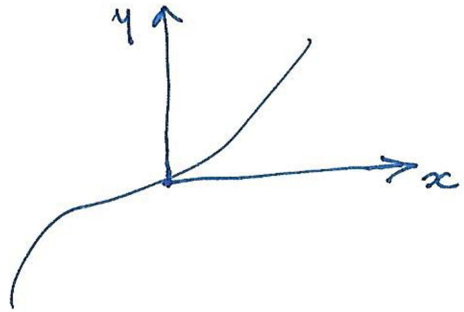
Implicit Functions theorem

Pb: Solve $f(x,y)=0$ w.r.t. y.

(A) given $f(0,0)=0$ in the neighborhood of $x=0$



Taylor:

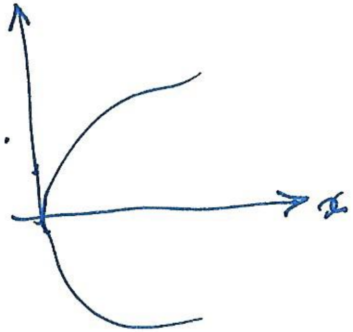
$$y = y_0 + y'_x\Big|_0 x + \frac{1}{2} y''_{xx}\Big|_0 x^2 + \dots \text{ etc.}$$

$$y'_x\Big|_0 = -\frac{f'_x}{f'_y}\Big|_0$$ No problem as far as $f'_y = 0$

In pendulum-like resonant problems: $f'_y = 0$ (but $f''_y \neq 0$).

$\boxed{\text{If } f'_y = 0}$

$y'_x \to \infty$ @ origin.



We may solve the inverse problem (find $x = x(y)$).

$$x = \underset{\substack{\shortparallel \\ 0}}{x_0} + \underbrace{x'_y\Big|_0}_{\text{this } = 0} y + \frac{1}{2} x''_{yy}\Big|_0 y^2 + \dots$$

So when we invert to obtain $y(x)$ (invert by iterating).

$$y = \sqrt{\frac{2x}{x''_y}} + \dots \quad \boxed{y \sim \sqrt{x}}.$$

then we see that we have to expand in $\sqrt{x}$ instead of x to solve the given Pb. (A)

$\boxed{\text{If } f'_y = 0,\ f''_y = 0}$

the same with cubes and $\sqrt[3]{\ }$

$$y = y_0 + y'_{\sqrt{x}}\Big|_0 \sqrt{x} + \frac{1}{2} y''_{\sqrt{x}\sqrt{x}}\Big| x + \dots + \dots x\sqrt{x} + \dots$$

N.B. $y'_{\sqrt{x}} = -\frac{2 f'_x \sqrt{x}}{f'_y} \sim \frac{0}{0}$ indetermination to solve with l'hospital $\to y'_{\sqrt{x}} = \frac{\#}{f''_{yy}}$

or if $f''_y \neq 0$