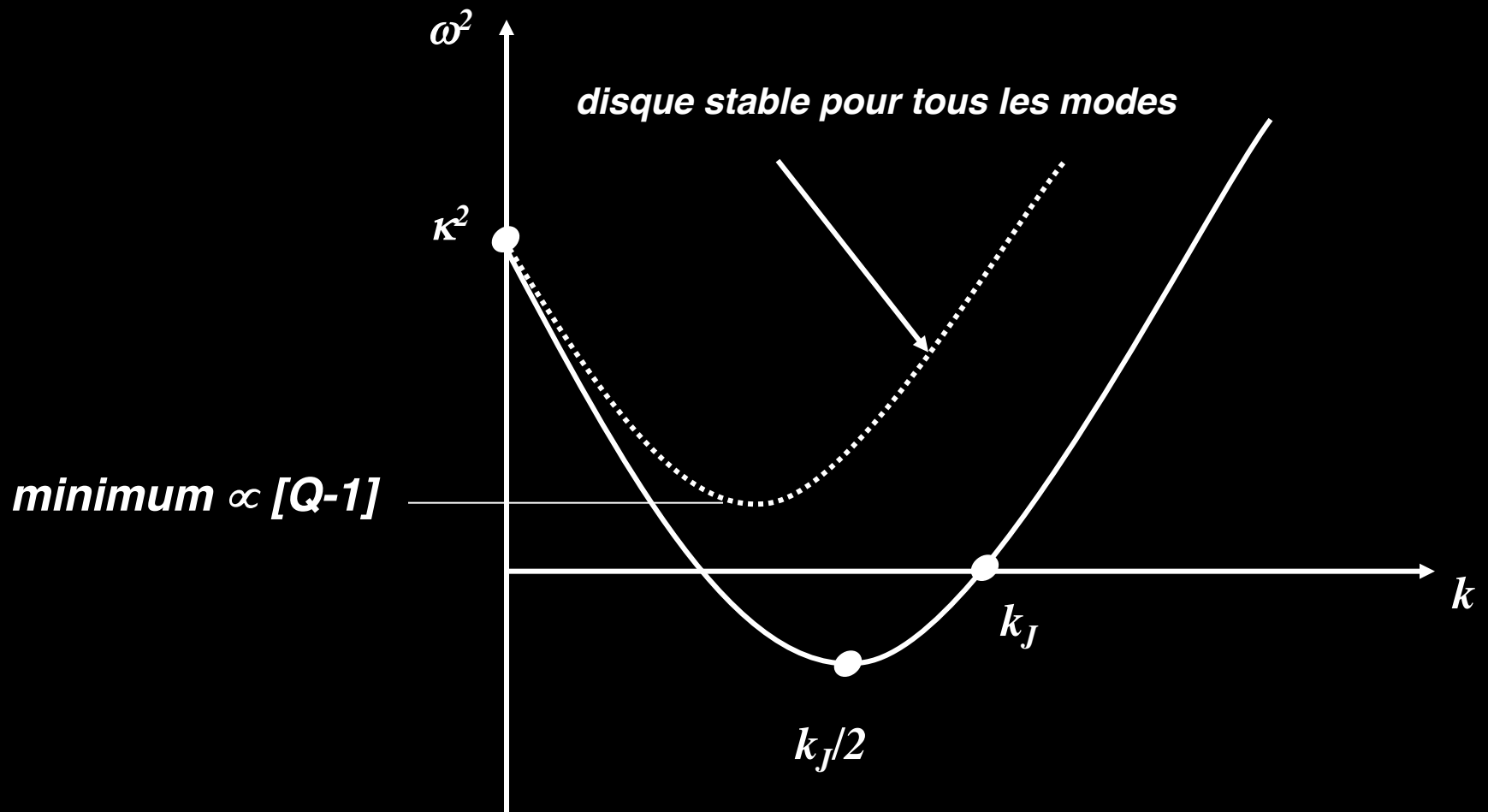
énergie totale →

$$U = U_G + U_R + U_T \sim \Sigma R^2 \left[ \Omega^2 R^2 - G\Sigma R + c_s^2 \right]$$
$$\sim \Sigma \frac{R^2}{k^2} \left[ c_s^2 k^2 - G\Sigma k + \Omega^2 \right]$$

où:  $k \sim 1/R$

est le nombre d'onde de la perturbation

relation de dispersion



où

$$Q = \frac{c_s \Omega}{\pi G \Sigma_0}$$

est le **paramètre de Toomre (1964)**



**stabilité pour tous les modes si:**

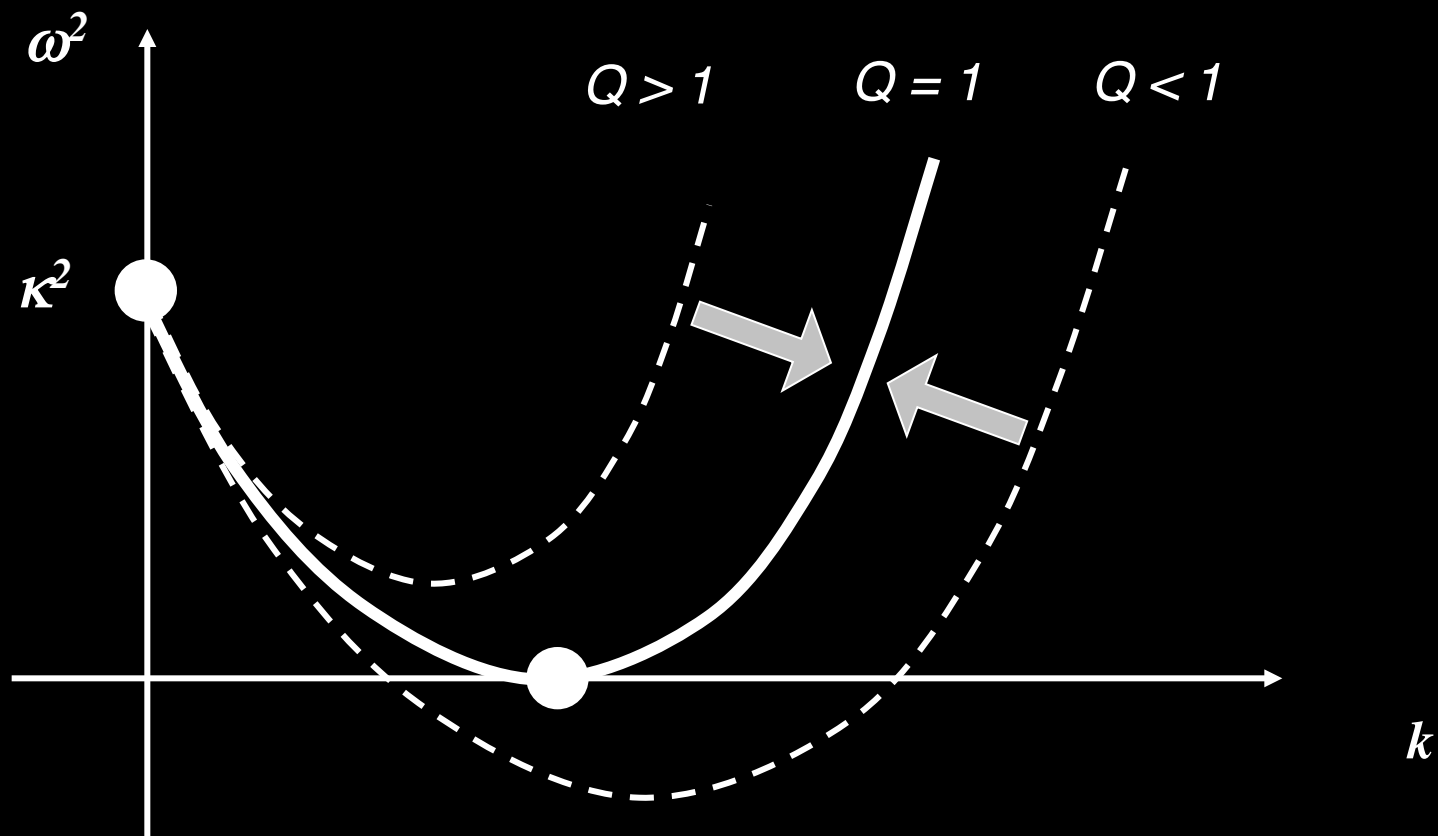
$$Q = \frac{c_s \Omega}{\pi G \Sigma_0} > 1$$

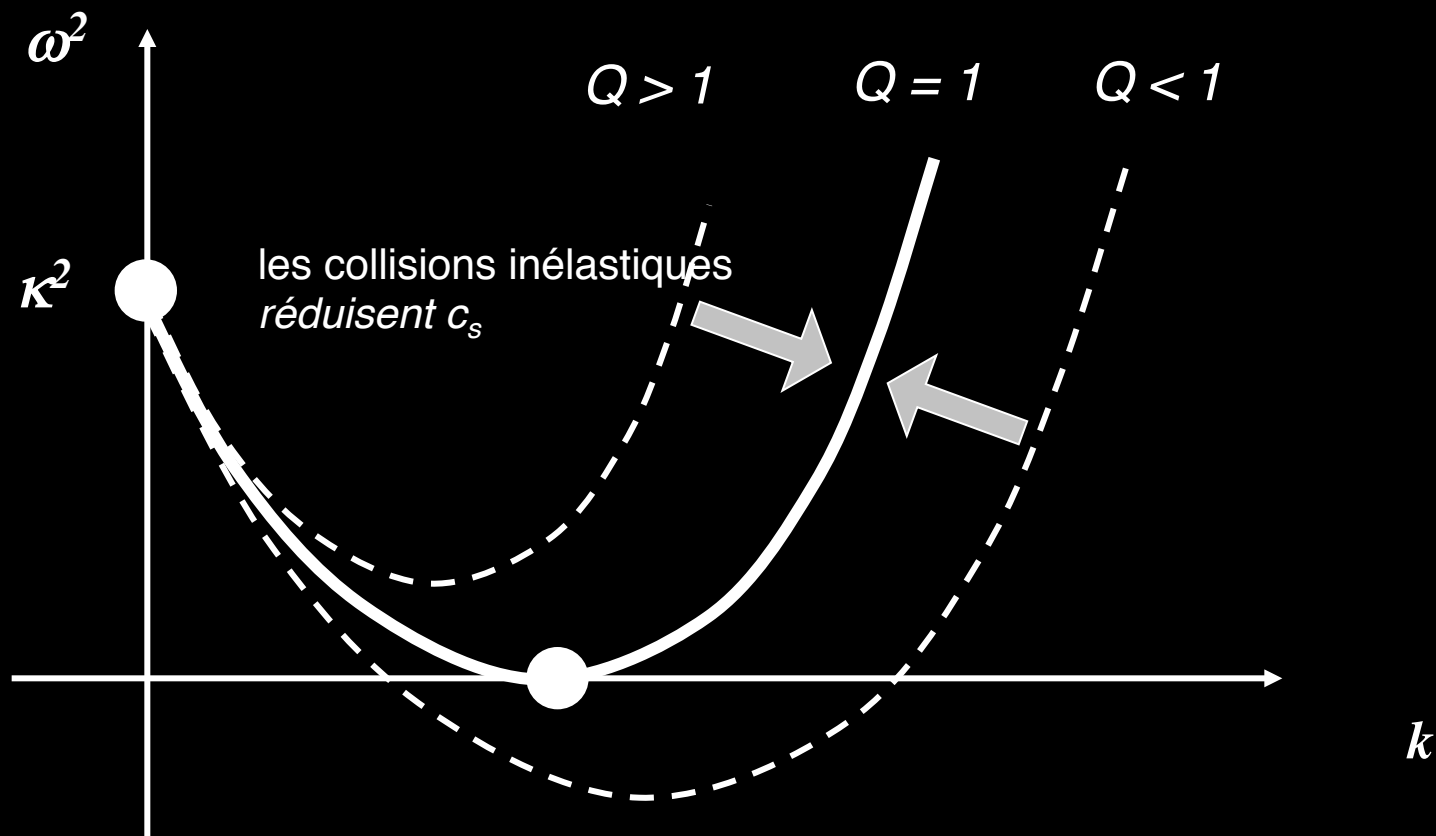
→ critère de **stabilité de Toomre** pour les disques minces

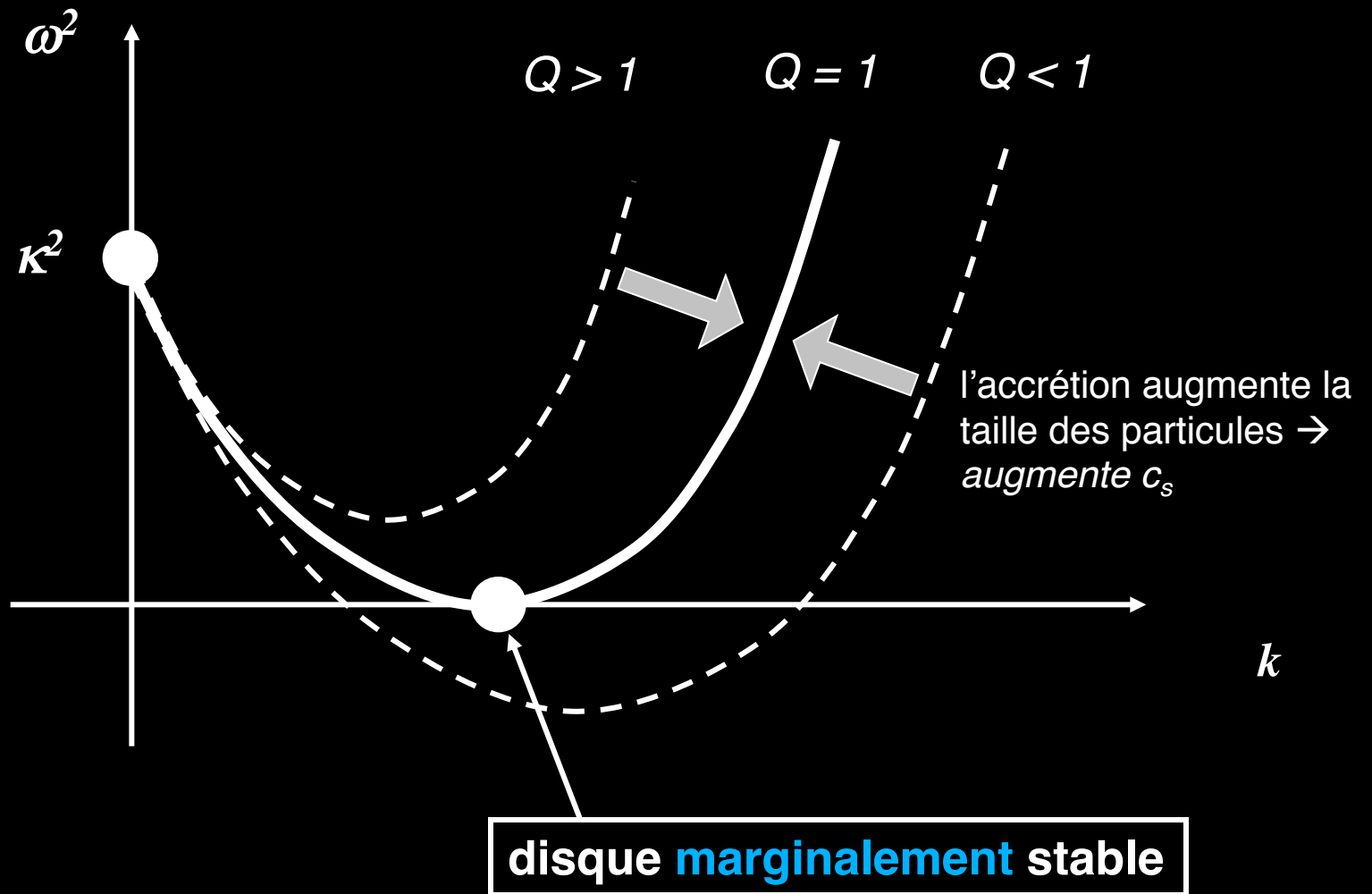


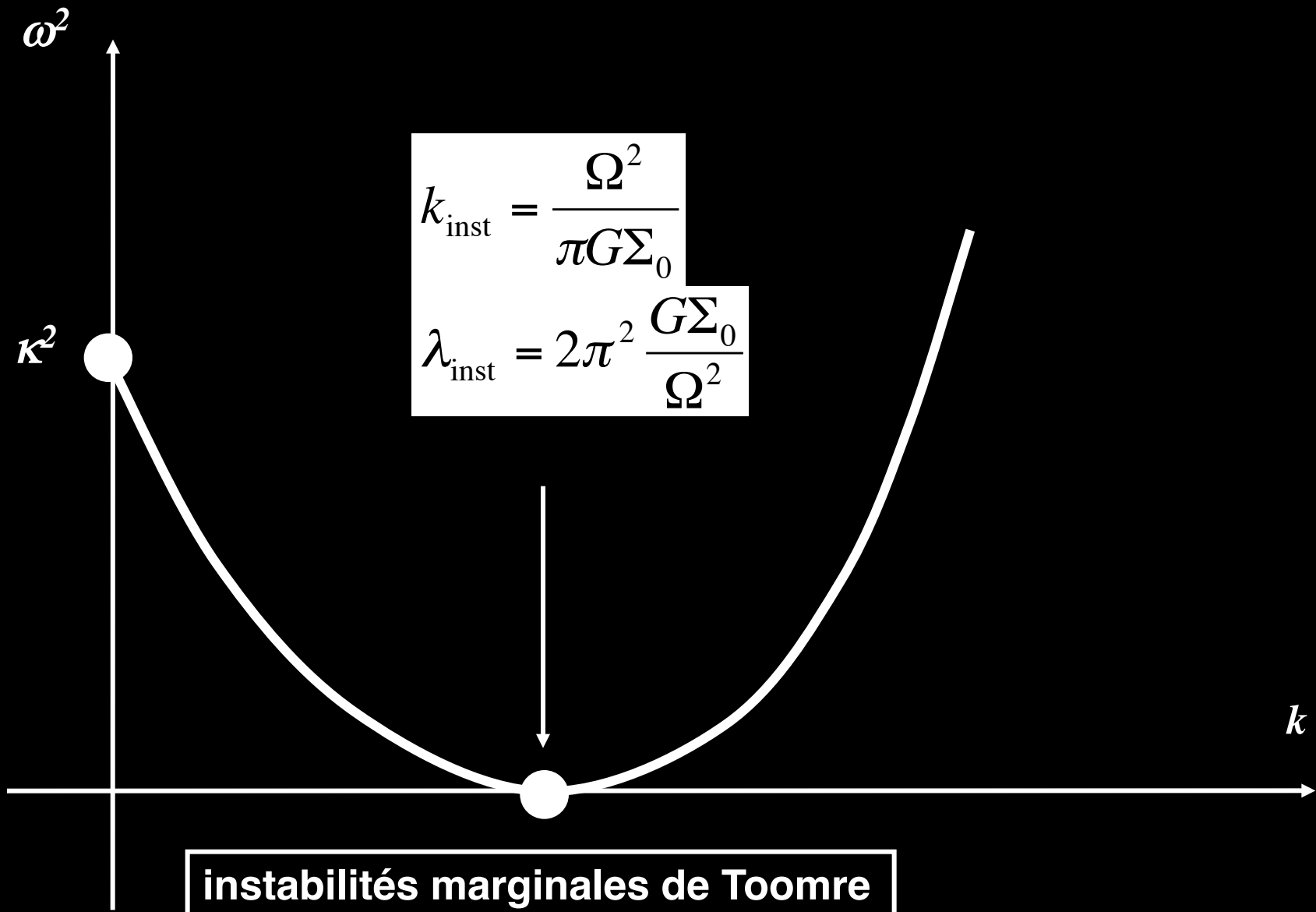
- Suffisamment « chaud » pour stabiliser les petites échelles
- Suffisamment « tournant » pour stabiliser les grandes échelles











## exemple: anneaux de Saturne

$$\lambda_{inst} = 2\pi^2 \frac{G\Sigma_0}{\Omega^2}$$

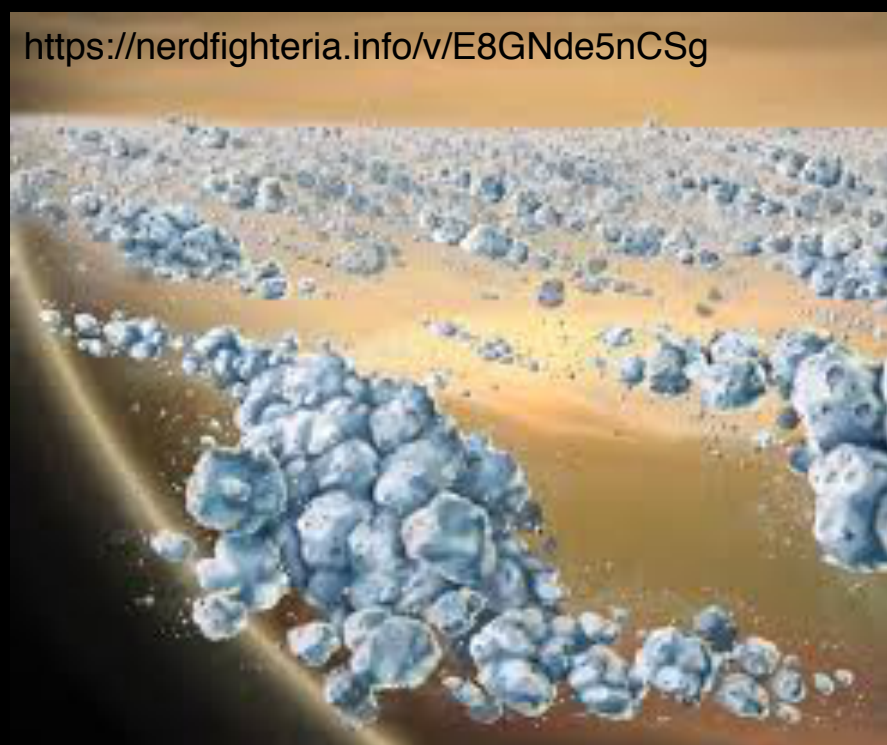
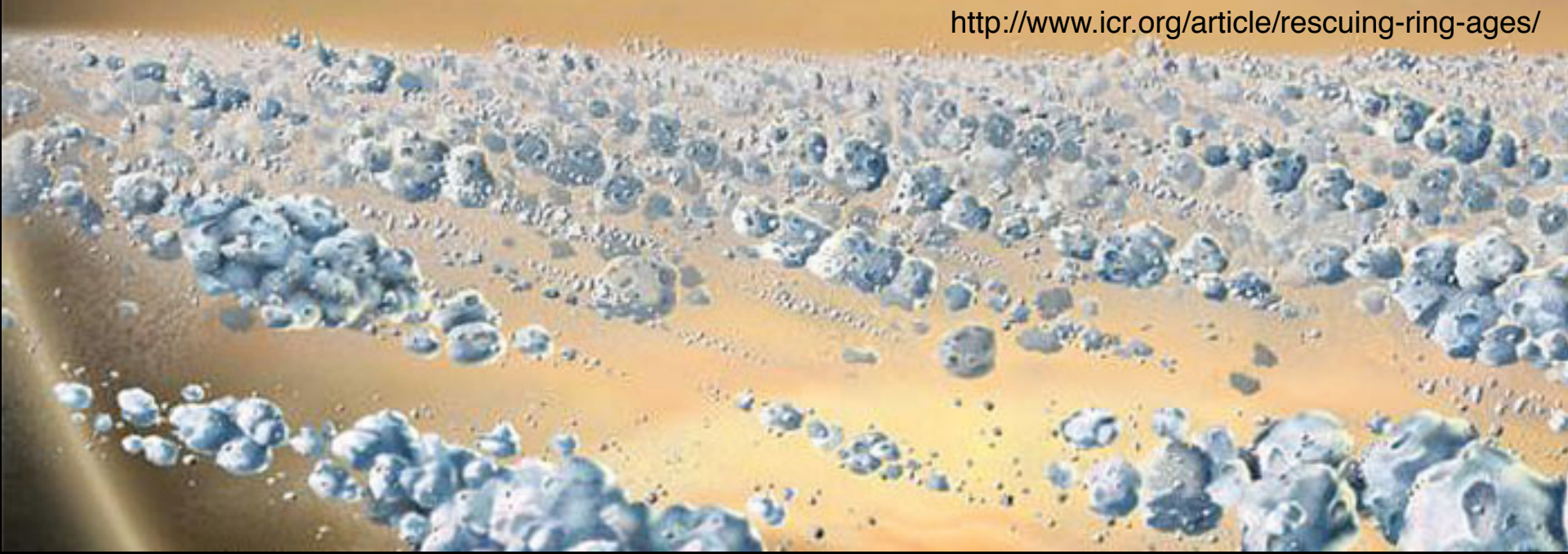
$$\Omega = 2\pi/P \sim 2 \times 10^{-4} \text{ sec}^{-1}$$

$$\Sigma_0 \sim 1000 \text{ kg m}^{-2}$$

$$\rightarrow \lambda_{inst} \sim 30 \text{ m}$$

$$\rightarrow m_{inst} \sim \Sigma_0 \lambda_{inst}^2 \sim 10^6 \text{ kg}$$

$$\rightarrow R_{inst} \sim 10 \text{ m avec } \rho \sim 1000 \text{ kg m}^{-3}$$



stabilité marginale :

$$Q = \frac{c_s \Omega}{\pi G \Sigma_0} \sim 1$$

pour un disque large homogène  $r$ :  $m_a \sim \pi \Sigma_0 r^2$ , d'où:

$$\frac{m_a}{M_p} \sim \frac{1}{Q} \cdot \frac{c_s}{r \Omega} \sim \frac{h}{r}$$

exemple de Saturne:  $\frac{m_a}{M_p} \sim 10^{-7} \Rightarrow h = \text{qqs mètres}$

forçage résonant du disque

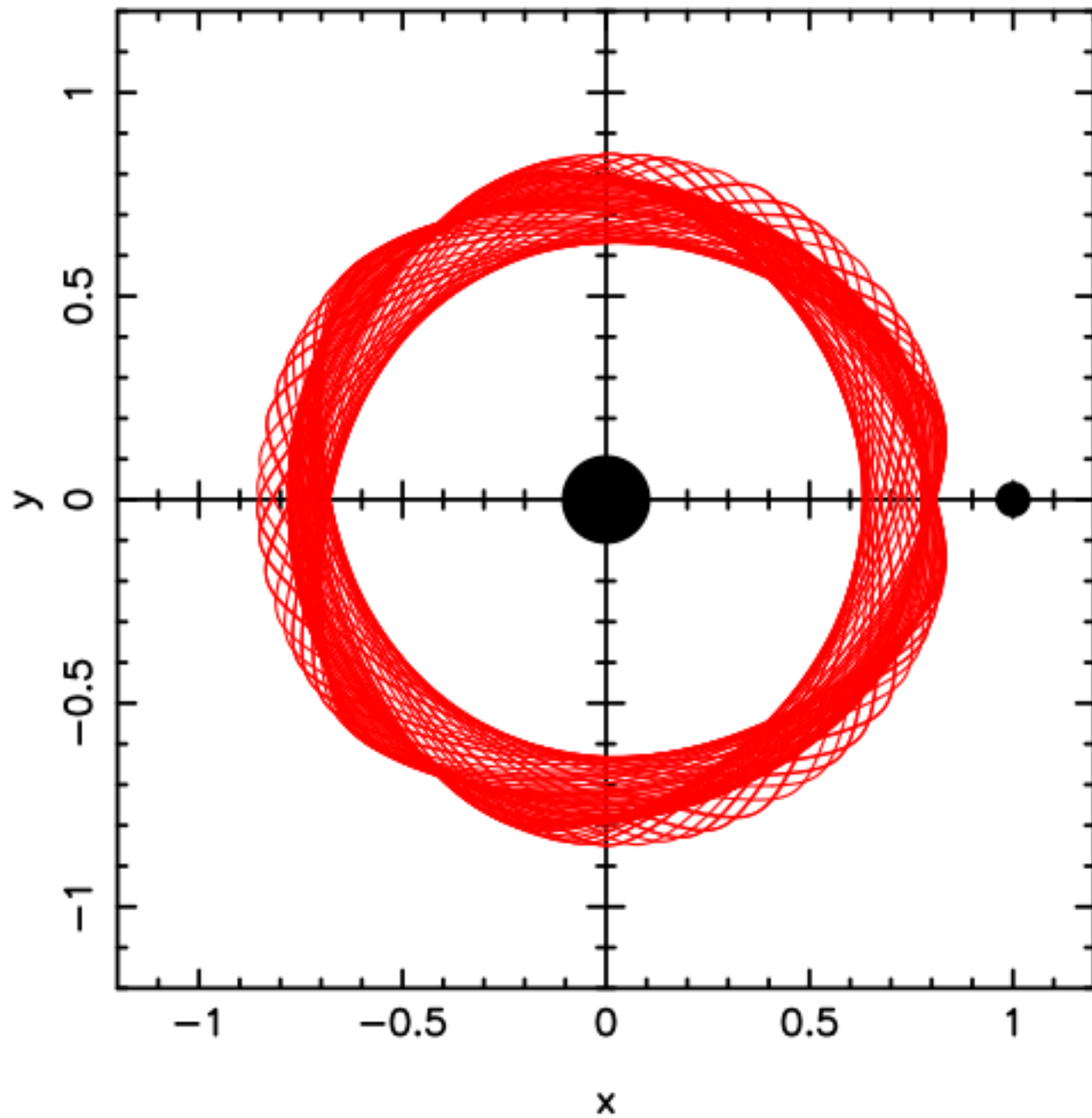


Spiral Galaxy NGC 4622

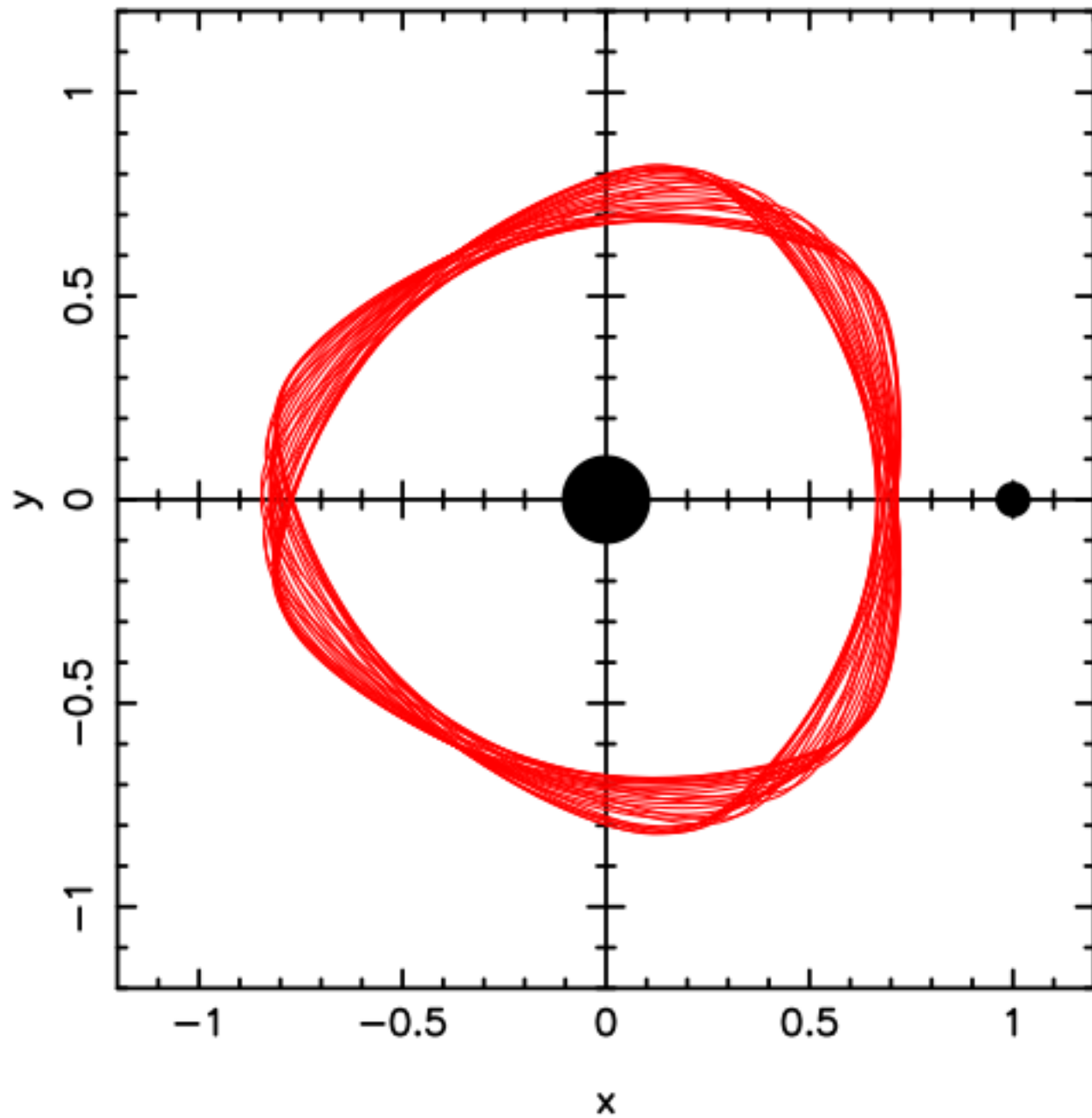


Hubble  
Heritage

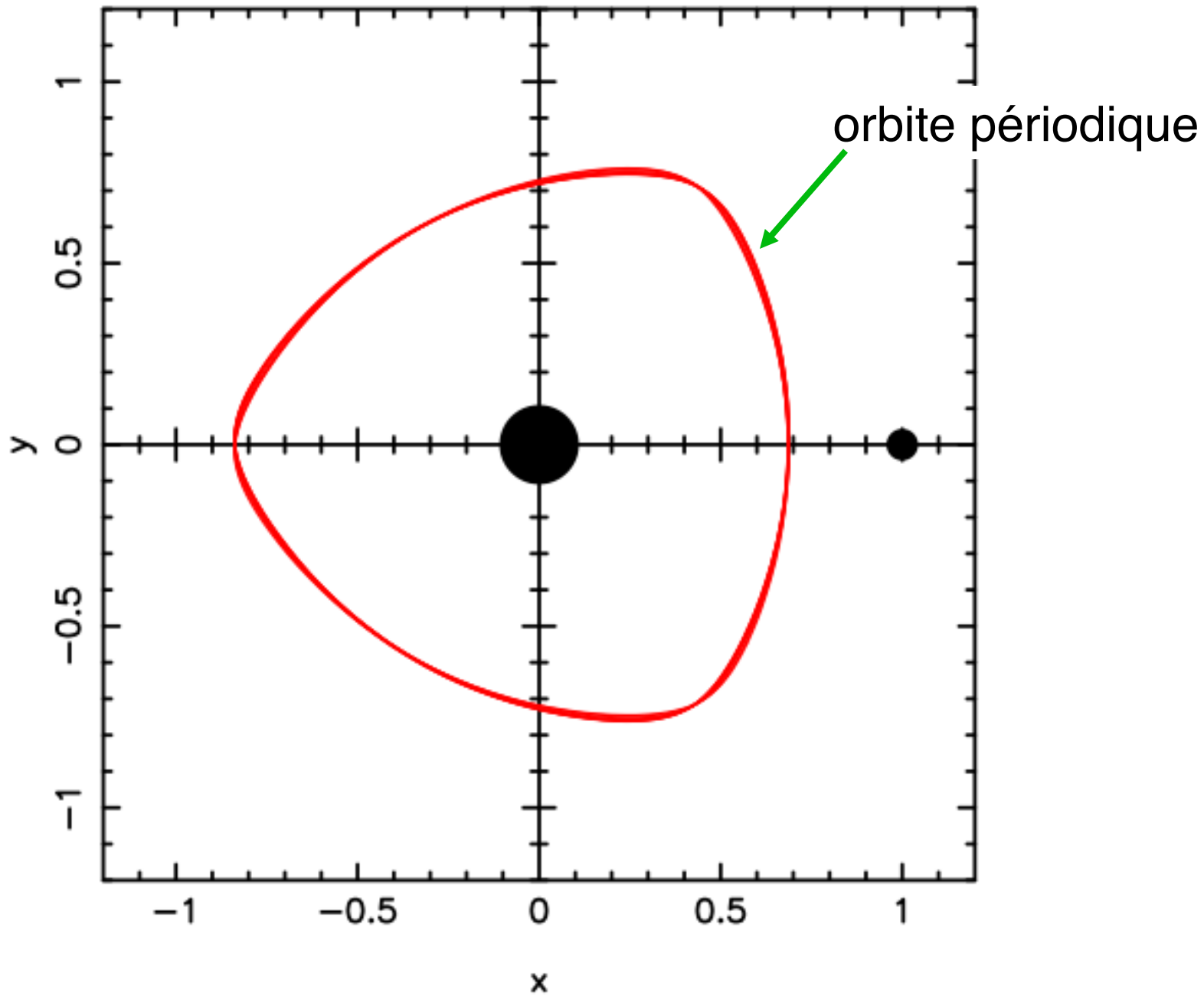
$m_{\text{sat}}=0$ ,  $m_s=0.001$ ,  $J=-1.52512540960337$ ,  $n/n_s=3:2$

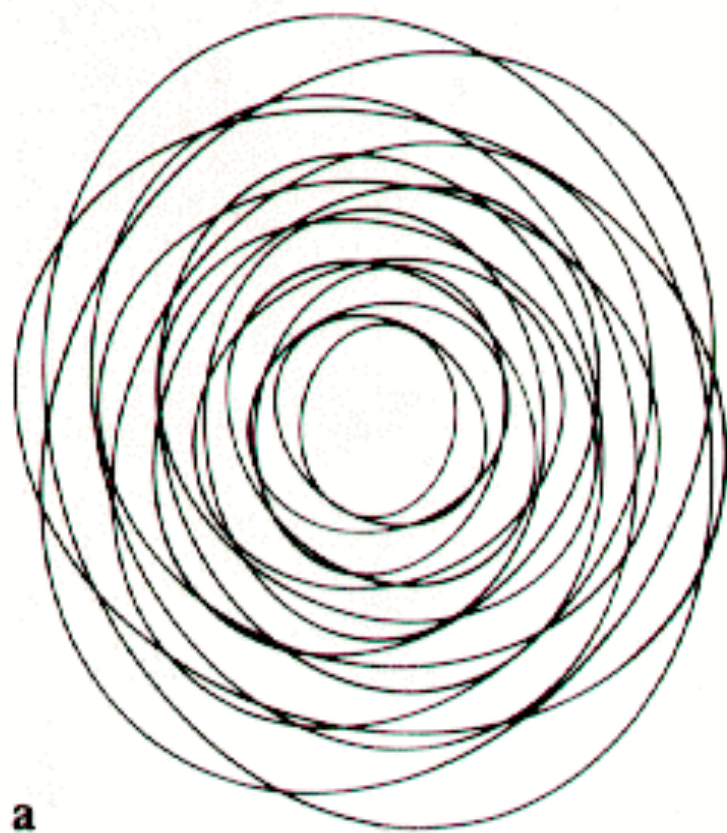


$m_{\text{sat}}=0$ ,  $m_s=0.001$ ,  $J=-1.52512540960337$ ,  $n/n_s=3:2$

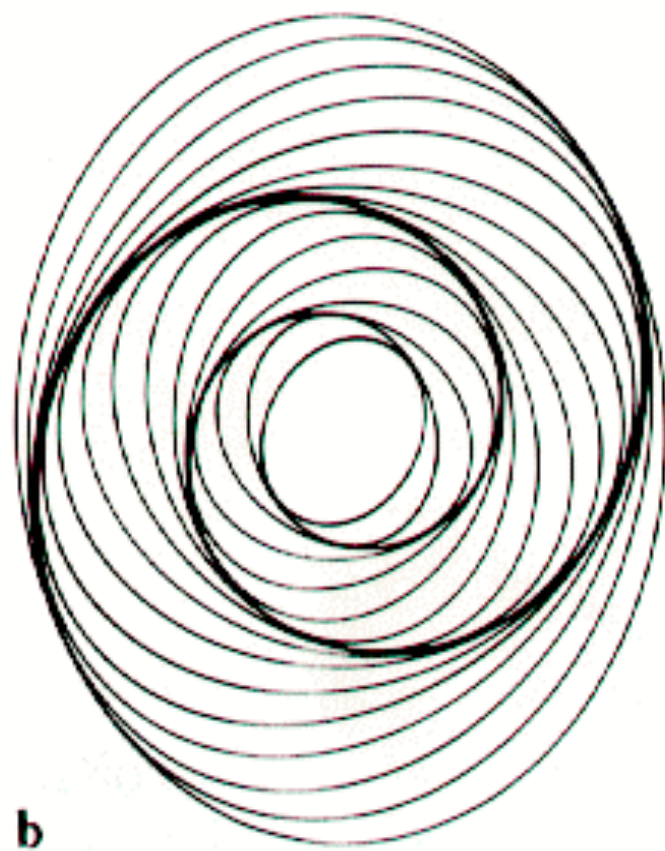


$m_{\text{sat}}=0$ ,  $m_s=0.001$ ,  $J=-1.52512540960337$ ,  $n/n_s=3:2$





a

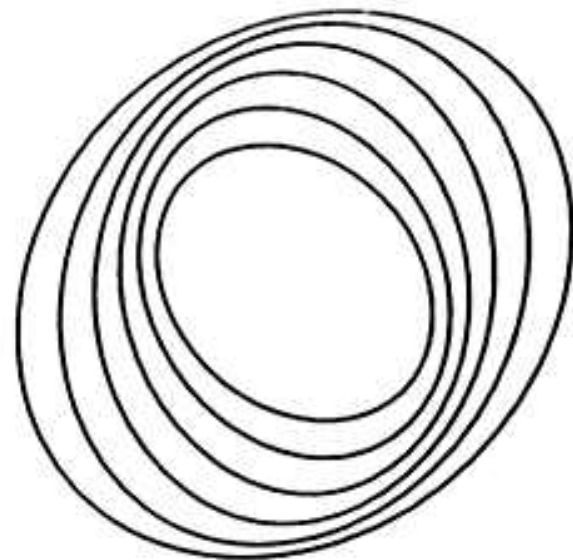
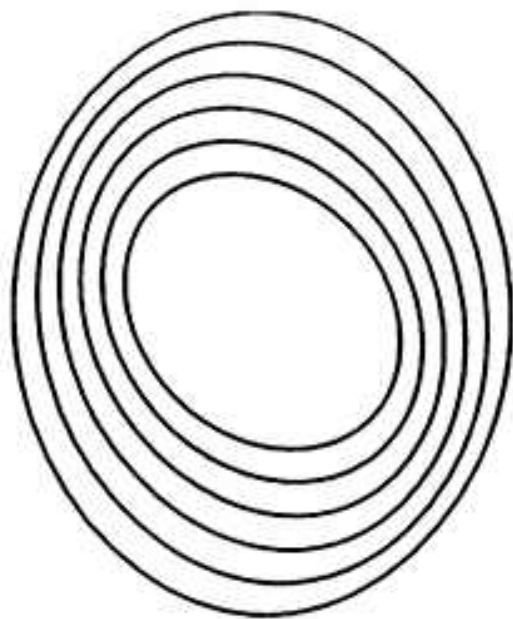
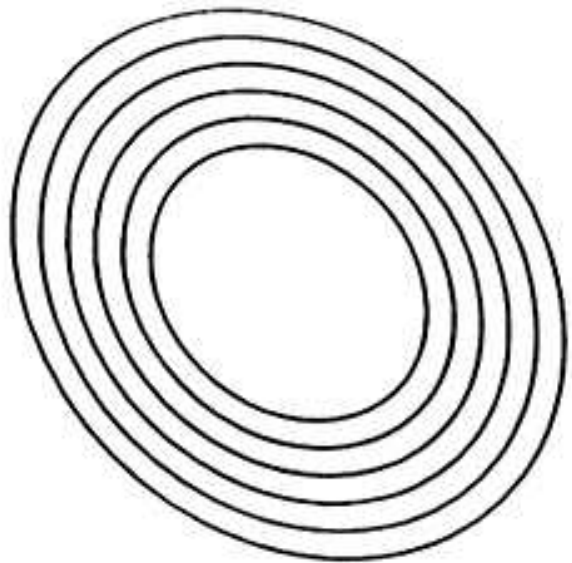


b

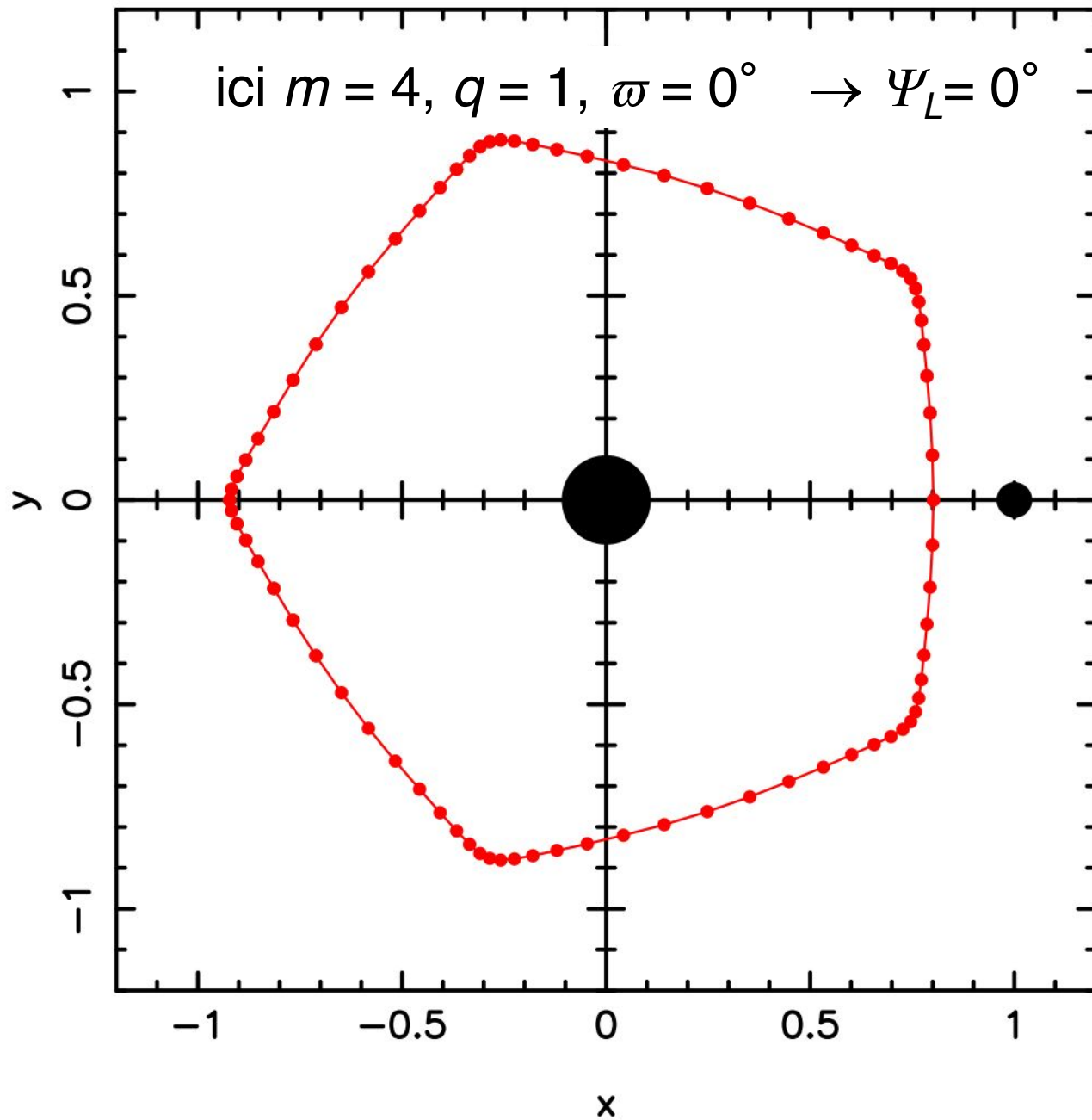
### **The Origin of Spiral Density Waves**

Both drawings have exactly the same number of ellipses, each one representing the orbit of a star. (a) Randomly oriented ellipses.

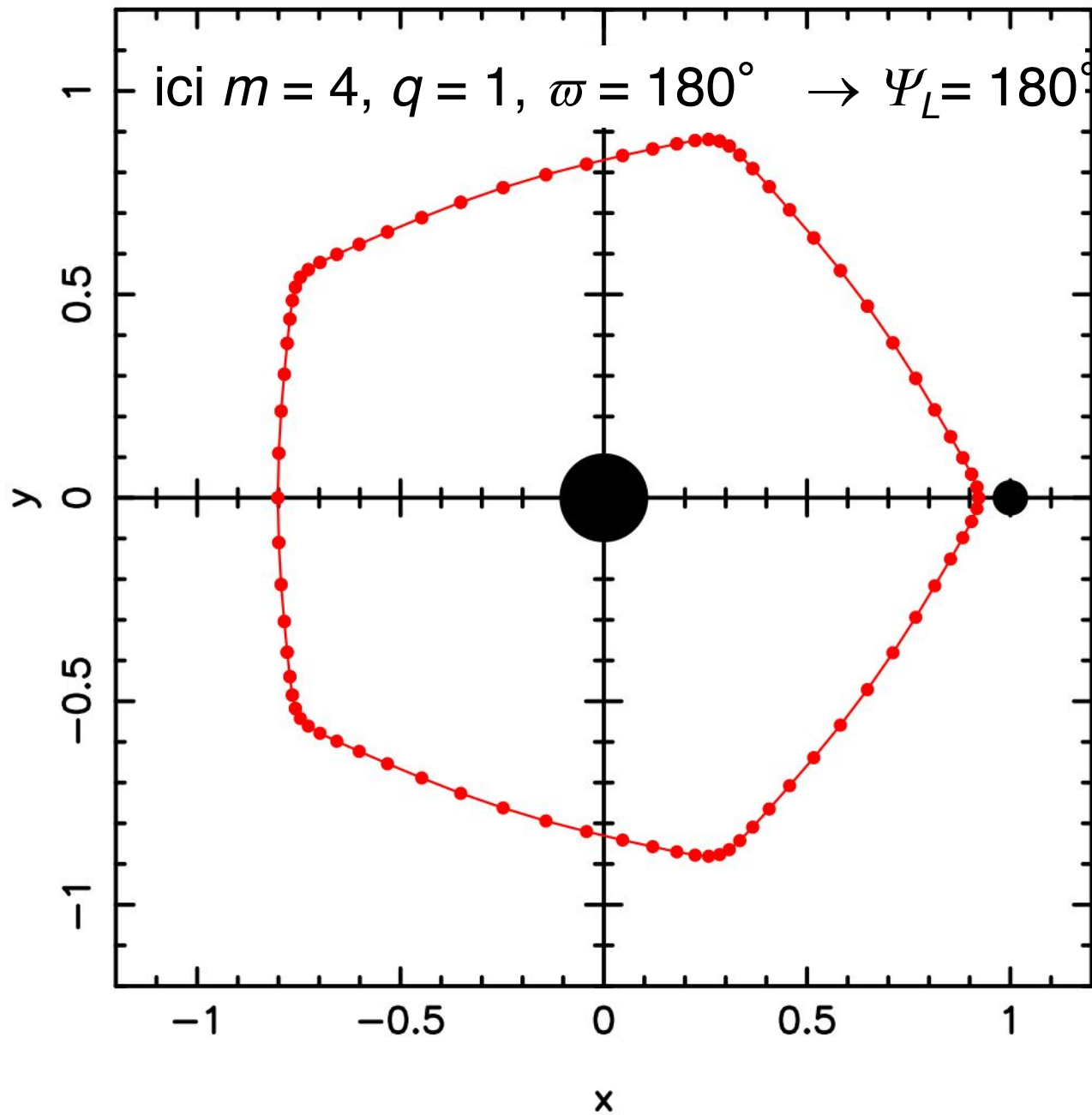
(b) Ellipses with a correlation between the orientations of adjacent ellipses.



$$\Psi_L = (m + q)\lambda_s - m\lambda - q\varpi$$

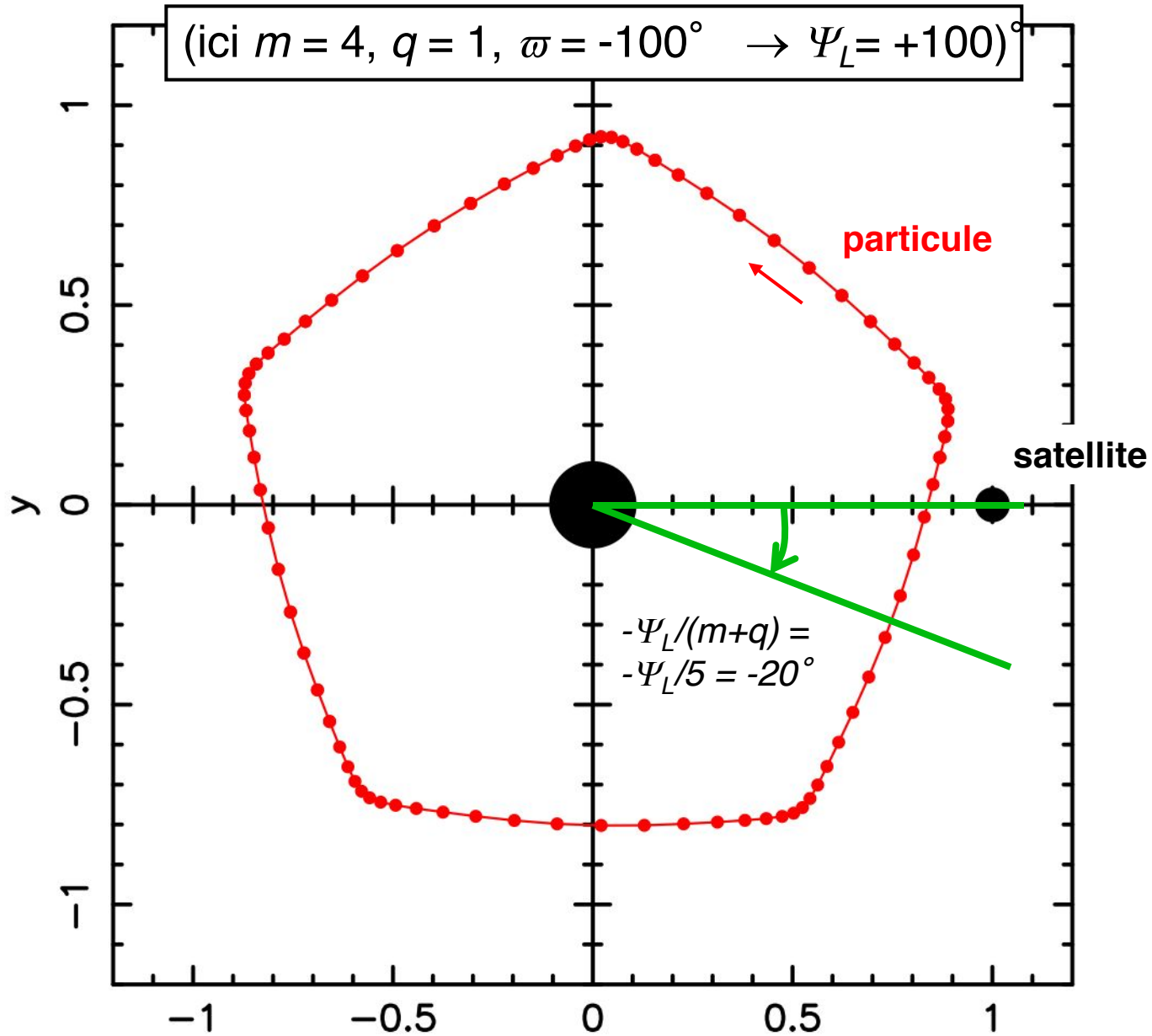


$$\Psi_L = (m + q)\lambda_s - m\lambda - q\varpi$$

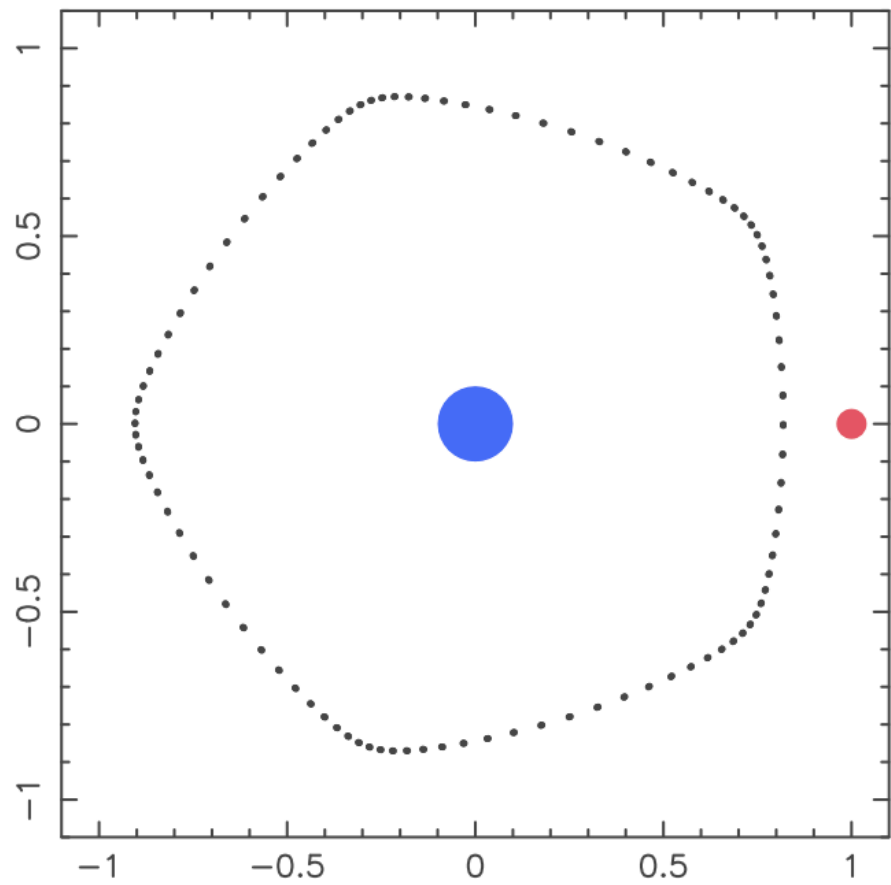




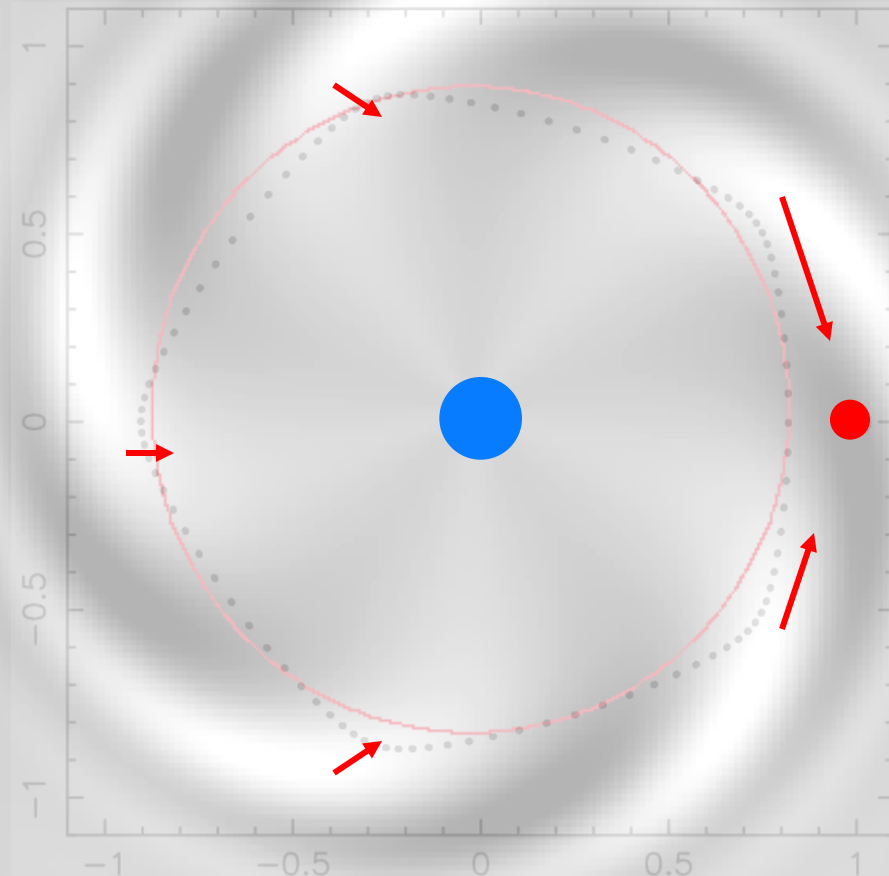
$$\Psi_L = (m + q)\lambda_s - m\lambda - q\varpi$$



le couple exercé par le satellite est  $\propto \mathbf{e} \times \sin(\Psi_L)$

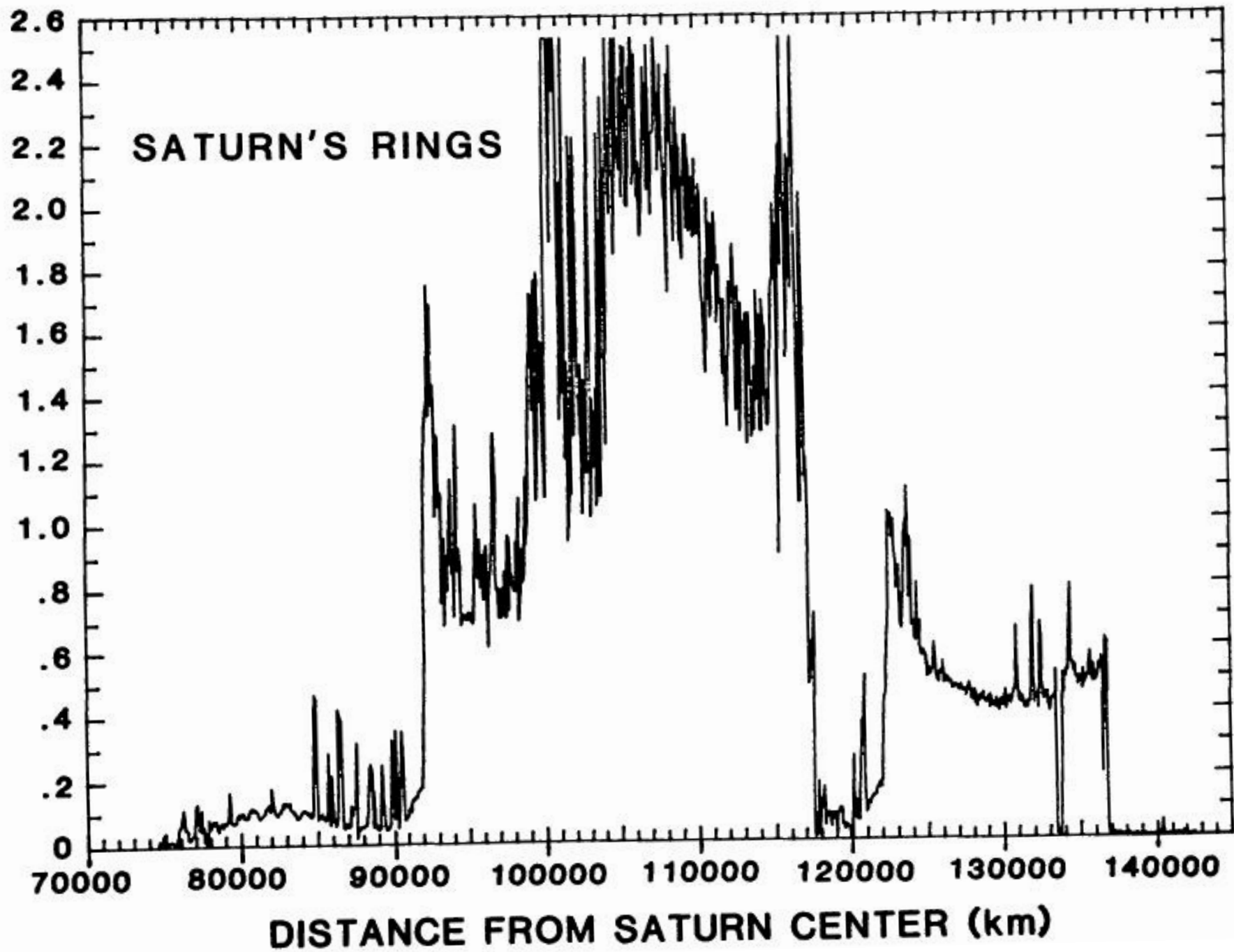


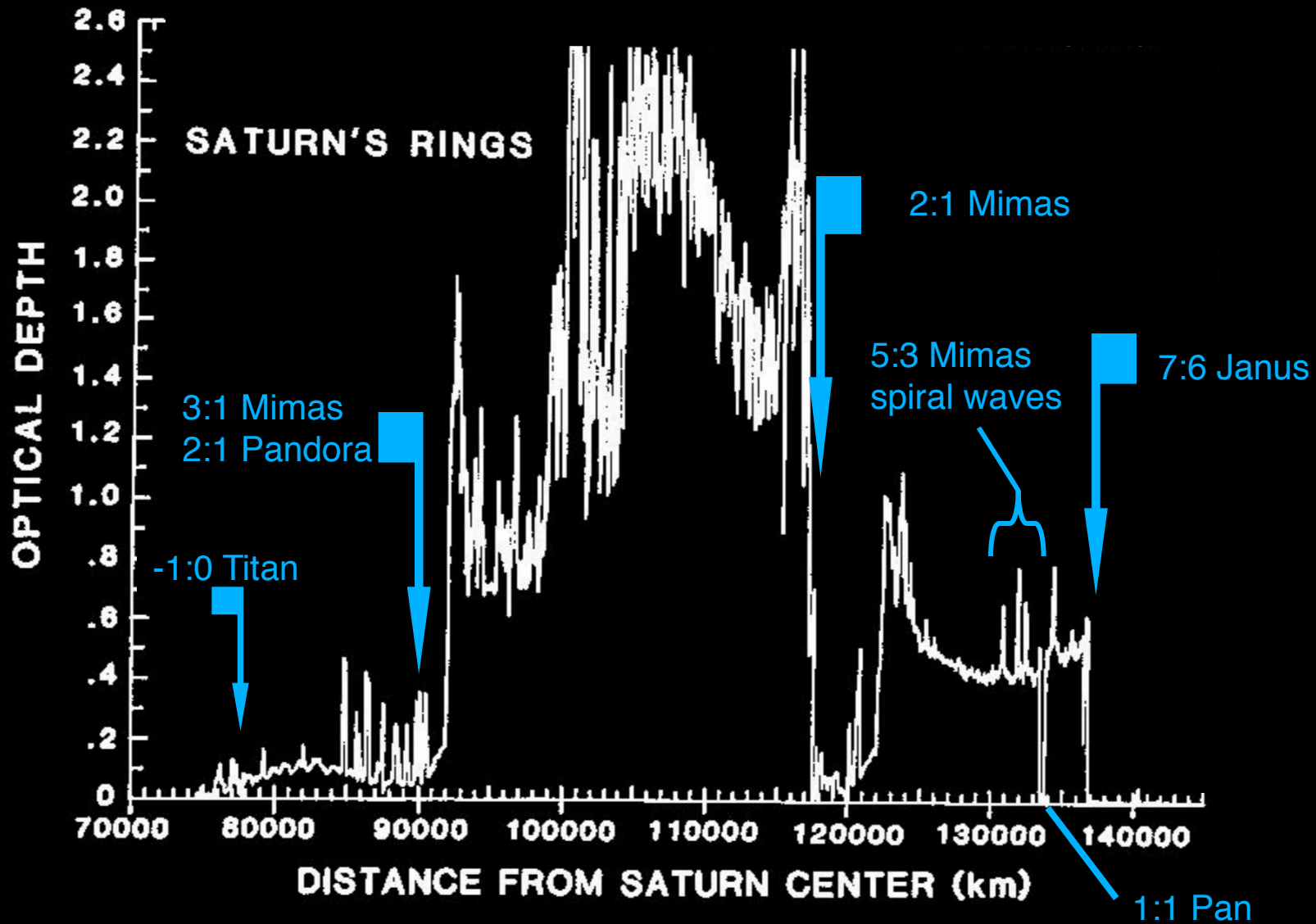
## exemple résonance 5:4



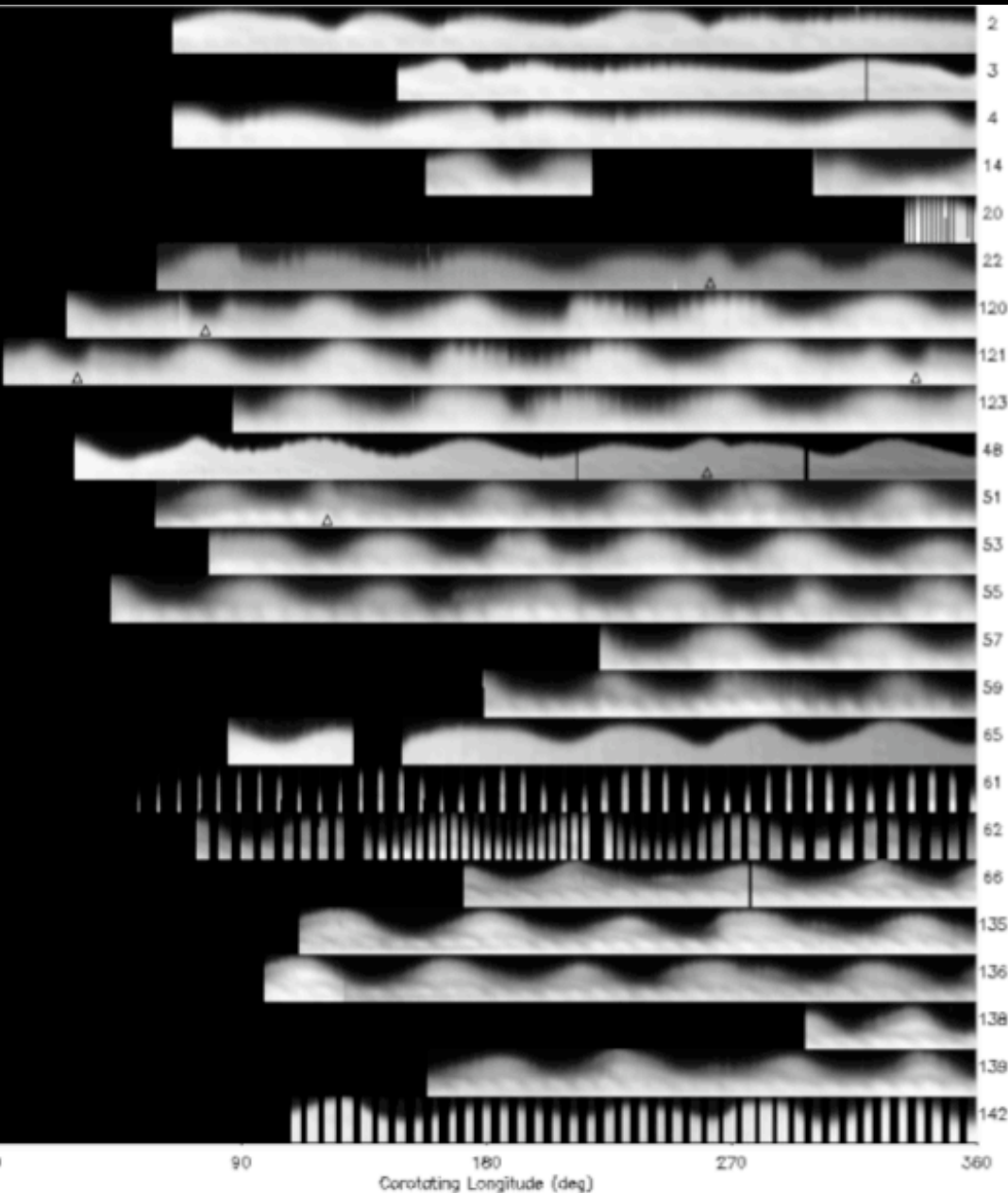
la planète exerce un couple sur le disque,  
et inversement  $\Rightarrow$  migration

OPTICAL DEPTH



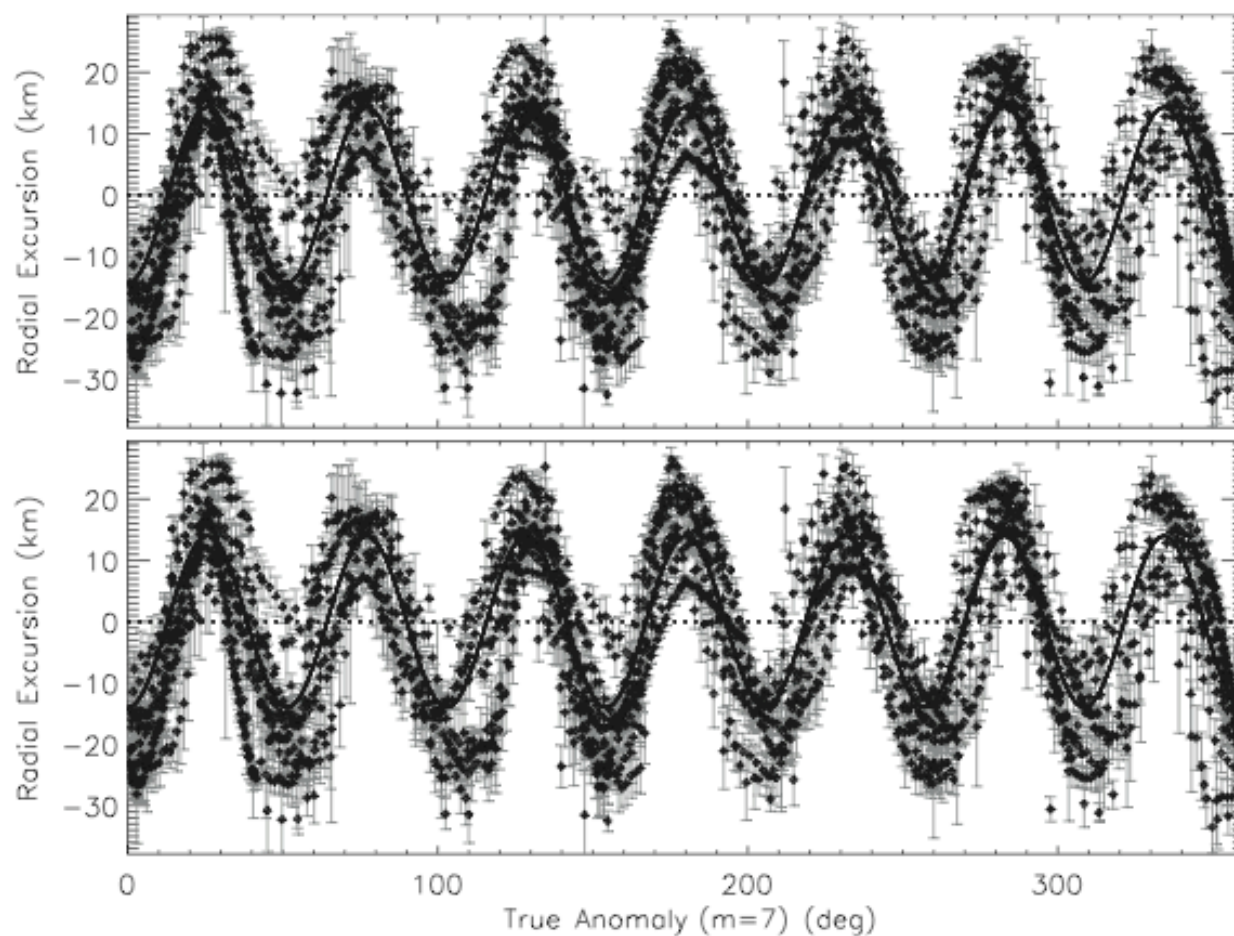


# bord externe de l'anneau A de Saturne (résonance 7:6 avec Janus)



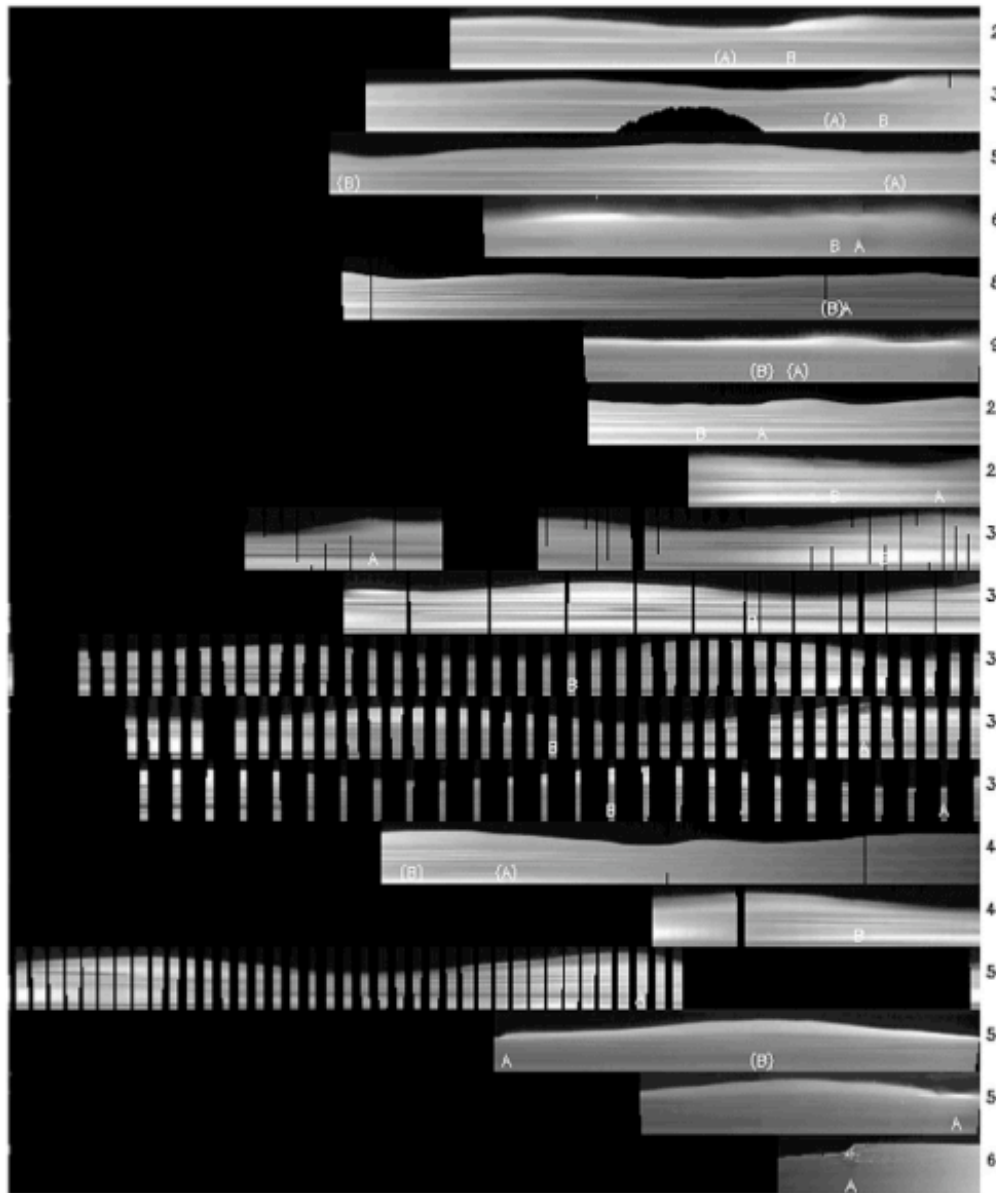
s. longitude for the Cassini data sets used in this study. The first image in each data set is assigned a longitude of zero. Later images are  
ft (i.e., decreasing in longitude from  $360^\circ$ ) according to their inertial longitudes, corrected for pattern rotation at Janus' present mean  
dge move through the mosaic from left to right. For each mosaic, radii range from 136,715 to 136,800 km. The expected seven lobes are  
here is additional structure, including short-wavelength undulations, as well as anomalous broad radial features, which are indicated by

Spitale & Porco  
*Astron. J.* 138  
1520-1528 (2009)



**Figure 3.** Simultaneous fits to all data sets in this study occurring on or after day 2006-268. In both plots,  $a$ ,  $e$ , and  $\varpi$  were allowed to vary. In the top plot, the pattern speed  $\Omega_p$  was also varied, so as to determine its mean value. In the bottom plot, the pattern speed was fixed at Janus' mean motion. The fits are nearly indistinguishable, consistent with the observation that the pattern is aligned with Janus.



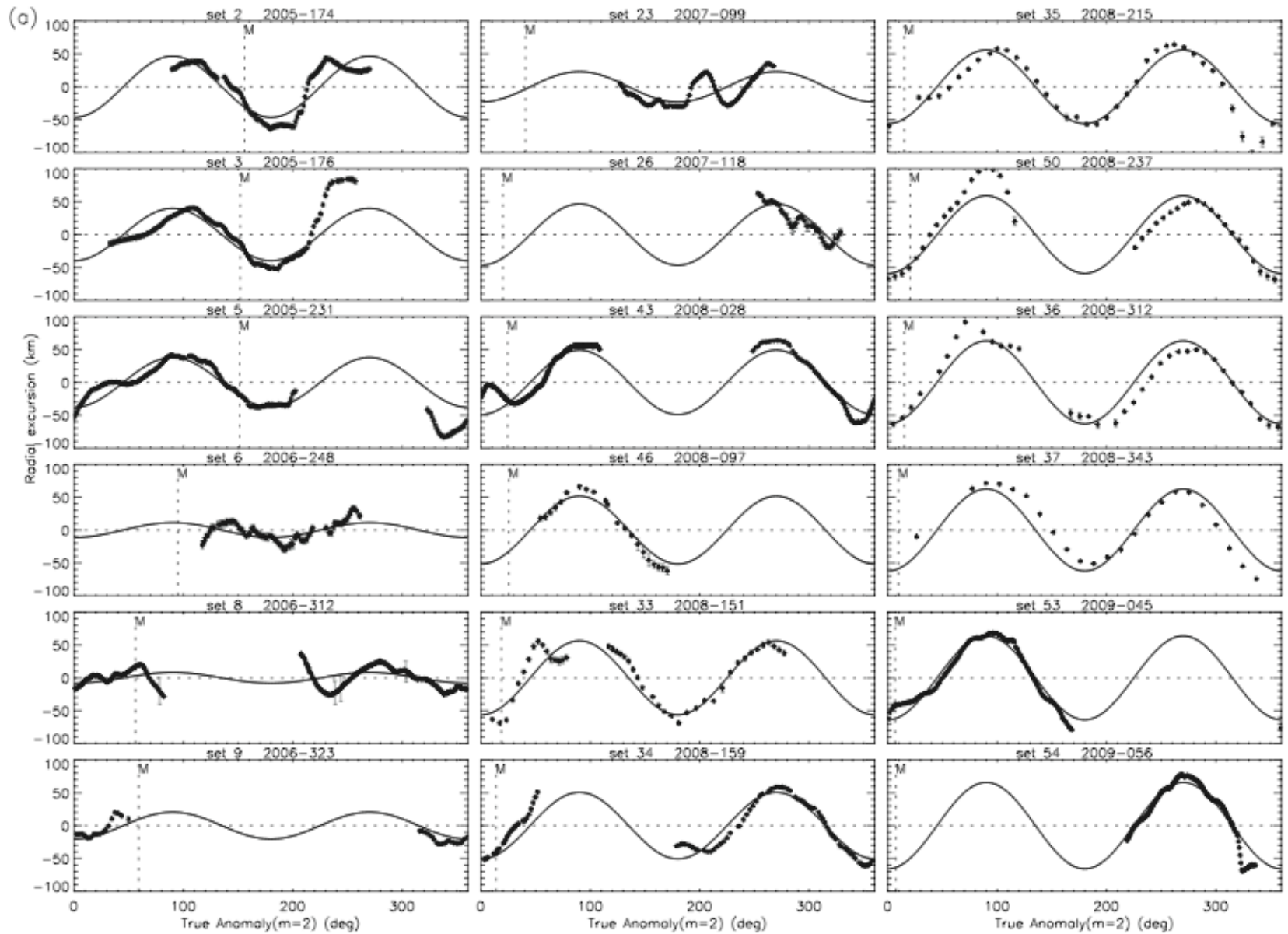


**bord externe  
de l'anneau B  
de Saturne  
(résonance  
2:1 avec Mimas)**

Spitale & Porco  
*Astron. J.* 140  
1747-1757 (2010)

mosaics (radius vs. longitude) for the *Cassini* data sets—either azimuthal scans or ansa-staring movies—used in this study. The first image had a longitude of zero. Later images are mapped successively to the left or right (i.e., decreasing or increasing in corotating longitude from inertial longitudes, corrected for pattern rotation at Mimas' average mean motion of  $381^{\circ}9835 \text{ day}^{-1}$ , and the direction in which the scan moves through the mosaic from left to right. Data set numbers appear to the right of each mosaic. For each map, radii range from 117,425 to 117,425 km. Regions A and B (see text) are marked; labels in parentheses indicate the predicted location of the feature, though it was not observed.



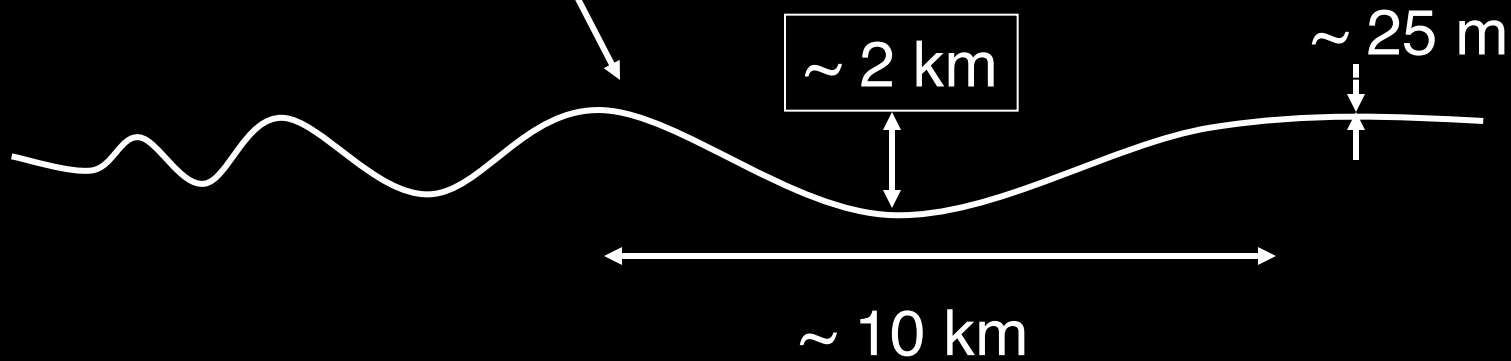


**Figure 3.** Radial excursion vs. true anomaly for total  $m = 2$  fits to each data set used in this study. The  $m = 1$  and  $m = 3$  patterns from Table 2 are present, but not shown. The solid line shows the best-fit model with  $a$ ,  $e$ , and  $\varpi_0^{(m)}$  free;  $\Omega_p$  was fixed at Mimas' average mean motion. Dashed vertical lines indicate the position of Mimas (M) at each epoch. Note that fixing  $\Omega_p$  at Mimas' speed does not imply that Mimas should maintain a constant phase in these plots because  $\varpi_0^{(m)}$  (i.e., the orientation at epoch) is a free parameter in each fit.

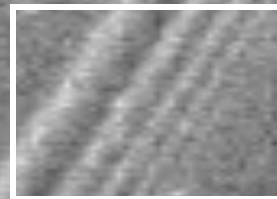
Lissauer *et al.*,  
Icarus, 1985

bending 5:3

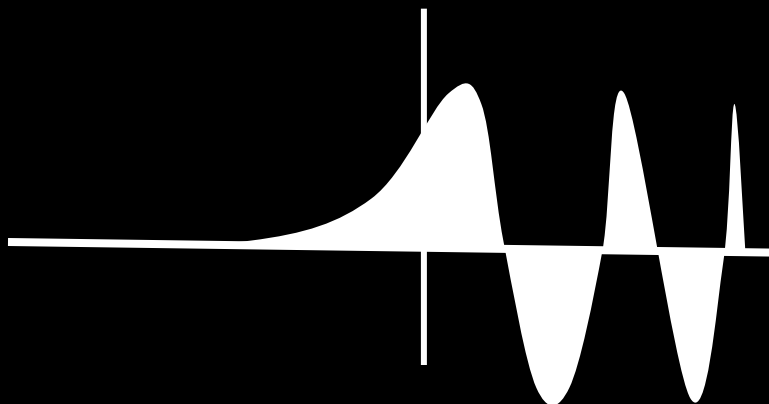
compression 5:3



Mimas 5:3  
resonances



damping  $\rightarrow$   
 $v \sim c_s^2/\Omega \sim h^2 \Omega$



surface  
density  
 $\Sigma_0$

retour au système initial, mais avec un terme de **forçage**:

$$\left\{ \begin{array}{l} \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = -\nabla (\Phi_P + \Phi_D + \Phi_S) - \frac{\nabla \cdot \mathbf{P}}{\Sigma} \\ \frac{\partial \Sigma}{\partial t} + \nabla(\Sigma \mathbf{v}) = 0 \\ \nabla^2 \Phi_D = 4\pi \Sigma G \delta(z) \\ p = \Sigma c_s^2, \end{array} \right.$$

potentiel d'un satellite sur orbite **circulaire**:

$$\Phi_S(r, \theta, t) = \sum_{m=-\infty}^{+\infty} \Phi_{Sm}(r) \cdot \exp [im (\theta - n_{St})]$$

détails dans le dossier “7- Résonances dans les disques“

[http://www.lesia.obspm.fr/perso/bruno-sicardy/ensei/m2\\_obs/index.html](http://www.lesia.obspm.fr/perso/bruno-sicardy/ensei/m2_obs/index.html)

et on cherche des solutions de la forme:

$$a_m(r, \theta, t) = A_m(r) \cdot \exp[im(\theta - n_s t)]$$

de plus, pour des disques de faibles masses, on peut montrer que l'on peut appliquer

l'approximation "WKB » (Wentzel-Kramers-Brillouin):

les variations radiales se font sur des distances *petites* par rapport à la taille du disque, *i.e.:*

$$A_m(r) = A_m \exp(ik \cdot r) \quad \text{où}$$

→  $k \cdot r \gg 1$       l'onde spirale est "resserrée" (tightly wound)

soit des solutions du type:

$$a_m(r, \theta, t) = A_m \exp[kr + im(\theta - n_s t)]$$

équation des points de *phase constante*:

$$kr + im(\theta - n_s t) = \text{cste}$$

→ équation d'une *onde spirale, avec  $m$  bras*, tournant à la vitesse angulaire  $n_s$ , de structure *serrée* (approx. WKB)

Alors: équations différentielles → équations algébriques

$$\partial/\partial t \Leftrightarrow -im.n_s$$

$$\partial/\partial \theta \Leftrightarrow im.\theta$$

$$\partial/\partial r \Leftrightarrow ik \text{ et}$$

$$\partial/\partial r \gg (1/r)\partial/\partial \theta$$

...et après linéarisation et passage en coordonnées polaires:

$$\left\{ \begin{array}{l} jm(n - n_s)v_{rm} - 2nv_{\theta m} = -\dot{\Phi}_{Sm} - \dot{\Phi}_{Dm} - \frac{c_s^2 \dot{\sigma}_m}{\Sigma_0} + \left( \mu + \frac{4\nu}{3} \right) \ddot{v}_{rm} \\ \frac{n}{2}v_{rm} + jm(n - n_s)v_{\theta m} = -jm \frac{\Phi_{Sm} + \Phi_{Dm}}{r} - jm \frac{c_s^2 \sigma_m}{r \Sigma_0} + \nu \ddot{v}_{\theta m} \\ \sigma_m = -\frac{\Sigma_0 \dot{v}_{rm}}{jm(n - n_s)} \\ \dot{\Phi}_{Dm} = -2\pi G j s \sigma_m \\ p_m = c_s^2 \sigma_m . \end{array} \right.$$

où le point désigne ici la dérivée radiale:  $\dot{a} = \partial a / \partial r$



en notant  $x$  la distance adimensionnelle à la résonance:  $x = (r - r_{res})/r_{res}$

$$-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jx u_{rm} = C_m$$

où:

$$\left\{ \begin{array}{l} \alpha_v^3 = j\alpha_P^3 + \alpha_\nu^3 \\ \alpha_P^3 = \mp \frac{c_s^2/n}{3ma_m^2 n_s} \\ \alpha_\nu^3 = \frac{\mu + 7\nu/3}{3ma_m^2 n_s} \\ \alpha_G^2 = \pm \frac{2\pi s G \Sigma_0}{3ma_m n n_s} \end{array} \right.$$

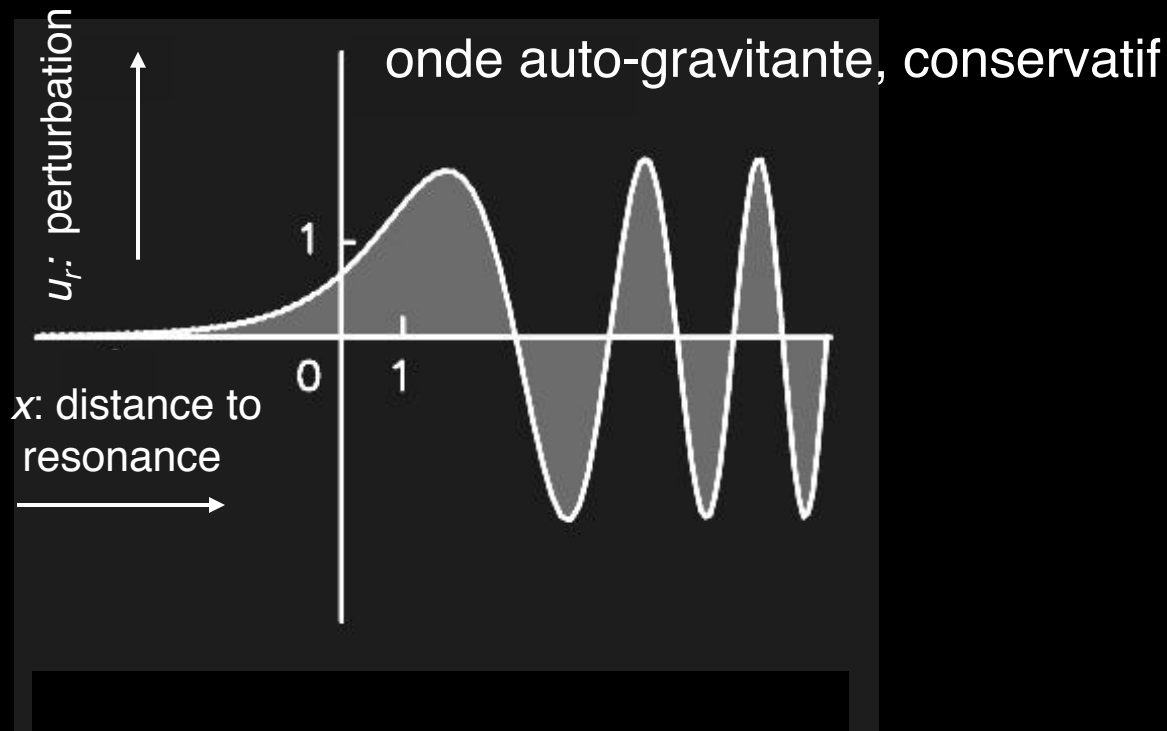
si tous les coefficients  $\alpha$  sont nuls (particules tests sans interactions):

$$\cancel{-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - j\mathcal{X}u_{rm} = C_m}$$

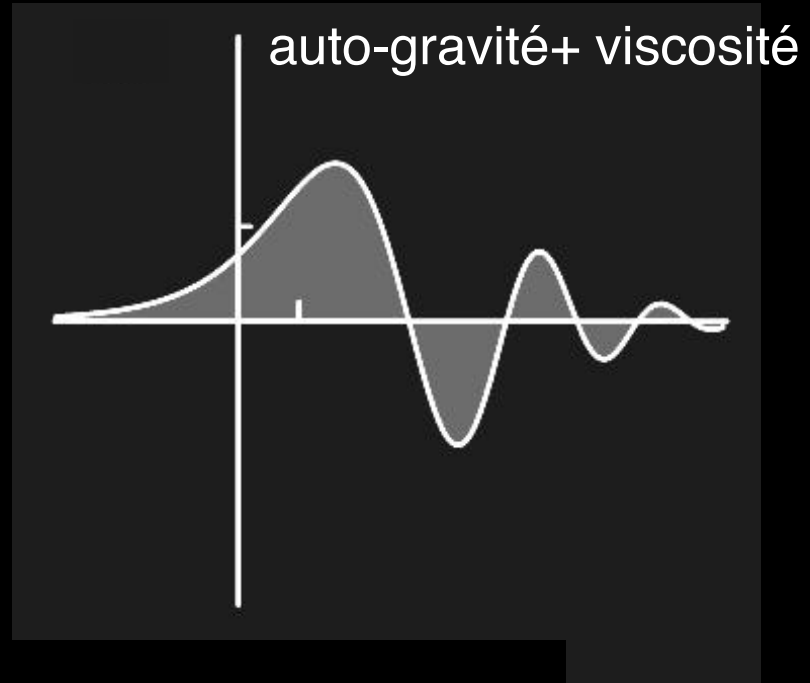
oscillateur harmonique!

solution de l'équation précédente obtenue par transformée de Fourier:

$$u_{rm}(x) = C_m \int_0^{+\infty} \exp[j(kx - \alpha_G^2 k^2 / 2 - \alpha_P^3 k^3 / 3) - \alpha_\nu^3 k^3 / 3] dk$$



~~$$-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jx u_{rm} = C_m$$~~



$$-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jx u_{rm} = C_m$$

$$\alpha_v^3 = \cancel{j\alpha_p^3} + \alpha_v^3$$



**other resonant waves**

**6:5 Janus resonant wave**

**instabilités locales à la Toon**

A grayscale image of Saturn's rings, showing a prominent wave-like disturbance. The rings are composed of many thin, parallel lines, with a distinct, bright, curved feature that resembles a wave or a gap. The background is a dark, textured gray.

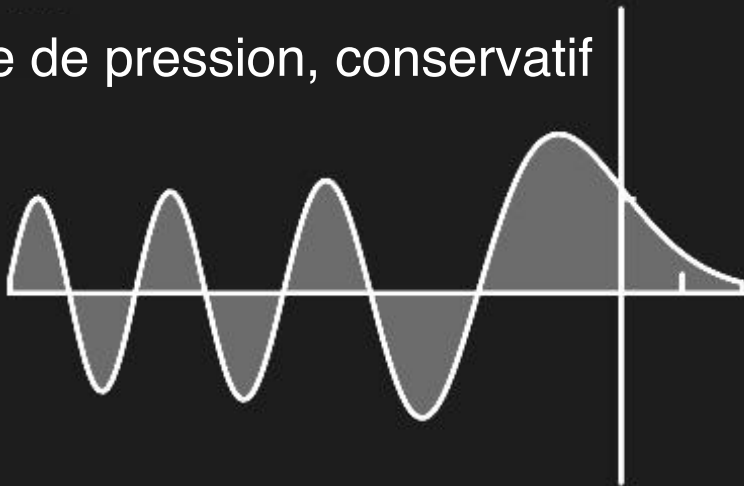
**12:11 Prometheus  
resonant wave**



$$-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jx u_{rm} = C_m$$

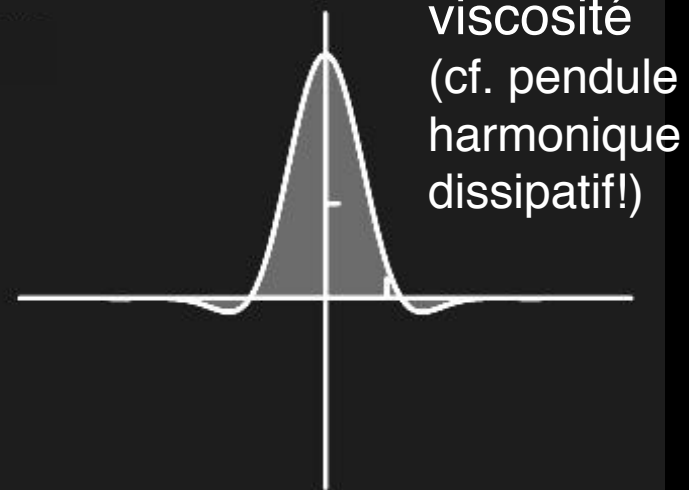
$$\alpha_v^3 = j\alpha_P^3 + \alpha_v^3$$

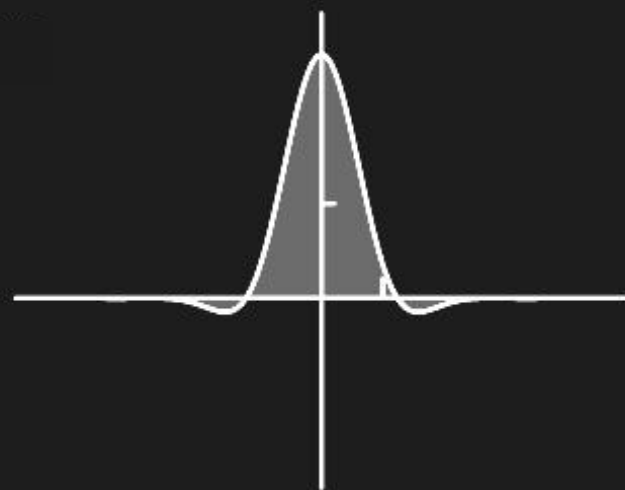
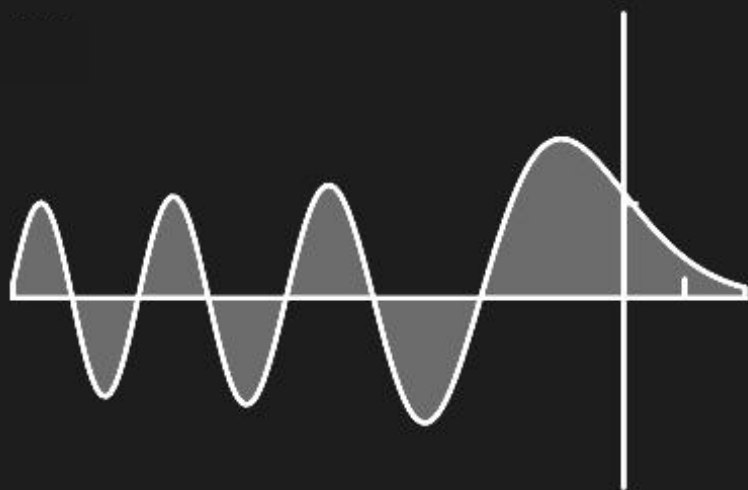
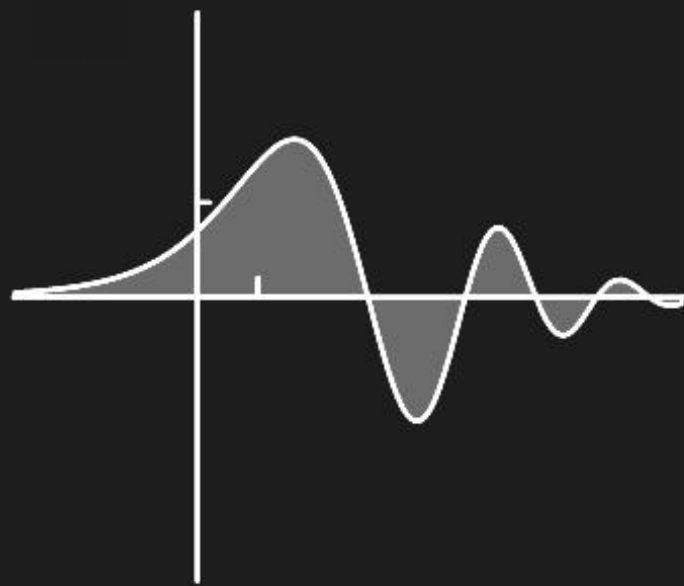
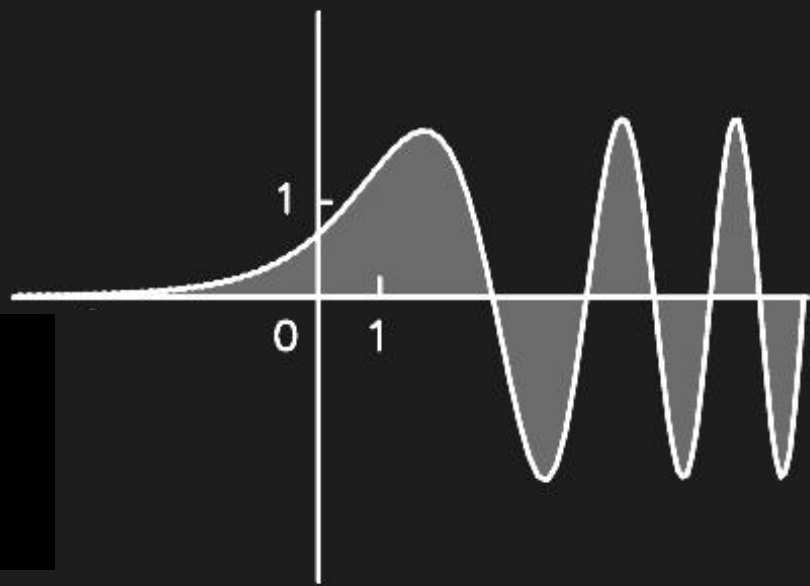
onde de pression, conservatif

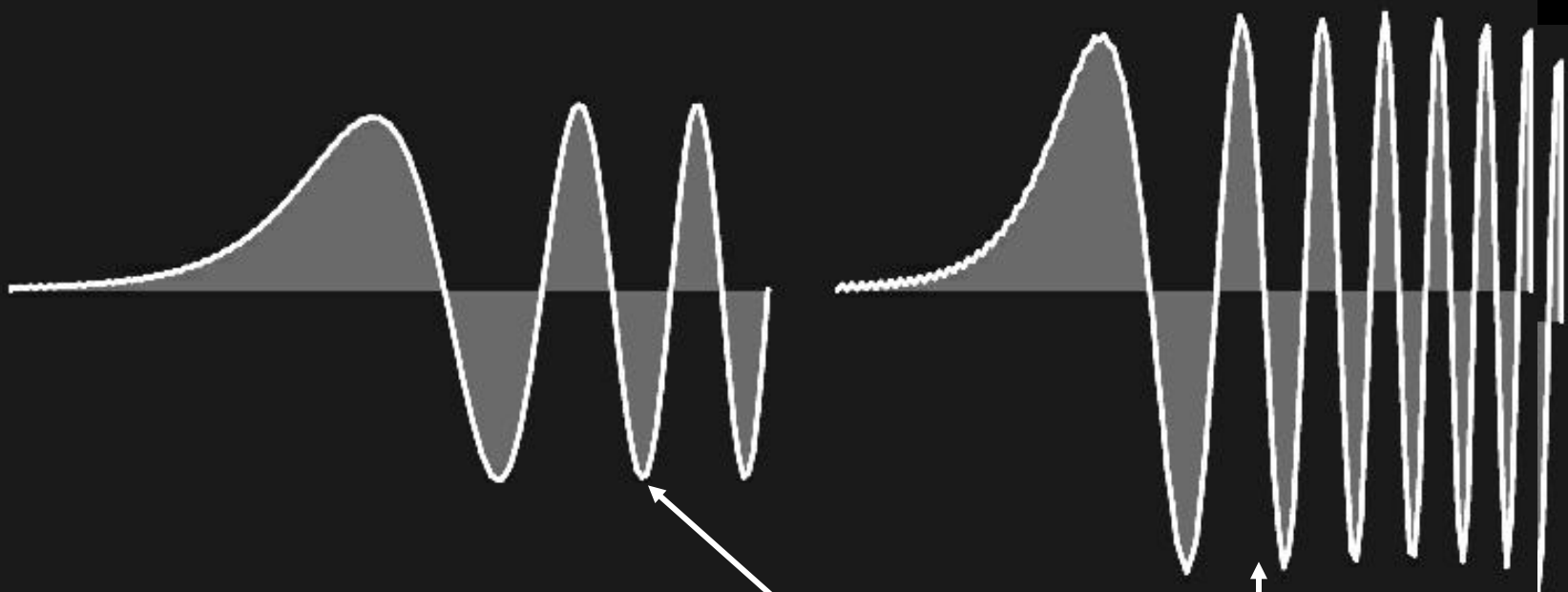


$$-\alpha_v^3 \frac{d^2}{dx^2}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jx u_{rm} = C_m$$

$$\alpha_v^3 = j\alpha_P^3 + \alpha_\nu^3$$







les anneaux comme sondes **macroscopiques** de propriétés **microscopiques**

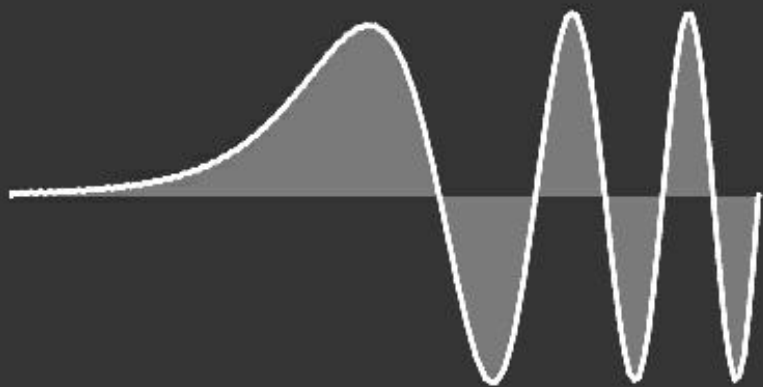
anneau moins massif

densité de surface donnée

anneau plus massif

$$-\cancel{\alpha_v^3 \frac{d^2}{dx^2}}(u_{rm}) + \alpha_G^2 \frac{d}{dx}(u_{rm}) - jxu_{rm} = C_m$$

où:  $\alpha_G^2 = \pm \frac{2\pi s G \Sigma_0}{3ma_m n n_s}$



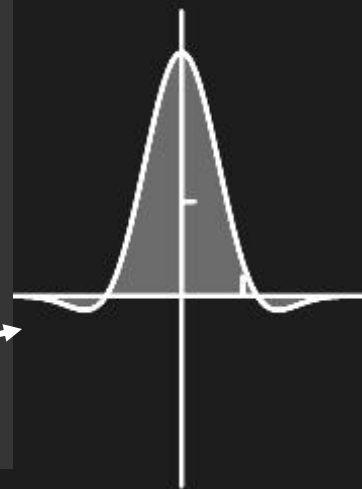
avec viscosité

onde auto-gravitante  
conservative



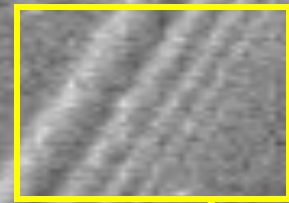
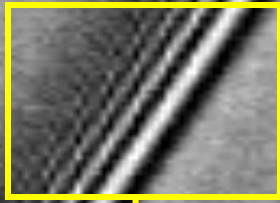
avec encore plus  
de viscosité

pure viscosité



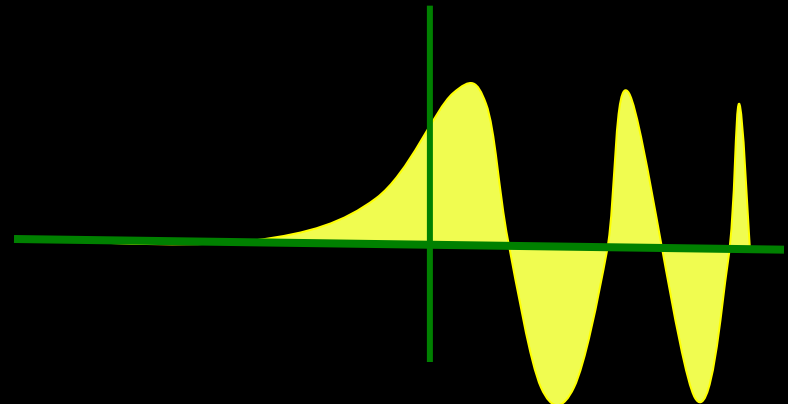
Lissauer *et al.*,  
Icarus, 1985

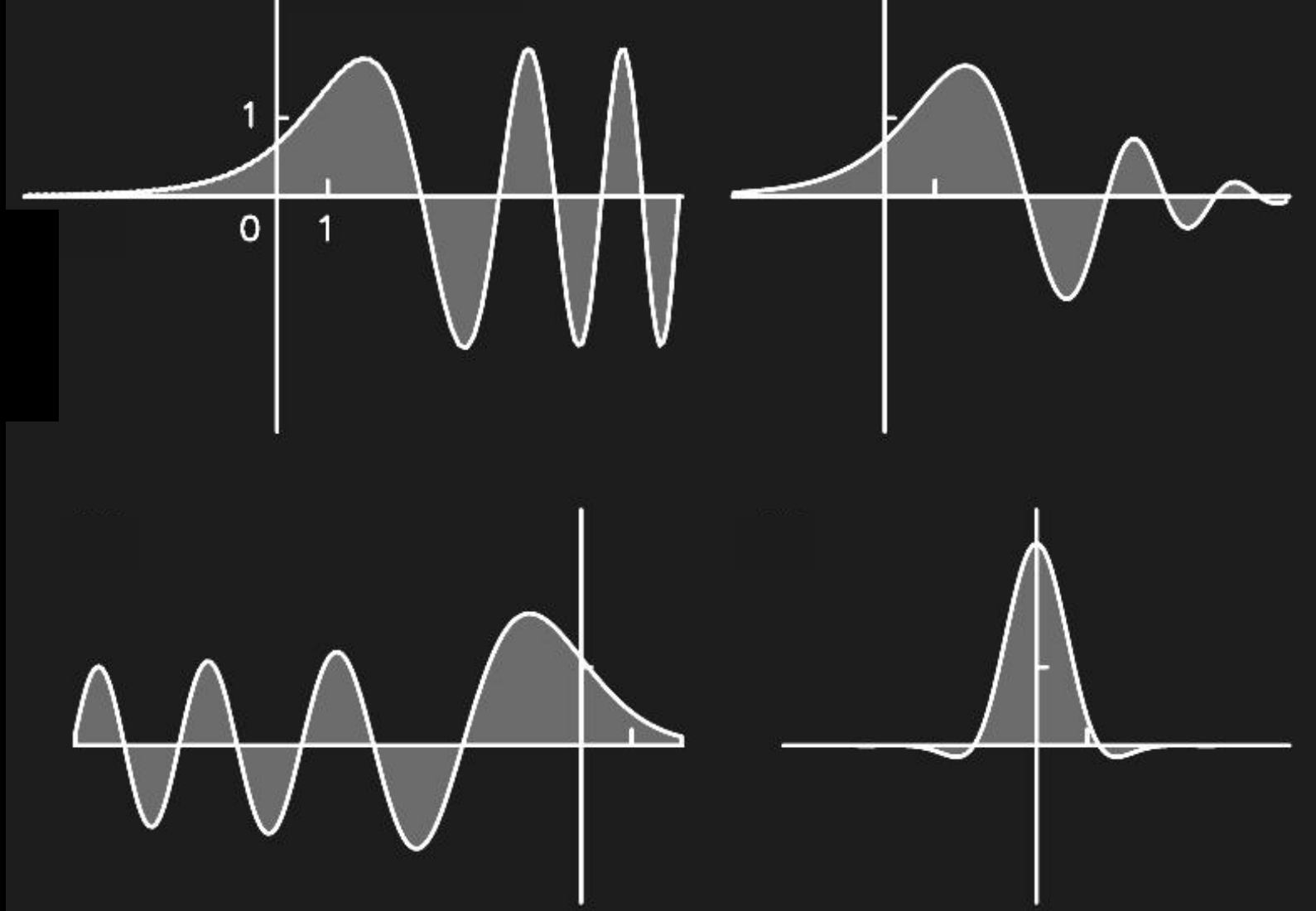
Mimas 5:3  
resonances



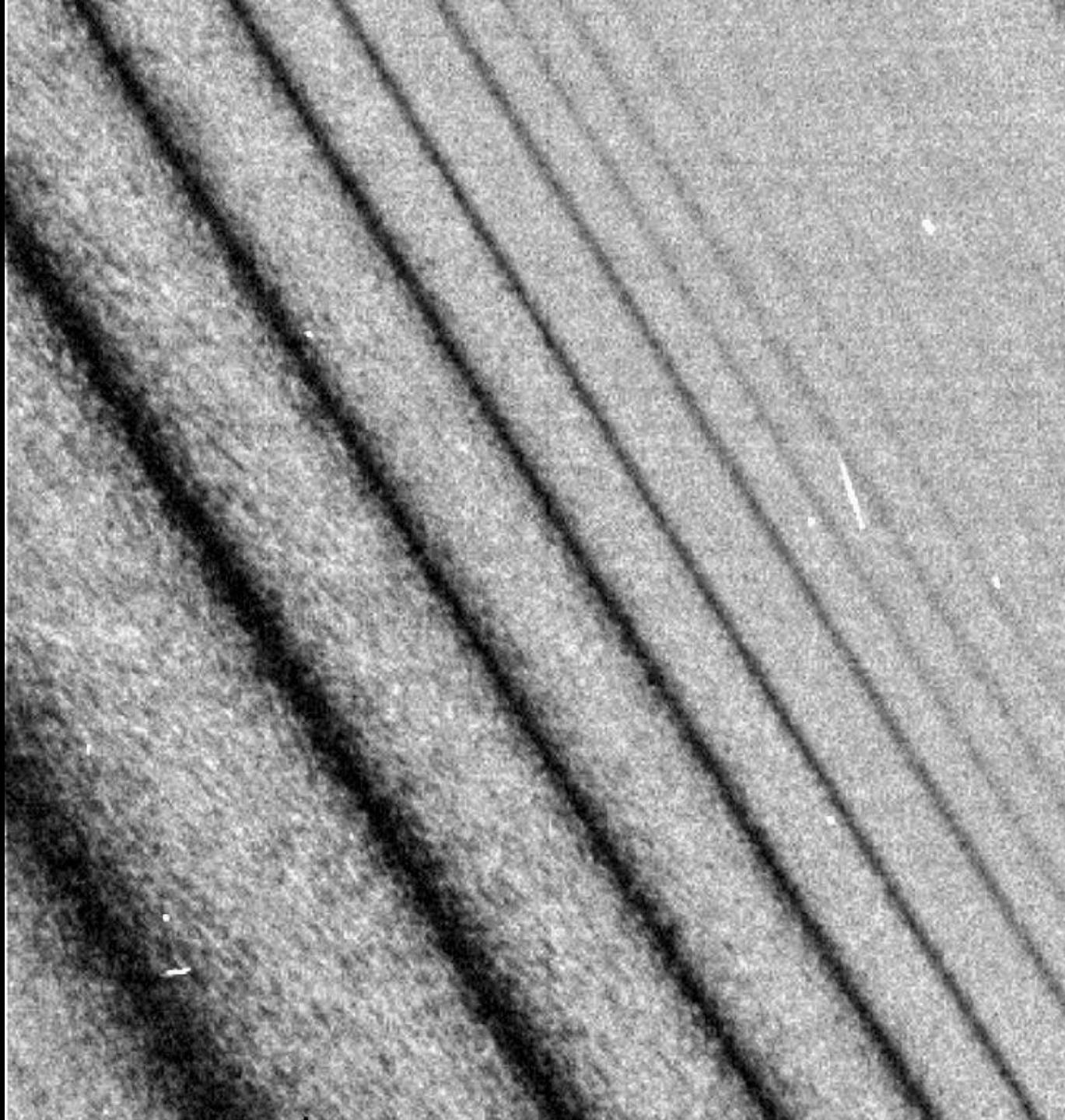
damping  $\rightarrow$   
 $v \sim c_s^2/\Omega \sim h^2 \Omega$

surface  
density  
 $\Sigma_0$

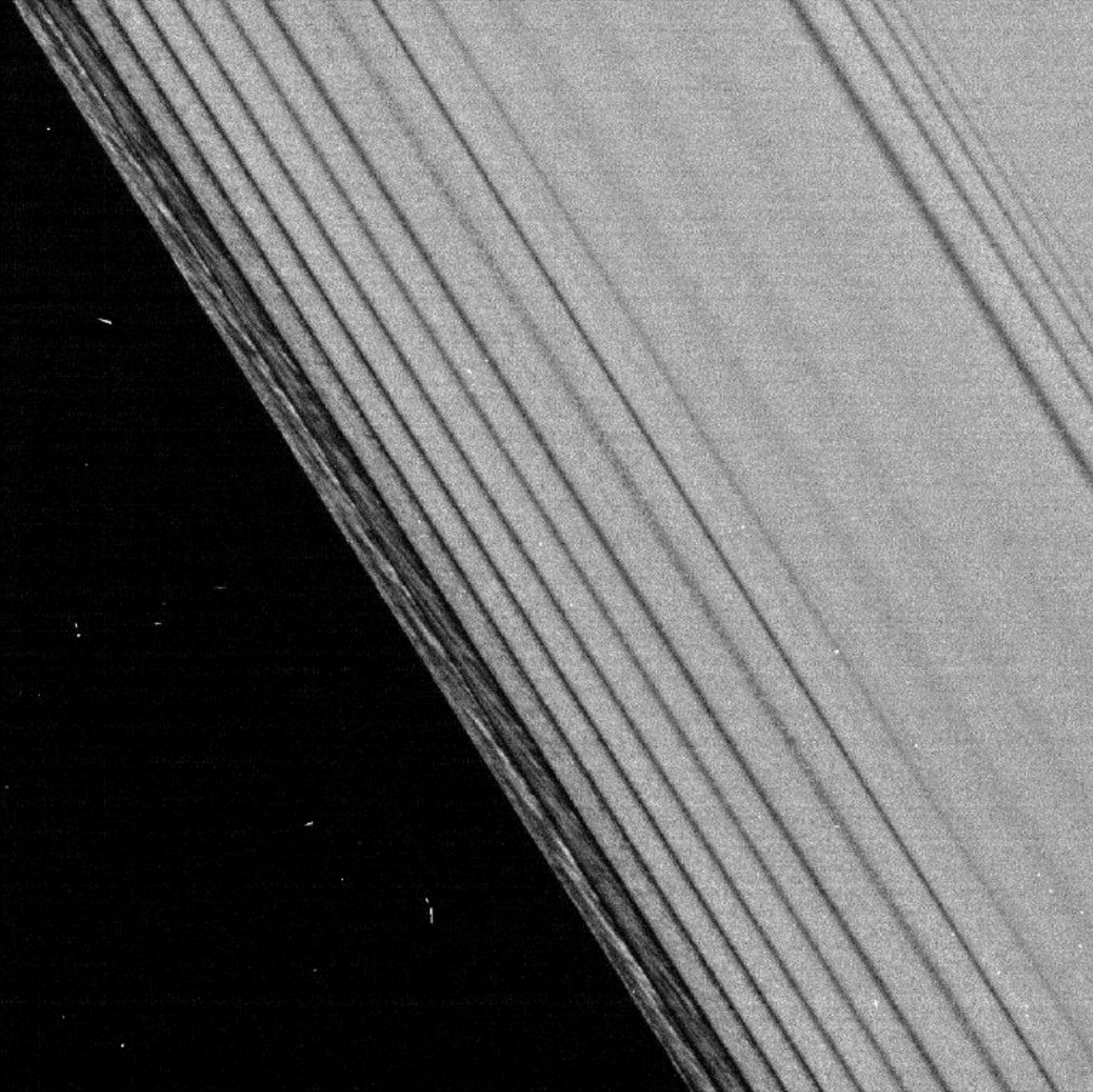




théorème: *toutes* les solutions ont la **même intégrale** → elles provoquent le **même couple** (échange de moment cinétique) entre le disque et le perturbateur (Meyer-Vernet & Sicardy, *Icarus* 1987)







오페리아  
1986U8

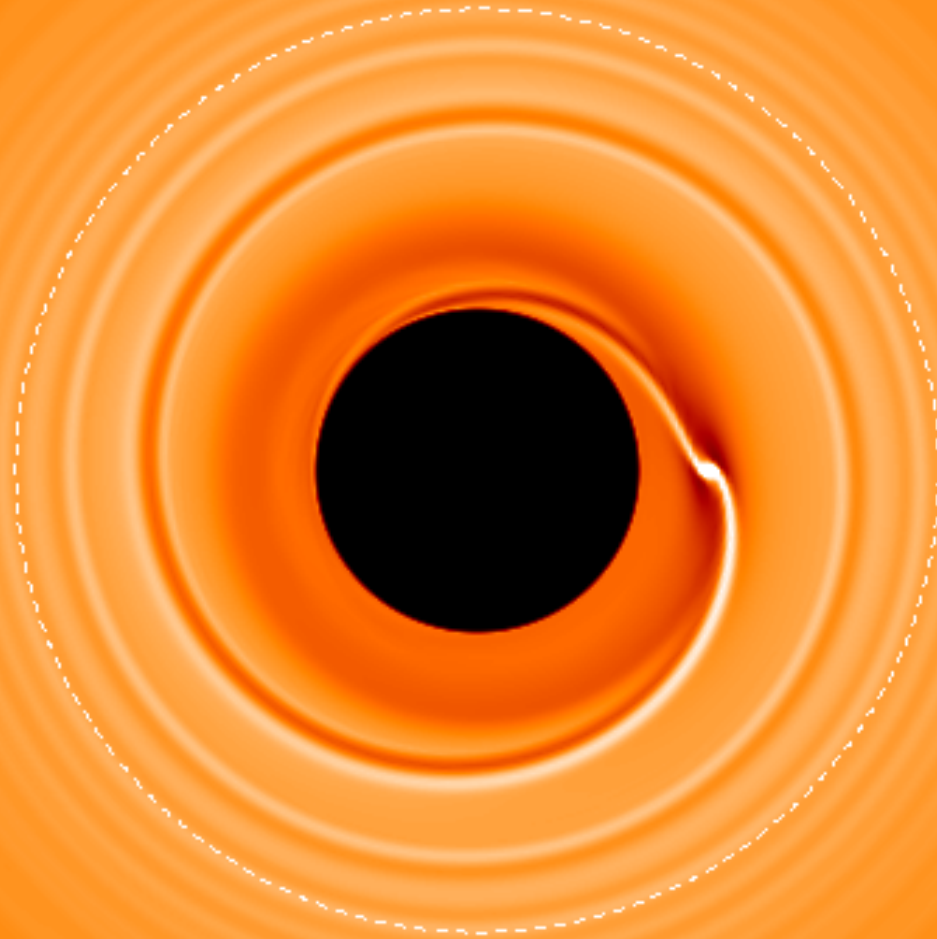


코르델리아  
1986U7



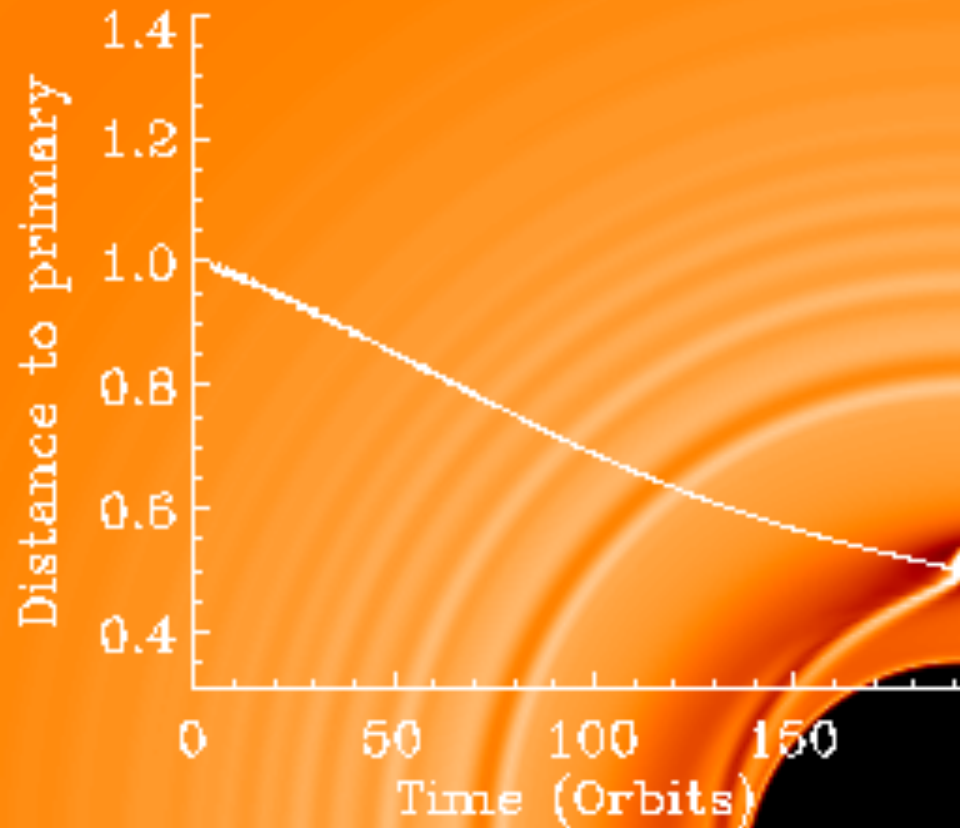
192.0 orbits

simulations F.Masset *et al.*  
migration type I



<http://www-star.qmul.ac.uk/~masset/moviesmpegs.html>

simulations F.Masset *et al.*  
migration type I

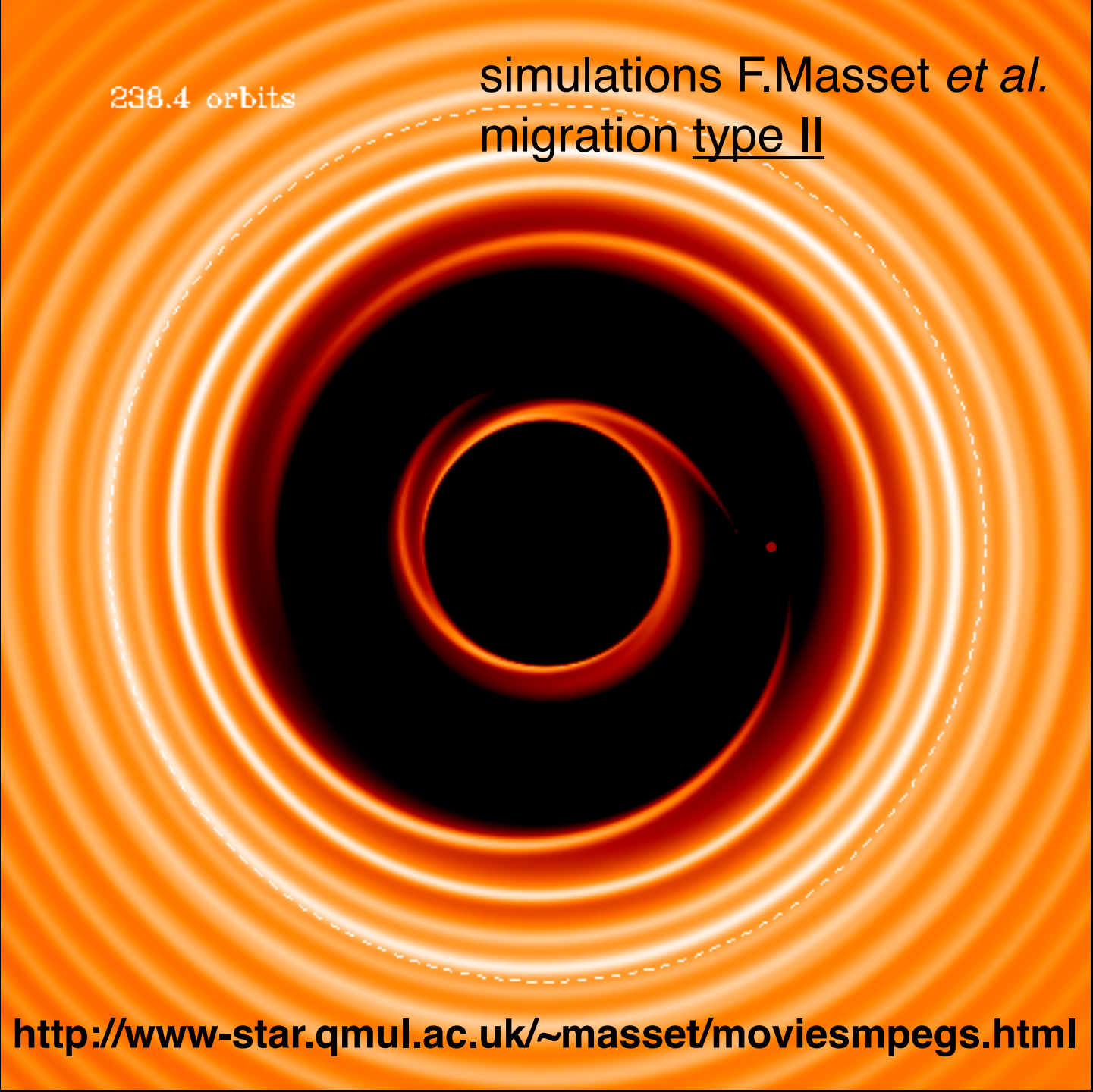




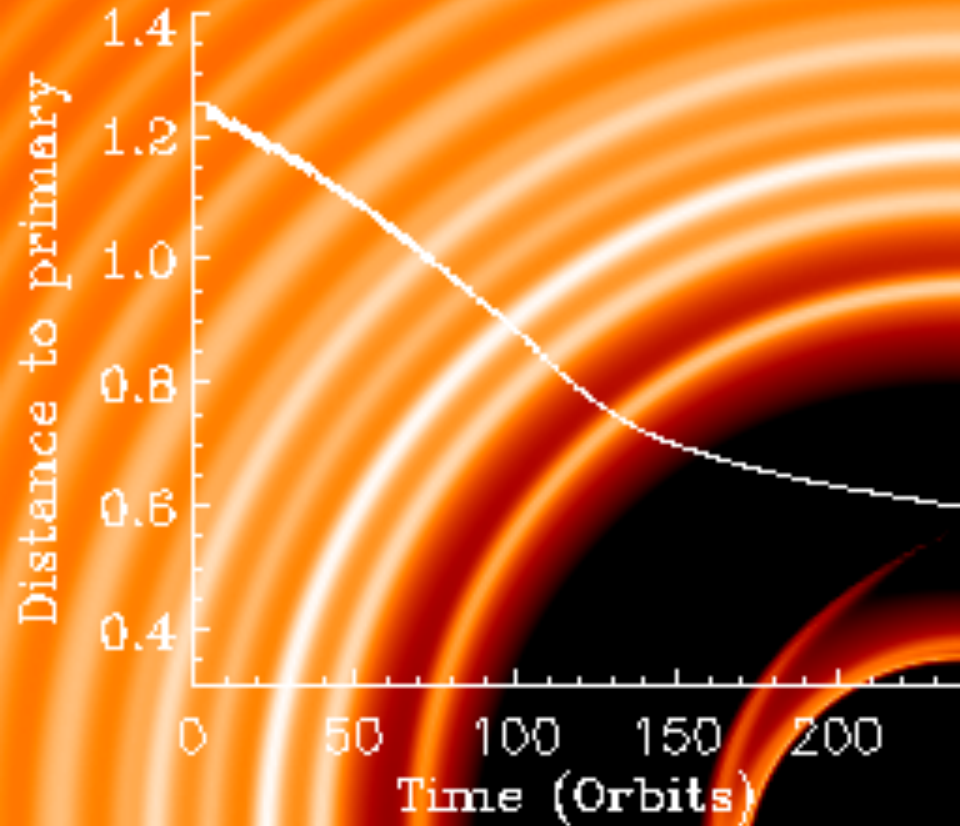
238.4 orbits

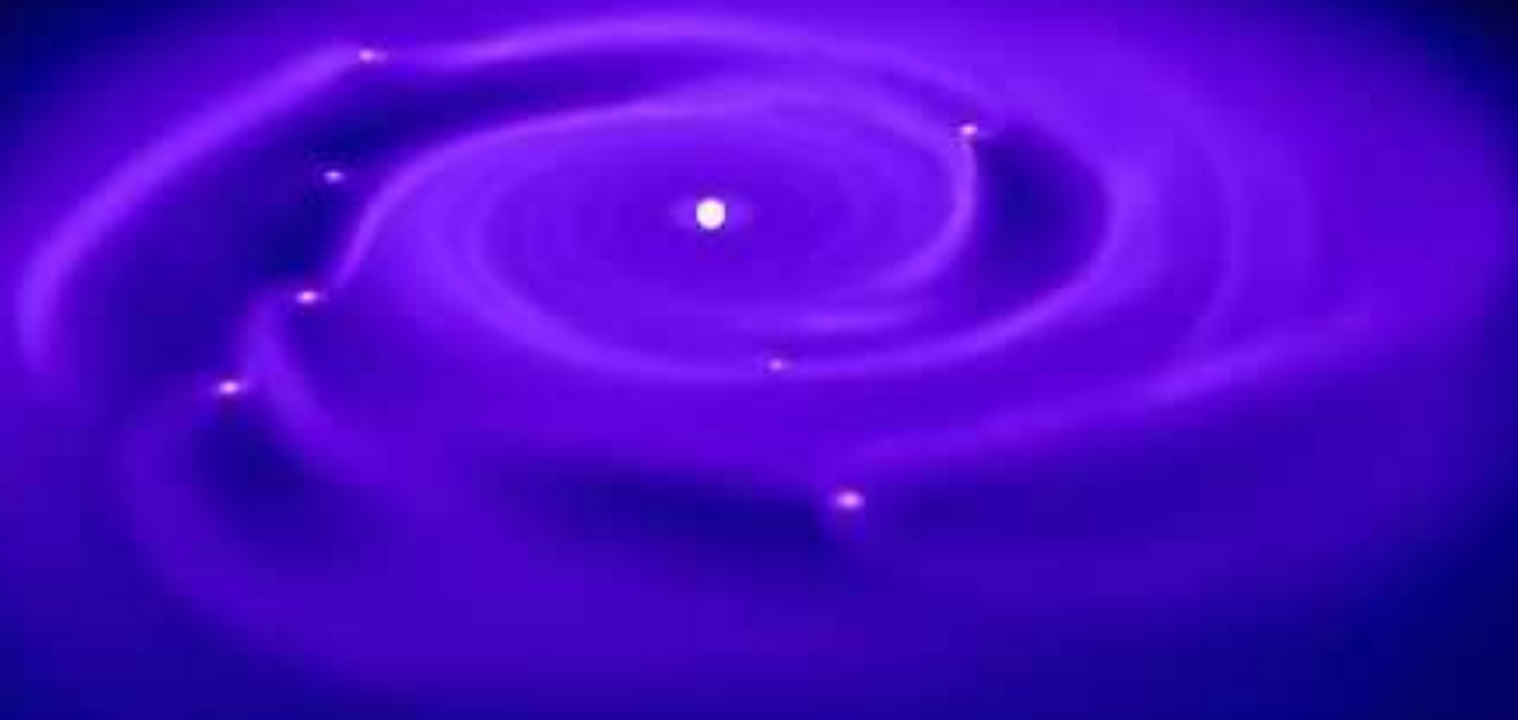
simulations F.Masset *et al.*  
migration type II

<http://www-star.qmul.ac.uk/~masset/moviesmpegs.html>

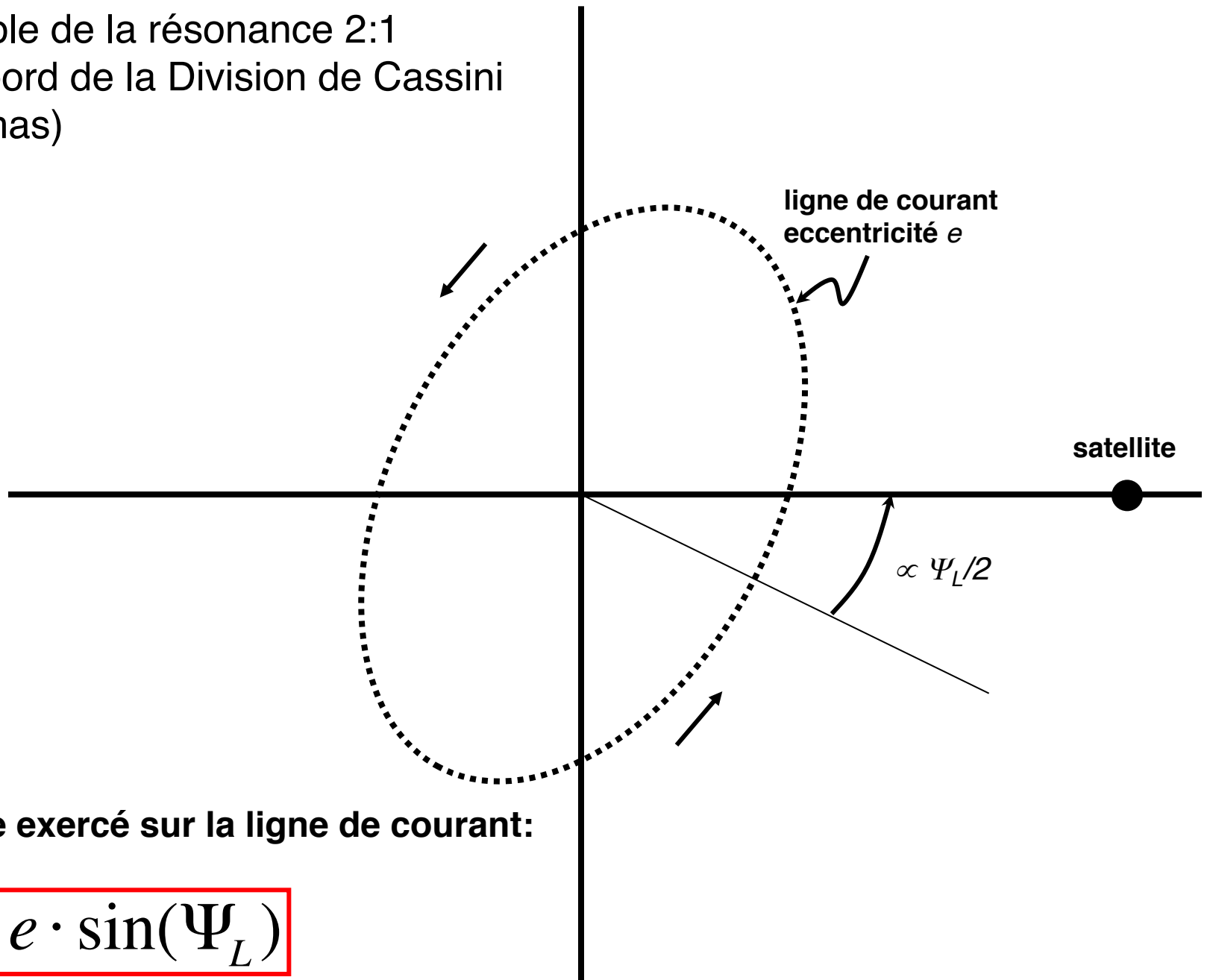


simulations F.Masset *et al.*  
migration type II



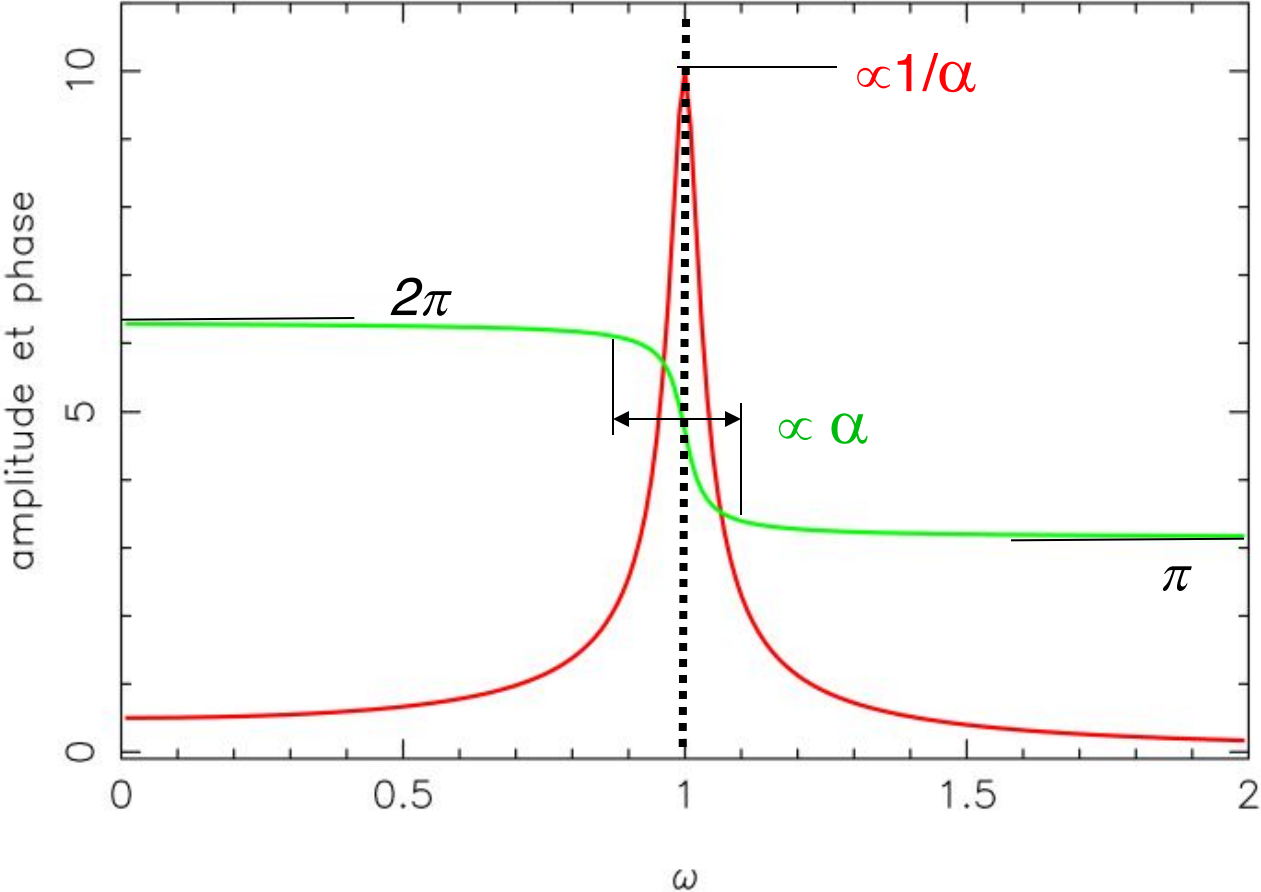


exemple de la résonance 2:1  
(e.g. bord de la Division de Cassini  
et Mimas)





Oscillateur harmonique,  $\alpha = 0.05$ ?



# Oscillateur harmonique, $A=0.5$ , $\alpha=0.05$ & $\alpha=0.15$

