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ENERGY LOSS BY SLOW MAGNETIC MONOPOLES IN A THERMAL PLASMA

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ABSTRACT

When calculating the deceleration of a magnetic monopole in a thermal plasma, one cannot neglect the nonlocal dispersive properties if the plasma is conductive and the monopole's velocity is subthermal. This yields values of the stopping power in astrophysical plasmas smaller than previously found.

Subject headings: elementary particles — plasmas

I. INTRODUCTION

The deceleration of massive nonrelativistic magnetic monopoles in a classical thermal plasma has recently been calculated (Hamilton and Sarazin 1983) in view of astrophysical applications. However, that calculation does not take into account the nonlocal plasma dispersive properties. This approximation is expected to be correct in a conductive plasma if the relevant phase velocity is much larger than the plasma thermal velocity, i.e., if the monopoles are suprathermal. Since this condition does not hold for monopoles moving at galactic virial velocities ($\sim 10^{-3}c$) in most diffuse astrophysical plasmas, the spatial dispersion should be taken into account. This is an example of the so-called anomalous skin effect (see, e.g., Akhiezer *et al.* 1975), where the electromagnetic characteristic length is much larger than the "normal" value c/ω_p. Thus, we expect that the spatial dispersion will reduce the losses.

The present short paper calculates this effect, in the simple case where the monopoles are very much subthermal with respect to the plasma electrons, and the plasma is highly conductive. These approximations hold for monopoles moving at $10^{-3}c$ in dilute astrophysical plasmas.

The plasma description is given in § II; the losses are calculated in § III and discussed and compared with previous results in § IV; SI units are used throughout the paper, unless otherwise stated.

II. PLASMA DESCRIPTION

Let us neglect the plasma large-scale magnetic field and nonlinearity. Thus, the transverse and longitudinal plasma dispersion properties decouple from each other. The former, relevant when the source is a magnetic monopole, is given by (see, e.g., Sitenko 1967)

$$\epsilon_T(k, \omega) = 1 - \frac{\omega_p^2}{\omega^2} \times \left\{\psi(z) + \frac{1}{\mu}\psi(\mu z) - i\pi^{1/2}z\left(e^{-z^2} + \frac{1}{\mu}e^{-\mu^2 z^2}\right)\right\},$$
$$\psi(z) = 2ze^{-z^2}\int_0^z e^{x^2}dx\ , \quad z = \frac{\omega}{kv_{\rm th}}\ , \tag{1}$$

where ω_p and $v_{\rm th} = (2KT/m)^{1/2}$ are respectively the electron (angular) plasma frequency and thermal velocity, and $\mu^2 = M/m$ (M and m are respectively the ion and electron mass). Equation (1) holds for a classical, nonrelativistic, thermal and collisionless plasma, in the Vlasov description (many particles in a cubic Debye length, i.e., $nL_D^3 \gg 1$).

Since the source is a monopole with velocity $V \ll v_{\rm th}$, we need an approximation of ϵ_T for $z = \omega/kv_{\rm th} = \boldsymbol{k} \cdot \boldsymbol{V}/kv_{\rm th} \ll 1$. We take

$$\epsilon_T(k, \omega) \sim 1 + i\pi^{1/2}(\omega_p^2/\omega^2)z\ . \tag{2}$$

It is easily seen that equation (2) is correct in both cases $\mu z \ll 1$ and $\mu z \gg 1$; the ions contribute only in the intermediate situation $\mu z \approx 1$, which for simplicity we will not consider here.

Writing the dispersion equation $k^2c^2/\omega^2 = \epsilon_T$ with $\omega = \boldsymbol{k} \cdot \boldsymbol{V}$ yields the characteristic attenuation length

$$l = |k|^{-1} = \pi^{-1/4}(v_{\rm th}/V)^{1/2}c/\omega_p\ , \tag{3}$$

larger than the value c/ω_p obtained by using the local approximation of ϵ_T (i.e., $z \to \infty$ in eq. [1]).

III. MONOPOLE LOSSES

The moving monopole yields the charge and current density

$$\rho^M(\boldsymbol{r}, t) = e^M\delta(\boldsymbol{r} - \boldsymbol{V}t)\ ; \quad \boldsymbol{J}^M(\boldsymbol{r}, t) = \boldsymbol{V}\rho^M(\boldsymbol{r}, t)\ , \tag{4}$$

where $e^M = (137/2)ec$ is the minimum Dirac monopole charge.

The corresponding magnetic field is given, in Fourier space, by Maxwell equations as

$$\boldsymbol{B}(\boldsymbol{k}, \omega) = \mu_0 i\left[\frac{\omega\epsilon_T \boldsymbol{J}^M(\boldsymbol{k}, \omega)/c^2 - \boldsymbol{k}\rho^M(\boldsymbol{k}, \omega)}{k^2 - \epsilon_T(\boldsymbol{k}, \omega)\omega^2/c^2}\right]. \tag{5}$$

The monopole power loss is

$$\frac{dW}{dt} = \int d^3r \boldsymbol{J}^M(\boldsymbol{r}, t)\boldsymbol{B}(\boldsymbol{r}, t) = -\frac{\mu_0 ie^{M2}}{(2\pi)^3}V\int d^3k k_z \frac{1 - \beta^2\epsilon_T}{k^2 - \beta^2\epsilon_T k_z^2}\ , \tag{6}$$

where ϵ_T stands for $\epsilon_T(\boldsymbol{k}, \boldsymbol{k} \cdot \boldsymbol{V})$, $\beta = V/c$, and the z-axis is taken along $\boldsymbol{V}$. Using equation (2) and integrating in spherical coordinates yield, since $\beta^2 \ll 1$,

$$\frac{dW}{dt} = -\frac{\mu_0 e^{M2}V}{4\pi^2 l^2}\int_0^\infty dk k^3 \int_{-1}^1 du \frac{1 - u^2}{k^4 + u^2/l^4}\ . \tag{7}$$

The logarithmic divergence in k is avoided in the usual way (see, e.g., Bekefi 1966; Sitenko 1967) by truncating the k integral at $k_{\rm Max}$. (This corresponds to the failure of the Vlasov equation to describe short-range interactions; in the absence of quantum effects, $k_{\rm Max}$ is customarily taken as the inverse of the impact parameter, where the charged particle electrostatic energy equals the thermal energy, i.e., $k_{\rm Max} \sim r_0^{-1} \sim 4\pi n L_{\rm D}^2$.) The actual value of $k_{\rm Max}$ relevant to the present problem could be discussed but is not very important, since it enters logarithmically in the result, and in any case the inequality $k_{\rm Max}\, l \gg 1$ holds. Thus equation (7) yields

$$\frac{dW}{dt} = -\frac{\mu_0 e^{M2} V}{3\pi^2 l^2}\left[\ln (k_{\rm Max}\, l) + \frac{2}{3}\right], \tag{8}$$

where the characteristic length l is defined in equation (3).

Thus the stopping power is

$$\begin{aligned} P_M &= \frac{dW}{dx} \\ &= -\frac{4}{3}\pi^{1/2}\frac{n}{m}\hbar^2\frac{V}{v_{\rm th}}\left[\ln (k_{\rm Max}\, l) + \frac{2}{3}\right], \quad \frac{V}{v_{\rm th}} < 1 \\ &= 10^{-17}\left(\frac{n}{\rm cm^{-3}}\right)\left(\frac{T}{\rm K}\right)^{-1/2}\beta\left[\ln (k_{\rm Max}\, l) + \frac{2}{3}\right] {\rm GeV\ m^{-1}}, \end{aligned} \tag{9}$$

where $\beta = V/c < 1.8 \times 10^{-5} T^{1/2}$, n is the plasma electron density, and

$$k_{\rm Max}\, l \sim 10^6\left(\frac{n}{\rm cm^{-3}}\right)^{-1/2}\left(\frac{T}{\rm K}\right)^{5/4}\beta^{-1/2}.$$

Physically, these losses (for $V/v_{\rm th} < 1$) stem from a Cerenkov-like interaction between the nonpropagating electromagnetic waves with phase velocity $\omega/k = V$ and electrons with the same velocity, as in the usual Landau damping of plasma waves. (On the contrary, if $V/v_{\rm th} \gg 1$, this mechanism is negligible, since there are few electrons with velocity V, and the local dispersion relation of propagating electromagnetic waves can be used.)

IV. DISCUSSION

a) *Limitations of the Calculations*

Equation (8) (or eq. [9]) does not take into account interactions closer than $k_{\rm Max}^{-1}$. The latter can be calculated by summing over single-particle short-range collisions, and yields a contribution of the same order as equation (8) or smaller, for $V/v_{\rm th} < 1$.

Since our starting point is the dielectric function given by the collisionless Vlasov equation, an implicit assumption is that the effective collision frequency ν satisfies $\nu/kV \ll 1$ for any $k \gtrsim l^{-1}$, i.e., $\nu/\omega_p \ll (V/v_{\rm th})^{1/2}\beta$; thus

$$\beta^{3/2} T^{5/4} \omega_p^{-1} (\ln \Lambda)^{-1} \gg 4 \times 10^{-12}, \tag{10}$$

where $\Lambda \sim k_{\rm Max} L_{\rm D}$. However, the calculation is expected to be approximately valid if $\nu/kv_{\rm th} \ll 1$, i.e.,

$$\beta^{1/2} T^{7/4} \omega_p^{-1} (\ln \Lambda)^{-1} \gg 2 \times 10^{-7}. \tag{11}$$

Besides, we have assumed implicitly that $\hbar/mV \ll k_{\rm Max}^{-1}$.

b) *Comparison with Other Works*

Most calculations of monopole deceleration have been undertaken in view of applications to detecting devices, and are concerned with neutral media or condensed materials (see Martem'Yanov and Khakimov 1972; Ahlen and Kinoshita 1982; Ford 1982; Drell *et al.* 1983). It is interesting to note that, not unexpectedly, our result is formally similar (apart from a factor of order unity, and replacing the thermal velocity by the Fermi velocity, $v_{\rm F}$) to that found for slow monopoles ($V/v_{\rm F} \ll 1$) in a degenerate Fermi gas, when damping and spin are neglected (see eq. [33] in Ahlen and Kinoshita 1982). In both cases, the nonlocal properties of the medium are important at low velocities. This yields a linear dependence of stopping power on velocity.

The deceleration of slow monopoles in classical plasmas has been studied by Hamilton and Sarazin (1983). They assume a local plasma dielectric function and include collisions. This corresponds to taking the limit $z = \omega/kv_{\rm th} \to \infty$ in equation (1), trivially generalized to a finite-conductivity plasma. As already noted, this is expected to be correct if either $V/v_{\rm th} \gg 1$ or $\nu/kv_{\rm th} \gg 1$ (where ν is the plasma effective collision frequency). Otherwise, their results (in eqs. [27] and [28]) and their applications to the interstellar medium and the Sun are overestimated by approximately the factor $v_{\rm th}/V$.

c) *Effect of an Electric Charge*

If the particle has electric (magnitude Ze) as well as magnetic charge, the stopping power becomes $P^M + P^E$, where P^E is due to the excitation of Landau damped longitudinal plasma waves (see, e.g., Sitenko 1967):

$$\begin{aligned} P^E = \frac{dW^E}{dx} &= P_0\left[1 + \left(\frac{3\pi^{1/2}}{4}\right)\mu^{-2}\left(\frac{v_{\rm th}}{V}\right)^3\right], \quad \mu^{-1} \ll \frac{V}{v_{\rm th}} \ll 1 \\ &= P_0\,\mu, \qquad \frac{V}{v_{\rm th}} \ll \mu^{-1}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} P_0 &= -\frac{1}{3\pi^{3/2}}\frac{ne^4Z^2}{\epsilon_0^2 m}\frac{V}{v_{\rm th}^3}\ln (k_{\rm Max} L_{\rm D}) \\ &\approx P_M\left(\frac{Z}{137/2}\frac{c}{v_{\rm th}}\right)^2 \end{aligned}$$

is the loss due to the plasma electrons for a subthermal electric charge with mass $\gg m$. Equation (12) takes into account both electron and ion contributions to the particle losses.

V. CONCLUSION

a) *Main-Sequence Stars*

It is tempting to apply our calculation to the interiors of main-sequence stars. However, in that case, equation (10) does not hold and equation (11) holds only partially, as does also the linearized Vlasov description. Thus our results should be taken with caution. Seeking only an order of magnitude, let us take $T \sim 10^7$ K, and an electron density of order $n \sim 10^{24}$ cm^{-3}. Equation (9) yields $P_M \sim 10^{-20}(n/{\rm cm}^{-3})\beta$ GeV m^{-1}.

This is rather close to the result quoted in Ahlen (1983), who considered only binary interactions. For a monopole with mass 10^{16} GeV and $\beta \sim 10^{-3}$, this yields a stopping length of the order of the solar radius (thus much larger than that found by Hamilton and Sarazin 1983), which allows trapping inside the Sun.

b) *Diffuse Plasmas*

The present calculation can be applied to heavy monopoles in the interstellar medium (ISM) or intergalactic medium

(IGM) plasmas, since inequality (10) generally holds. In the ISM, we take the "standard" velocity for heavy monopoles in the galaxy, $\beta \sim 10^{-3}$ (larger values are expected when the monopole mass is smaller than 10^{16} GeV/c^2, owing to acceleration by the galactic magnetic field; see, e.g., Turner, Parker, and Bogdan 1982). In the IGM, this estimate of β is still less secure, and we shall also take $\beta \sim 10^{-3}$, which holds approximately for the virial velocity in a cluster (larger values could be due to the intracluster magnetic field; see Rephaeli and Turner 1983). In both cases, $V/v_{\rm th} < 1$, so that equation (9) holds. As shown below, this yields negligible losses in present diffuse astrophysical plasmas.

i) *Warm Interstellar Medium*

Let us take $n \sim 0.03$ cm^{-3} and $T \sim 10^4$ K; thus $V/v_{\rm th} \sim 0.5$, $P_M \sim -10^{-22}$ GeV m^{-1}.

ii) *Hot Interstellar Medium*

We take $n \sim 0.003$ cm^{-3} and $T \sim 10^6$ K (McKee and Ostriker 1977; McCray and Snow 1979 and references therein); thus $V/v_{\rm th} \sim 0.05$, $P_M \sim -10^{-24}$ GeV m^{-1}.

iii) *Intergalactic Medium*

Here $n \sim 10^{-3}$ cm^{-3} and $T \sim 10^8$ K (Silk 1973; Bahcall 1977 and references therein); thus $V/v_{\rm th} \sim 0.005$, $P_M \sim -0.5 \times 10^{-25}$ GeV m^{-1}.

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