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SUPPLEMENTARY INFORMATION

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Ring dynamics around non-axisymmetric bodies with application to Chariklo and Haumea

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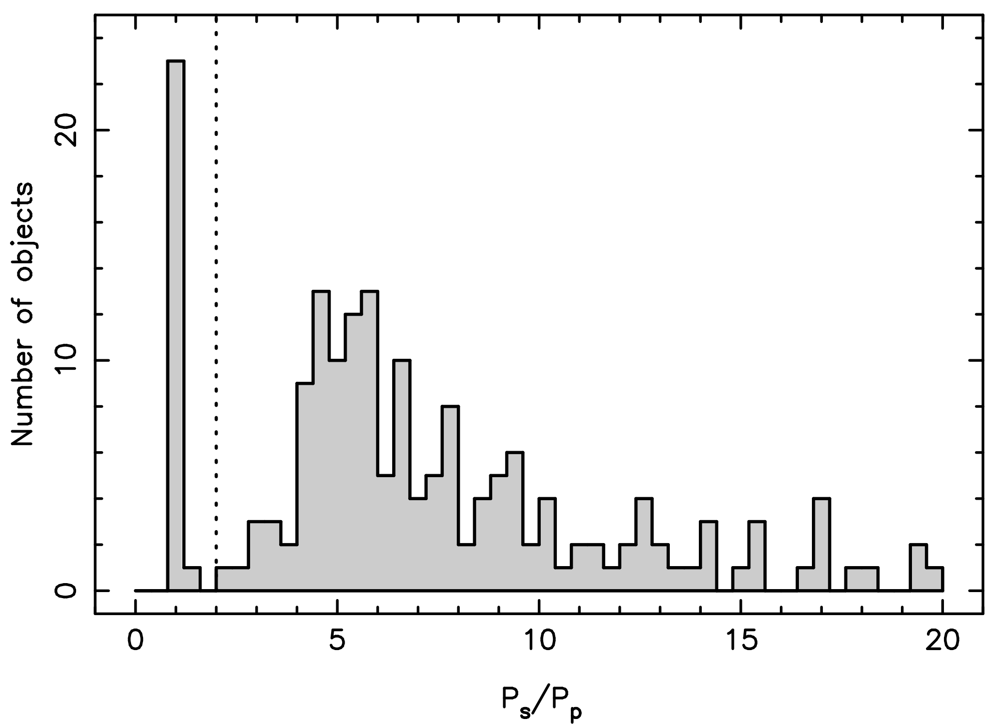
Supplementary Table 1

Azimuthal variation $f(\theta)$ of the corotation potential (Methods Eq. (26))	
Mass anomaly	$q^{-1/6}\left(\dfrac{1}{\sqrt{q^{1/3}+q^{-1/3}-2\cos\theta}} - q^{1/2}\cos\theta\right)\cdot\mu$
Triaxial ellipsoid[a]	$2\sum_{p=1}^{+\infty} q^{2p/3} S_p \epsilon^p \cos(2p\theta)$
Coefficients $\mathcal{A}_m(a)$ of the $m/(m-1)$ Lindblad resonances (Main Text Eq. (3) and Methods Eq. (39))	
Mass anomaly[b]	$\left\{\left[m + \dfrac{a}{2}\dfrac{d}{da}\right] b_{1/2}^{(m)}(a/R_{\mathsf{sph}}) + q\left(\dfrac{a}{2R_{\mathsf{sph}}}\right)\delta_{(m,-1)}\right\}\cdot\mu$
Triaxial ellipsoid[e] (with m even)	$[2m-(\lvert m\rvert+1)]\, S_{\lvert m/2\rvert}\left(\dfrac{R}{a}\right)^{\lvert m\rvert+1}\cdot\epsilon^{\lvert m/2\rvert}$

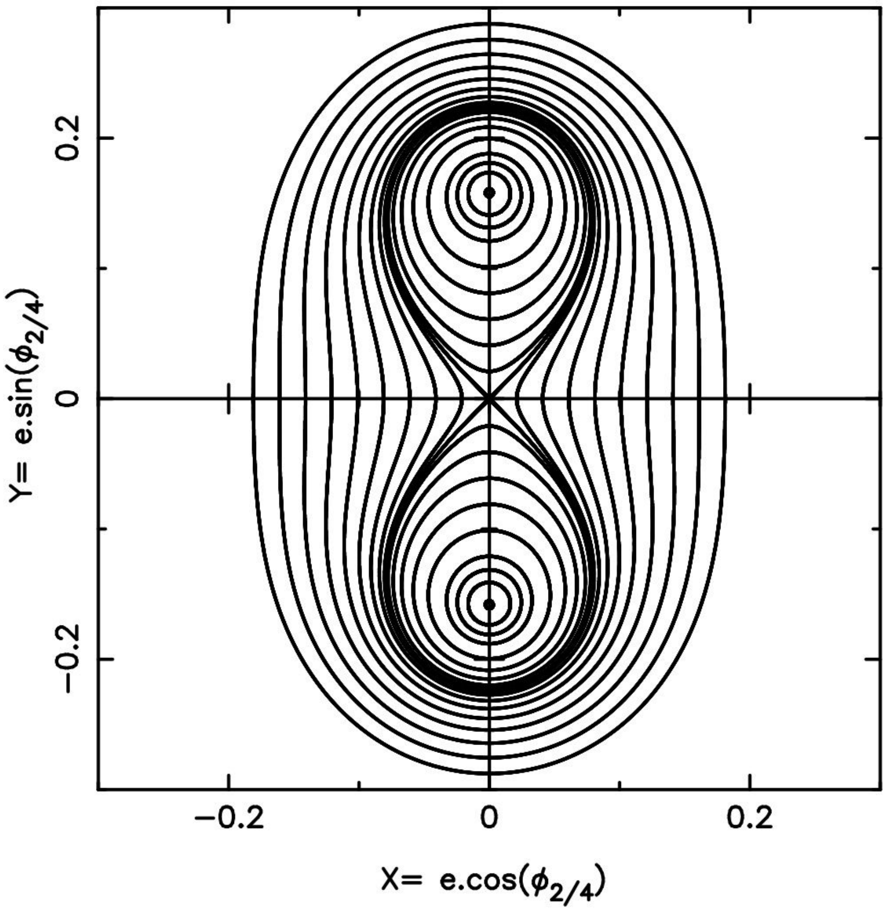
[a] The sequence S_p is defined by Eq. (20).
[b] Assuming a spherical body of radius R_{sph}. The terms $b_{1/2}^{(m)}$ are the Laplace coefficients and $\delta_{(m,-1)}$ is the Kronecker delta function.

Supplementary Figure 1



Supplementary Figure 1 | Distribution of orbital periods of satellites around asteroids and Trans-Neptunians Objects. The orbital period P_s of 179 satellites known around binary or multiple asteroids and Trans-Neptunians Objects (taken from http://www.johnstonsarchive.net/astro/astmoontable.html as of April 2018) are plotted in units of the rotation period P_p of their primaries. The resulting histogram of P_s/P_p shows a peak near unity, corresponding to tidally evolved systems, in which the primary rotates synchronously with the satellite orbital period. The vertical dotted line correspond to the outer $1/2$ resonance, where the satellite completes one revolution while the primary completes two rotations. The steady increase of satellite presence beyond that resonance is in line with the model presented in the text, i.e. satellite formation in a primordial collisional disk that has been pushed outwards by the resonant torque of the $1/2$ resonance.

Supplementary Figure 2



Supplementary Figure 2 | Phase portrait of the 2/4 outer spin-orbit resonance. The phase portrait of the 2/4 resonance is shown for an ellipsoidal Chariklo with elongation $\epsilon = 0.20$ (Table 1), with $X = e\cos(\phi_{2/4})$ and $Y = e\cos(\phi_{2/4})$, where e is the particle eccentricity, $\phi_{2/4} = 2\lambda - \lambda_A - \varpi$ is the resonant angle, and the various other angles are defined in the Methods. All the trajectories share the same Jacobi constant, see Murray & Dermott, Solar system dynamics, Cambridge University Press (1999) for details. This constant has been chosen so that the particle that starts at the origin $(X, Y) = (0, 0)$ is at exact resonance, i.e. with semi-major axis $a_{2/4} = a_{1/2}$, see Main Text. The origin is then an unstable hyperbolic point that forces particles initially on a circular orbit to acquire high eccentricities of the order of $e \sim 0.2$, see Methods. This kind of topology occurs for a narrow semi-major axis range of $a_{1/2}(1 - 0.25\epsilon) \lesssim a \lesssim a_{1/2}(1 + 0.25\epsilon)$ around the resonance.