Polytech' Orléans

Techniques numériques en radioastronomie

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PART A : Techniques de lutte contre les interférences radioélectriques (RFI mitigation)

1. RFI Mitigation Group
2. Signal utile/RFI
3. RFI/méthodes
4. Méthodes/calculateur

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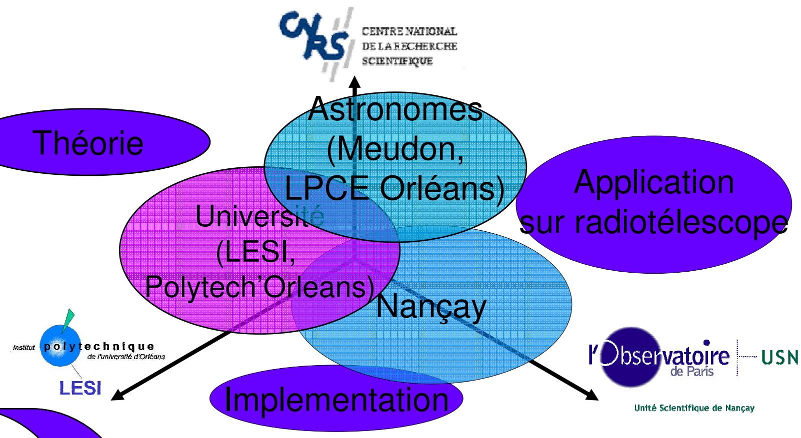
RFI Mitigation Group



Receiver design (Hardware, Digital Electronics)

Algorithms, Methods (Theory, Simulations)

Algorithm Implementation On the hardware (Digital Electronics and signal processing)

Tests and Validation

Le signal utile

Onde électromagnétique

Domaine radio : 3 kHz à 300 GHz

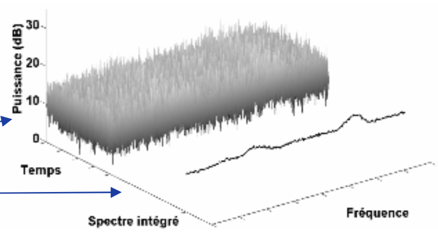
Radiotélescope décimétrique de Nançay (1 – 3 GHz)

Signal observé :

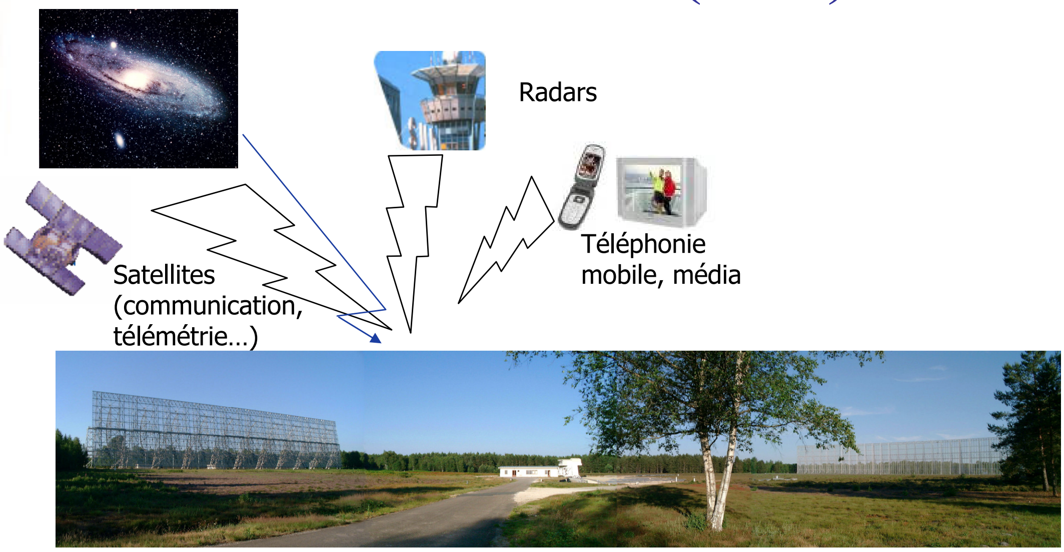
$s(t)=u_{sys}(t)+u_{source}(t)$

Gaussien

(Localement) Stationnaire

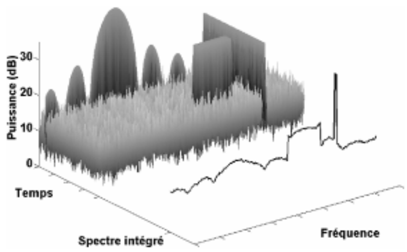


L'interférence (RFI)



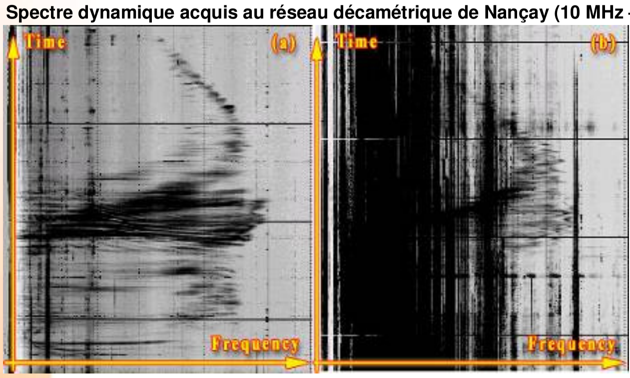
Signal observé :

$$s(t) = u_{sys}(t) + u_{source}(t) + rfi(t)$$
$$= \quad u(t) \quad + rfi(t)$$



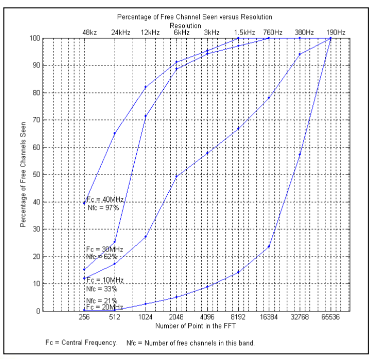
En bande décamétrique

Spectre dynamique acquis au réseau décamétrique de Nançay (10 MHz – 40 MHz)



- bandes très encombrées
- brouilleurs puissants
- effets ionosphériques
- le bruit du ciel élevé
- sources fortes et non statio.

Pourcentage des canaux disponibles en fonction de la résolution fréquentielle (thèse Vincent Clerc 2003)

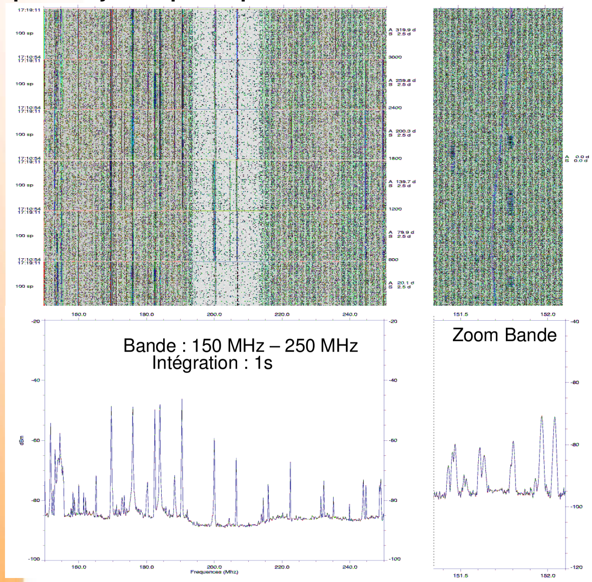


Un taux de disponibilité de 90% nécessiterait dans la bande :

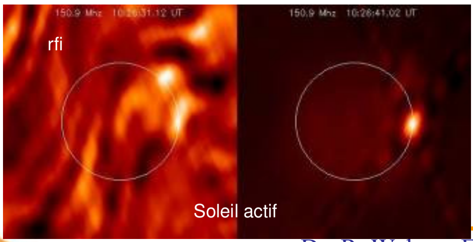
- 35-45 MHz, une résolution de 6.25 kHz.
- 25-35 MHz, une résolution de 1.6 kHz.
- 15-25 MHz, une résolution de 190 Hz.
- 5-15 MHz, une résolution de 6.25 kHz.

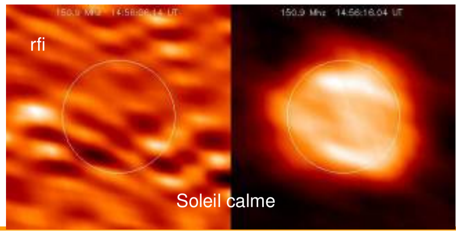
En bande métrique

Spectre dynamique acquis avec l'antenne de surveillance de Nançay (NSA)



- **source forte et non stat. (Soleil)**
- **brouilleurs puissants TV (> 80dB/ soleil)**
- **brouilleurs moyens**
  - **<40dB / soleil**
  - **plutôt < 300MHz**
  - **largeur : 5-20 kHz**
- **difficulté de trouver des bandes de 1MHz propres**

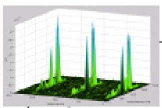


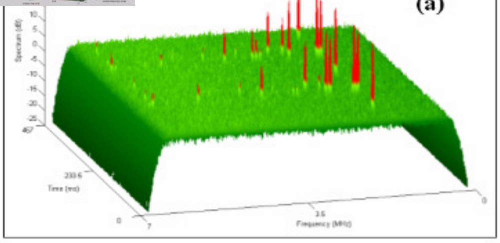


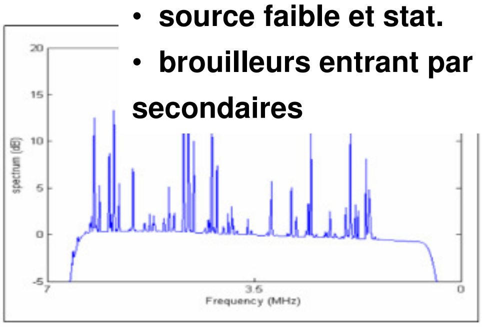
Integration 10s

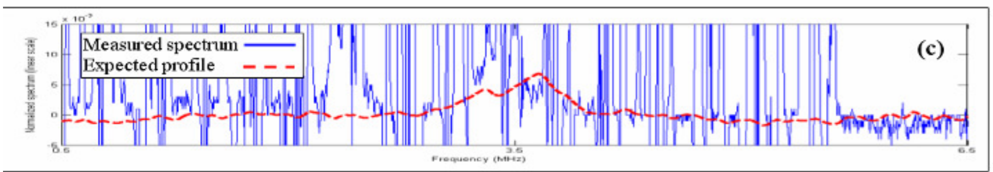
En bande décimétrique

- **source faible et stat.**
- **brouilleurs entrant par les lobes secondaires**

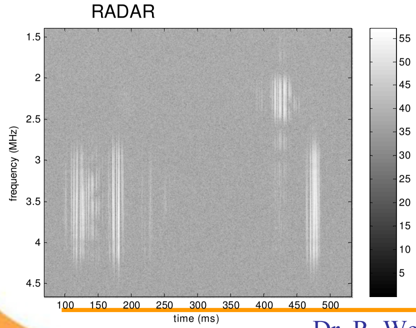


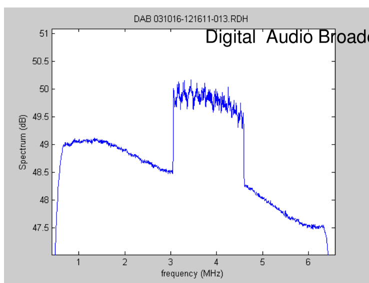






RADAR



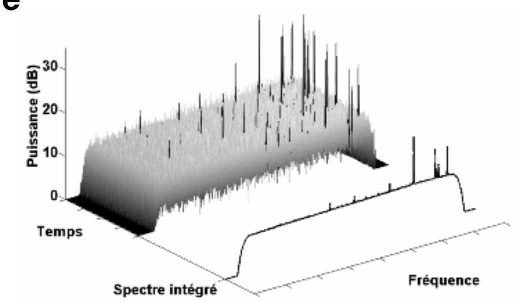


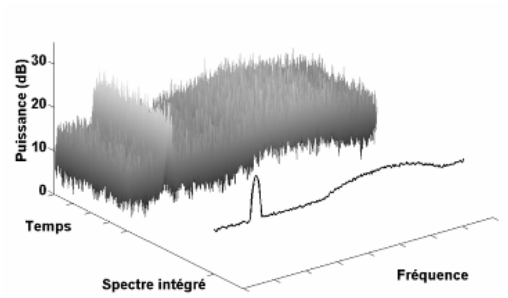
Digital Audio Broadcast

Le couple (RFI, méthode)

• **Les différentes classes de méthode**

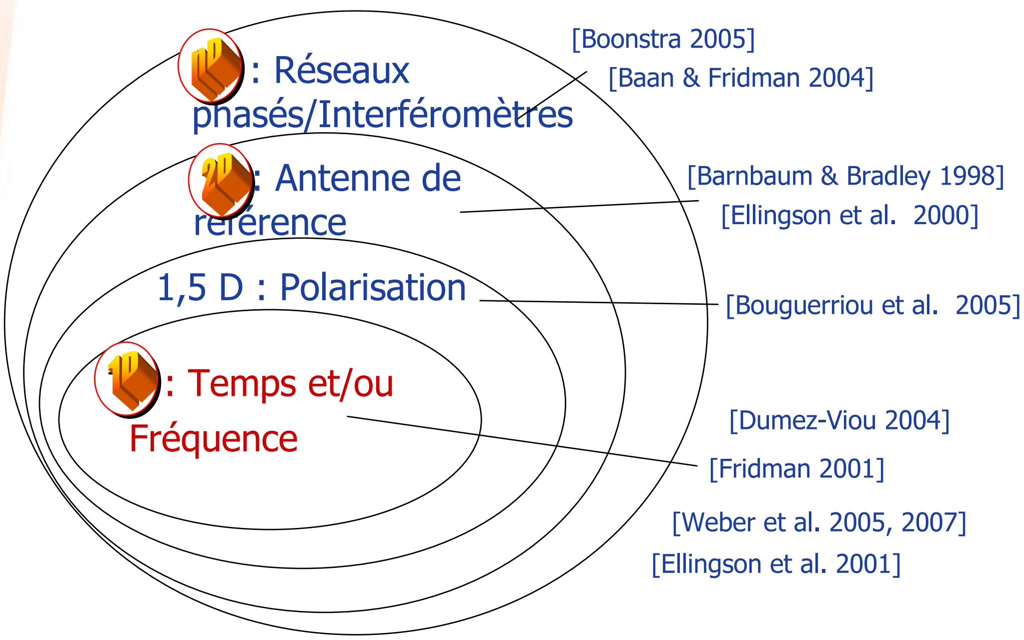
- Interférences intermittentes
  - Détection
    - →*Blanking*
- Interférences continues
  - Réduction/Annulation
  - Estimation
- Protection juridique





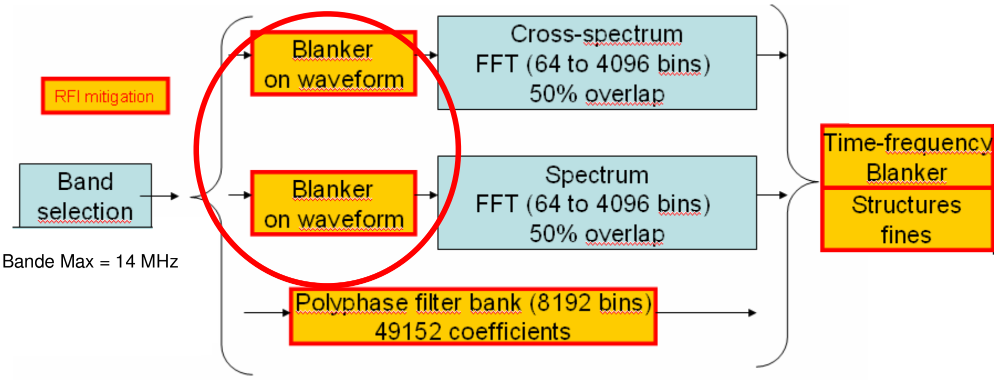
Hiérarchisation des méthodes

• **Quelles propriétés peut-on utiliser pour différencier le signal utile du brouilleur radio électrique ?**

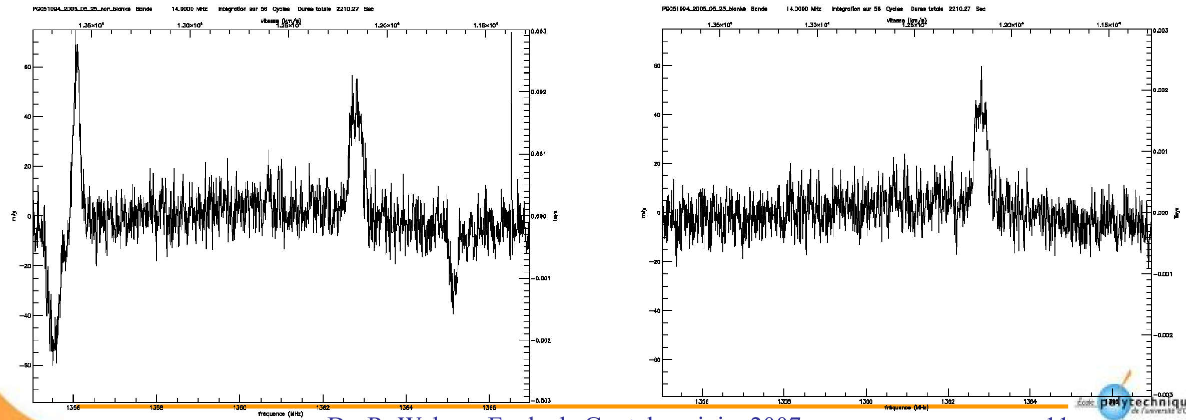


Détecteur temporel

•Le Blanking temps réel en puissance (Thèse C. Dumez-viou)

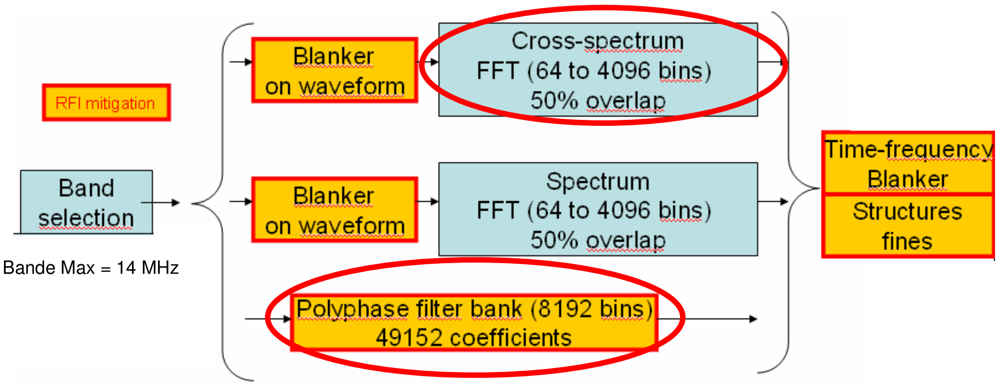


RT : Radar

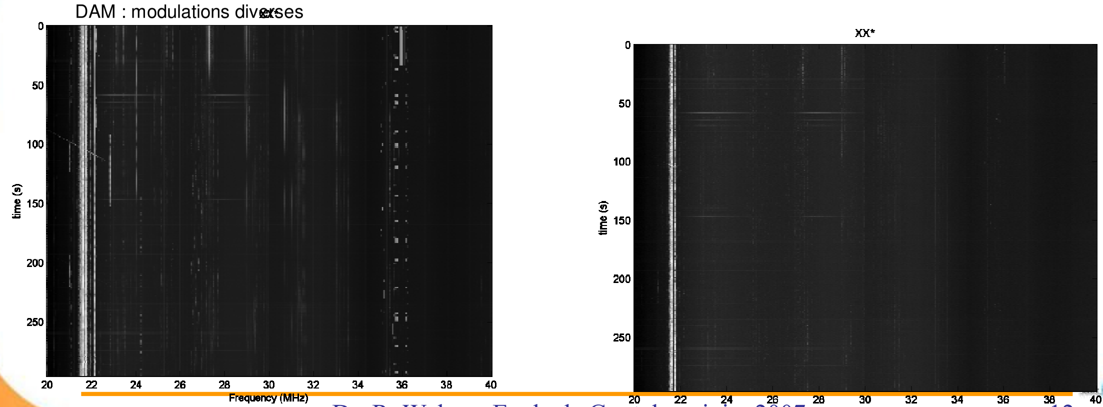


Détecteur fréquentiel

•Le Blanking temps réel en puissance (Thèse C. Dumez-viou)

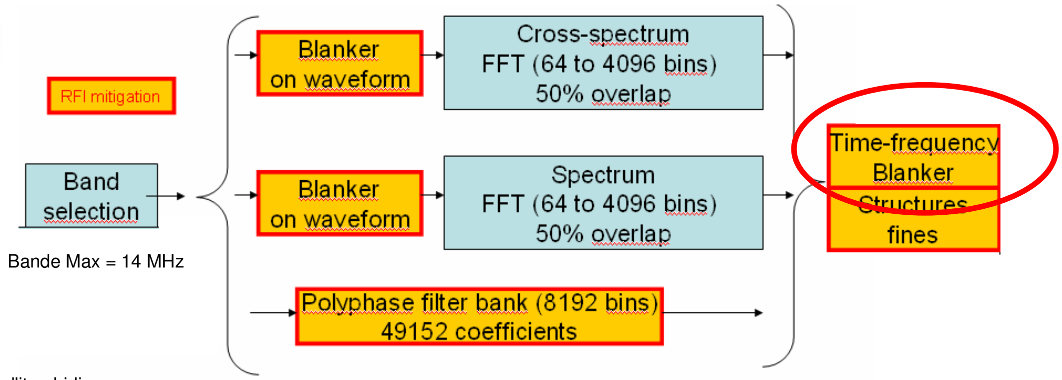


DAM : modulations diverses

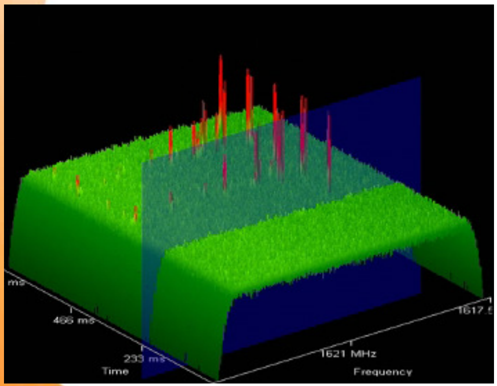


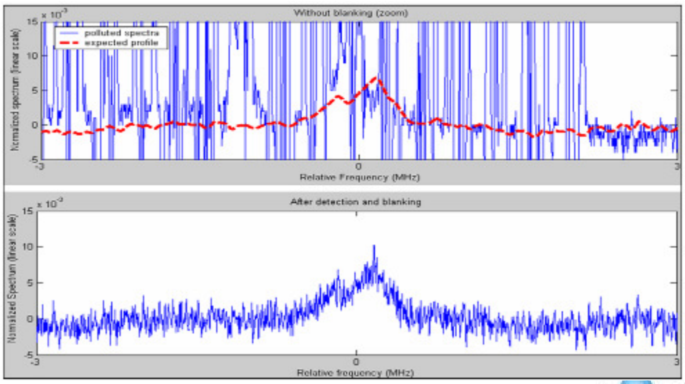
Détecteur T-F

•**Le Blanking temps réel en puissance (Thèse C. Dumez-viou)**



RT : Satellites Iridium





Quel critère pour le détecteur ?

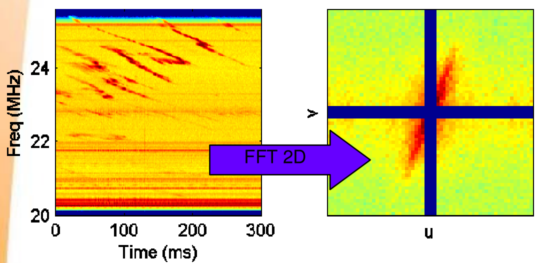
•**Le critère de puissance (Thèse C. Dumez-viou)**

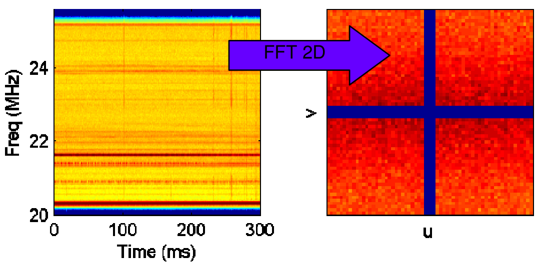
- Simple à mettre en œuvre
- problème du choix du seuil (calibration, non-stationnarité…)

$$threshold = 2\frac{\widehat{\mu}_{\chi^2(\nu)}}{\nu}P^{-1}\left(\frac{\nu}{2}, 1-\alpha\right)$$

Méthode d'estimation robuste qui dépend du contexte RFI

•**Exemple d'un critère topologique T-F (Thèse C. Dumez-viou)**





•**Exemple d'un critère cyclostationnaire (Thèse S. Bretteil)**

Hyp: RFI possède une fonction d'autocorrélation périodique

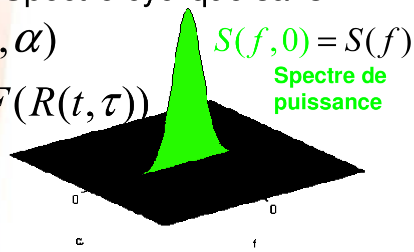
$$R_{rfi}(t+T,\tau) = R_{rfi}(t,\tau)$$

Détecteur cyclostationnaire

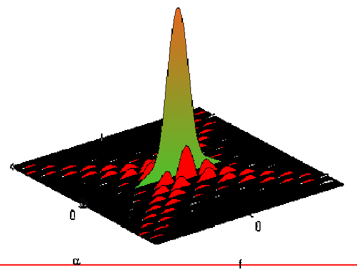
•**Blanking par détecteur cyclostationnaire (Thèse S.Bretteil)**

Spectre cyclique sans RFI

$$S(f,\alpha) = TF(R(t,\tau))$$

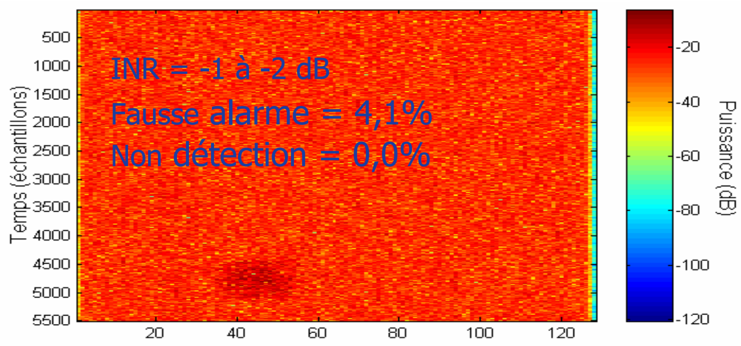


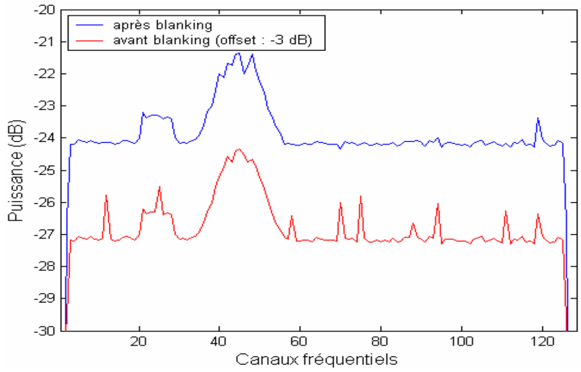
Spectre cyclique avec RFI



Conclusions
- Insensible au facteur de puissance
- Détecte les brouilleurs faibles

Exemple :





Traitement de RFI continues

•**Annulation par méthode cyclostationnaire (Thèse S.Bretteil)**

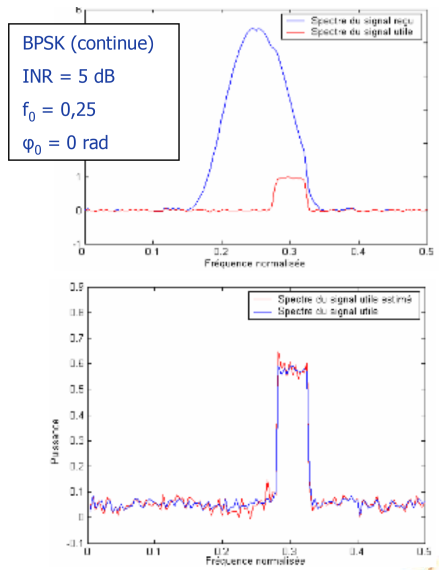
$$R(t,\tau) = R_{utile}(\tau) + \underbrace{R_{rfi}(t,\tau)}_{0}$$

pour certains couples (t,τ)

$$\Rightarrow \hat{R}_{utile}(\tau) = \int_{t\,/\,R_{rfi}(t,\tau)=0} R(t,\tau)dt$$

Conclusions
- Interférences continues
- Possibilité d'intégration forte
  - Parasites faibles
  - Traitement en temps différé
- Conditions d'observations strictes
- Perte d'information
- Innocuité sur le signal utile

Exemple



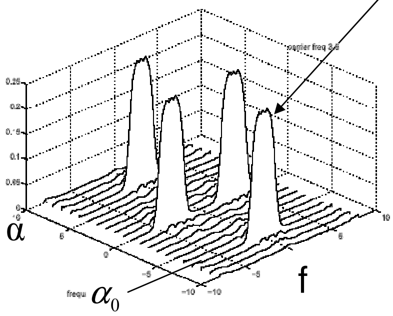
Estimation de RFI

•**Estimation par méthode cyclostationnaire (Thèse S.Bretteil)**

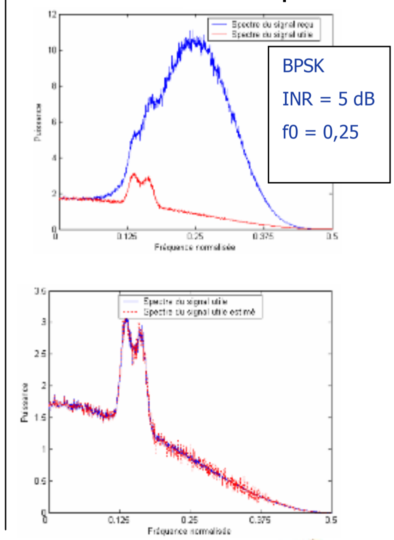
$$\hat{S}_{utile}(f) = S(f) - \hat{S}_{rfi}(f)$$

$$\hat{S}_{rfi}(f) \approx S(f, \alpha_0)$$

Redondance d'information dans le spectre cyclique :

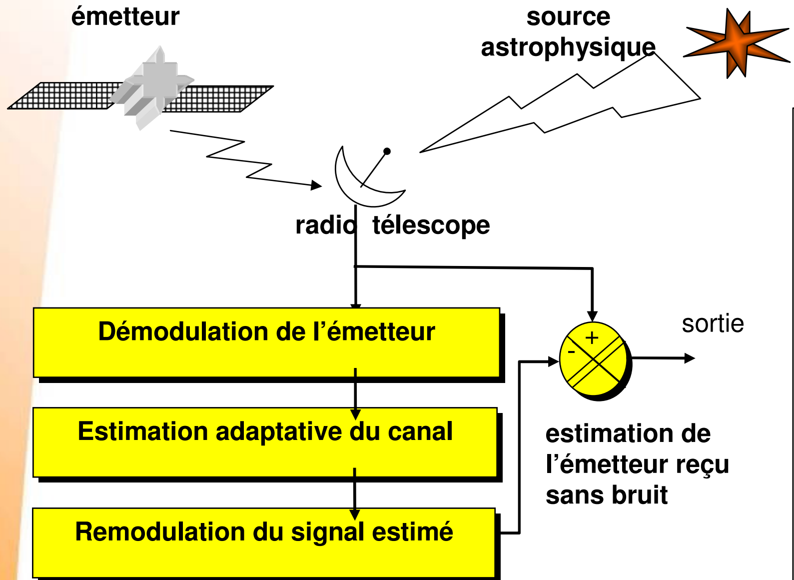


Exemple :

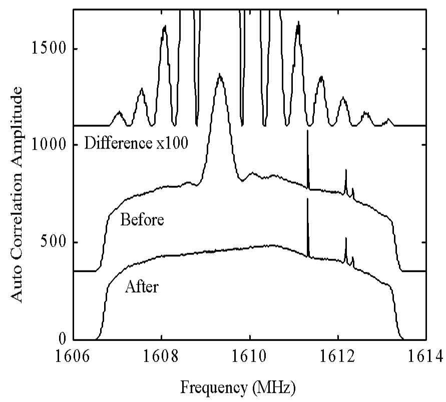


Conclusions
- Interférences continues
- Possibilité d'intégration forte
  - Parasites faibles
  - Traitement en temps différé
- Résultats encourageants
- Conditions d'observations moins strictes
- Dégradation de la sensibilité de la mesure

Estimation par démodulation

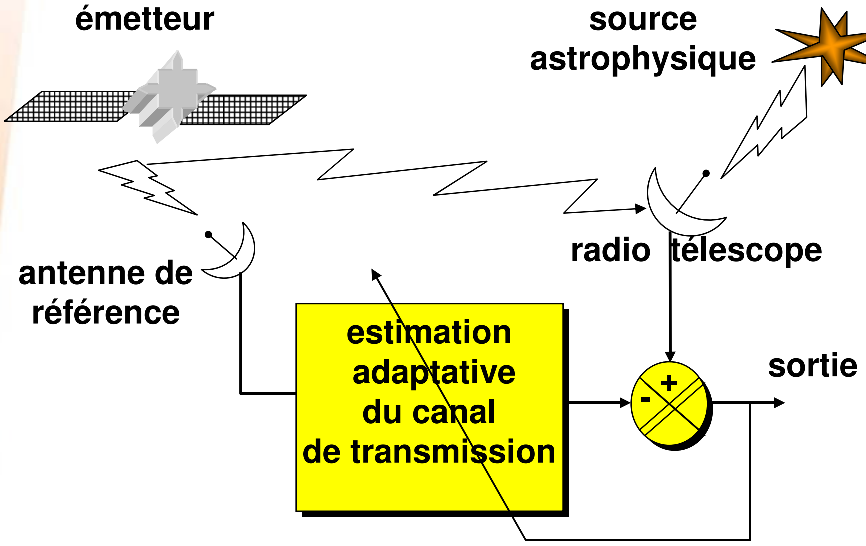


Exemple :

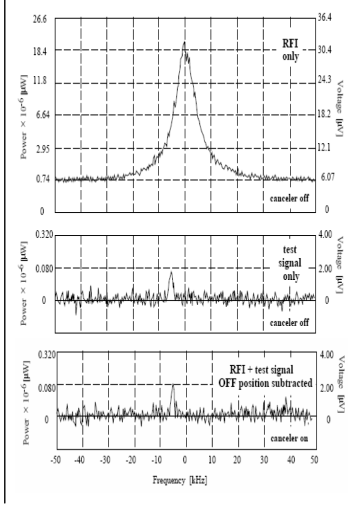


Ellingson S.W. , Bunton J. D., Bell J.F., "Removal of the GLONASS C/A signal from OH spectral line observations using a parametric modelling technique", *Astrophysical Journal Supplement*, **Vol. 135**, **No. 1**, Juillet 2001, 87-93.

Utilisation d'une antenne de référence

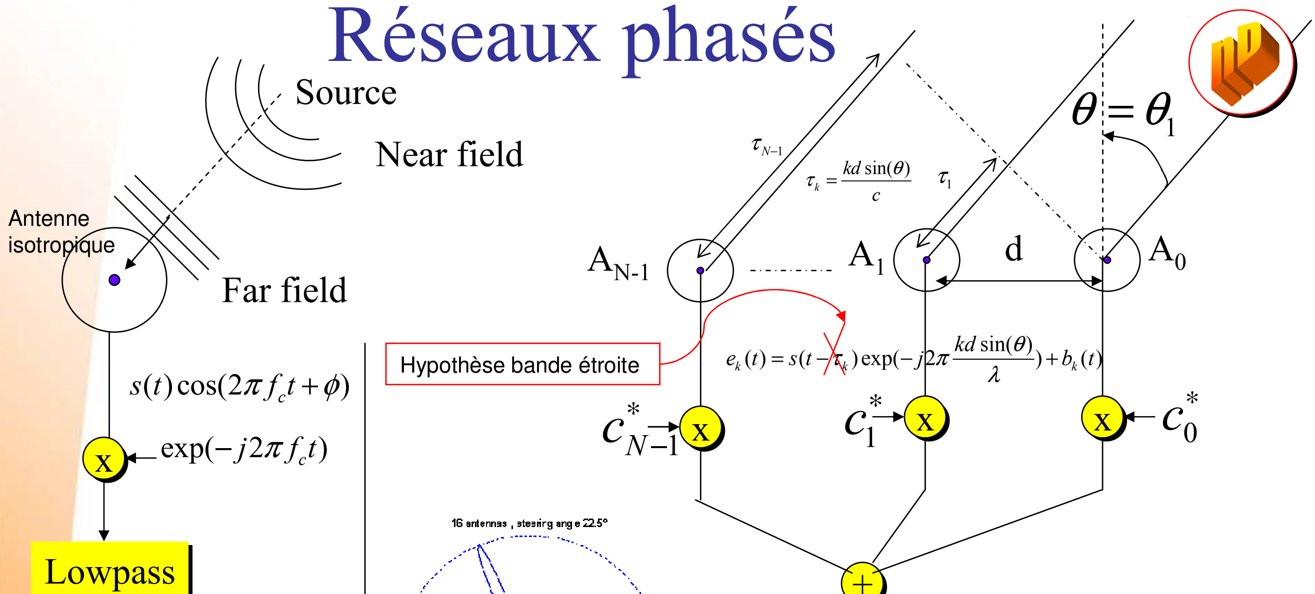


Exemple :



Barnbaum C., Bradley R.F., "A new approach to interference excision in radio astronomy: real-time adaptive cancellation", *The Astronomical Journal*, **115**, Novembre 1998, 2598-2614.

Réseaux phasés



$s(t)\cos(2\pi f_c t + \phi)$

$\exp(-j2\pi f_c t)$

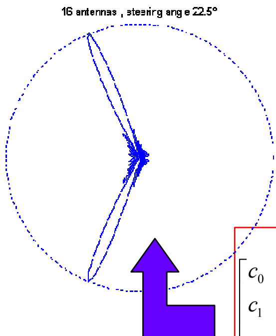
$b(t)$

$e(t) = s(t)\exp(j\phi) + b(t)$

$\tau_k = \frac{kd\sin(\theta)}{c}$

$e_k(t) = s(t - \cancel{\tau_k})\exp(-j2\pi\frac{kd\sin(\theta)}{\lambda}) + b_k(t)$

$$e(t) = \underbrace{\begin{bmatrix} c_0^* & c_1^* & & c_{N-1}^* \end{bmatrix}}_{C^H} . \underbrace{\begin{bmatrix} e_0(t) \\ e_1(t) \\ \\ e_{N-1}(t) \end{bmatrix}}_{E(t)} = C^H.E(t) = \sum_{k=0}^{N-1} c_k^* e_k(t)$$

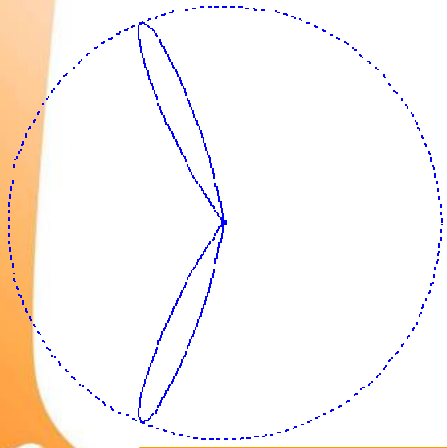


$$\boxed{\begin{bmatrix} c_0 \\ c_1 \\ \\ c_{N-1} \end{bmatrix} = \frac{1}{N}\begin{bmatrix} 1 \\ \exp(-j2\pi\frac{d\sin(\theta_1)}{\lambda}) \\ \\ \exp(-j2\pi(N-1)\frac{d\sin(\theta_1)}{\lambda}) \end{bmatrix}}$$

Beamforming et nulling

•Utilisation d'une fenêtre de pondération

$$\begin{bmatrix} c_0 \\ c_1 \\ \\ c_{N-1} \end{bmatrix} = \frac{1}{N} \underbrace{\begin{bmatrix} 1 \\ \exp(-j2\pi \frac{d\sin(\theta_1)}{\lambda}) \\ \\ \exp(-j2\pi (N-1)\frac{d\sin(\theta_1)}{\lambda}) \end{bmatrix}}_{A(\theta_1)} .* \begin{bmatrix} h_0 \\ h_1 \\ \\ h_{N-1} \end{bmatrix}$$



•Annulation dans les directions des RFI

1) Définir un critère d'optimisation :

Exemple:

$$e(t) = C^H E(t)$$

$$P = \left\langle \|y(t)\|^2 \right\rangle = C^H \underbrace{\left\langle E.E^H \right\rangle}_{} C$$

R matrice de cov.

Critère de Capon : $\min_C C^H RC$ avec $C^H A(\theta_1) = 1$

Solution : $C = \dfrac{R^{-1}A(\theta_1)}{A^H(\theta_1)R^{-1}A(\theta_1)}$

2) Méthodes haute résolution :

La diagonalisation de R permet de séparer l'espace signal de l'espace bruit

Conclusions

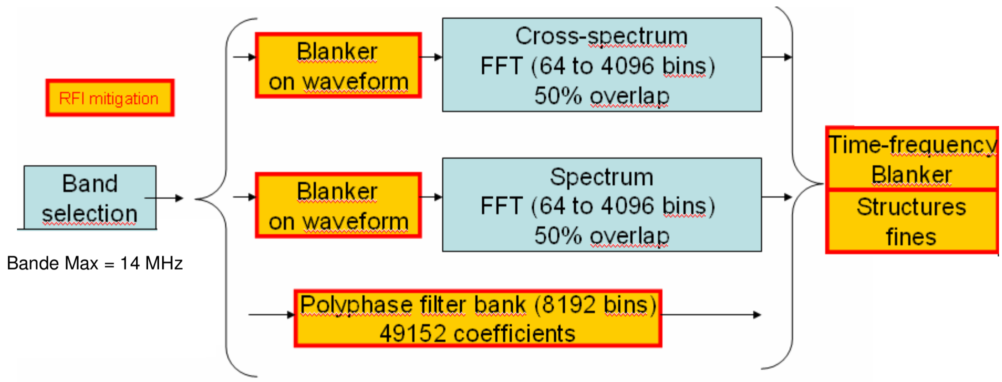
Beaucoup de solutions ou de pistes potentielles

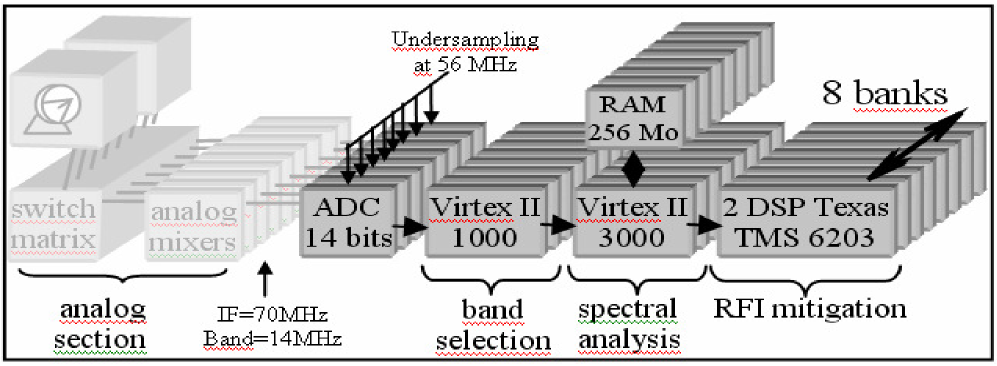
Mais

Nécessité de faire des choix stratégiques:
- Efficacité
- Innocuité
- Complexité
- Coût

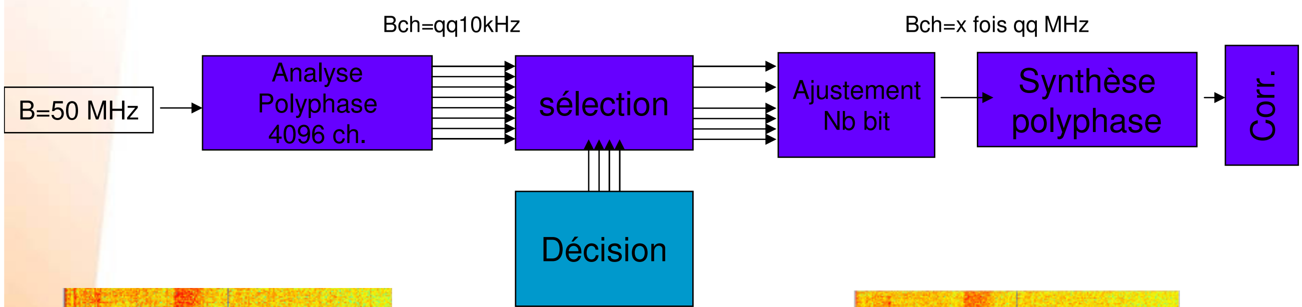
• Adéquation algorithme architecture
• Virgule fixe, Virgule Flottante ?
• Pré-intégré dans le Hardware ou reconfigurabilité (boîte à outils)?
• A quel niveau dans le radiotélescope ?
• Cluster PC ou Composants programmables ?
•…….

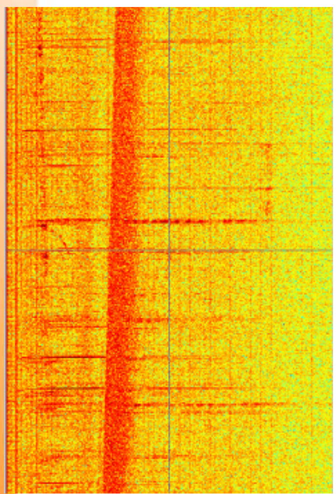
Exemples de stratégie (Nançay)

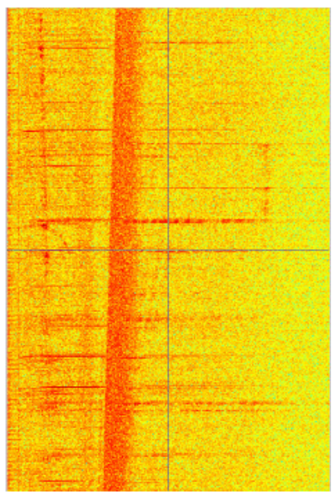




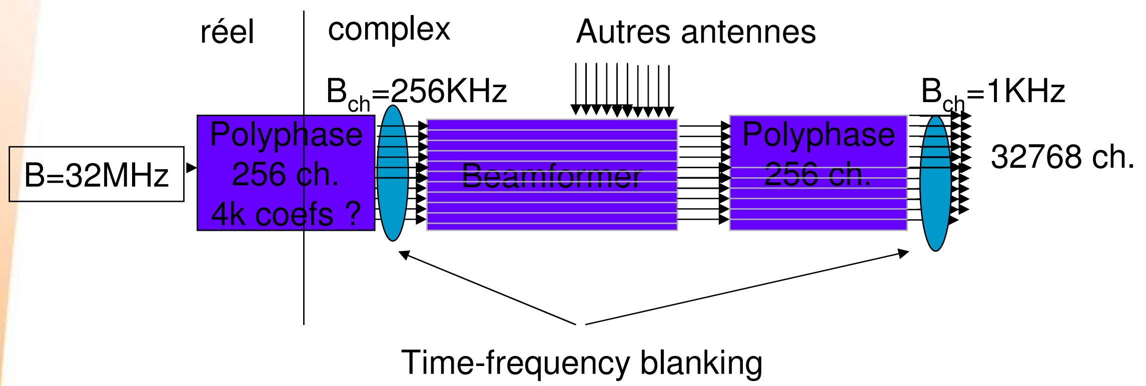
Exemples de stratégie (FASR)



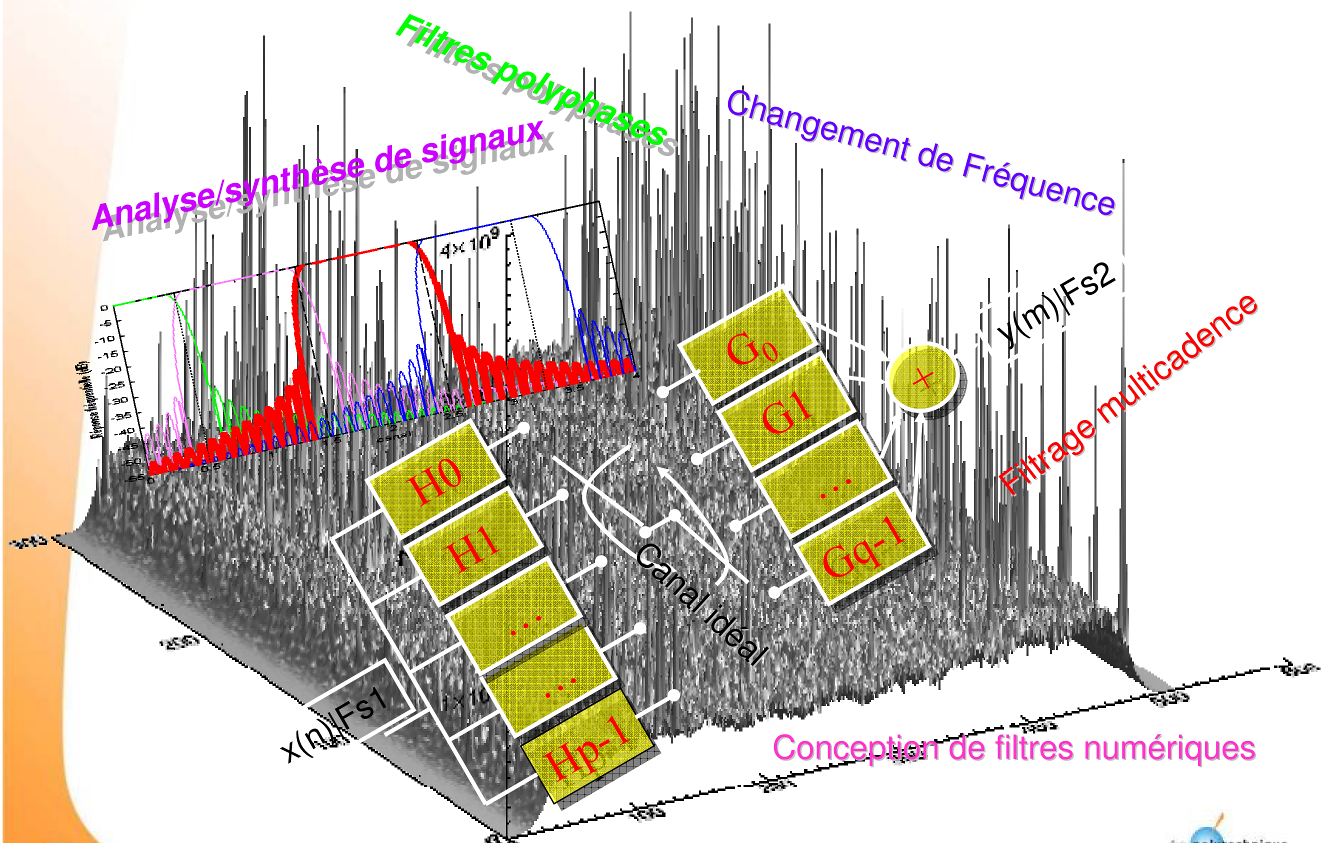




Exemples de stratégie (LOFAR)



Un point commun



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PART B: Filtrage multicadence

1. Description temporel et fréquentiel d'un signal numérique
2. Rappels sur les filtres numériques
3. Filtrage polyphase
4. Filtrage multicadence

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PART B.1: Description temporel et fréquentiel d'un signal numérique

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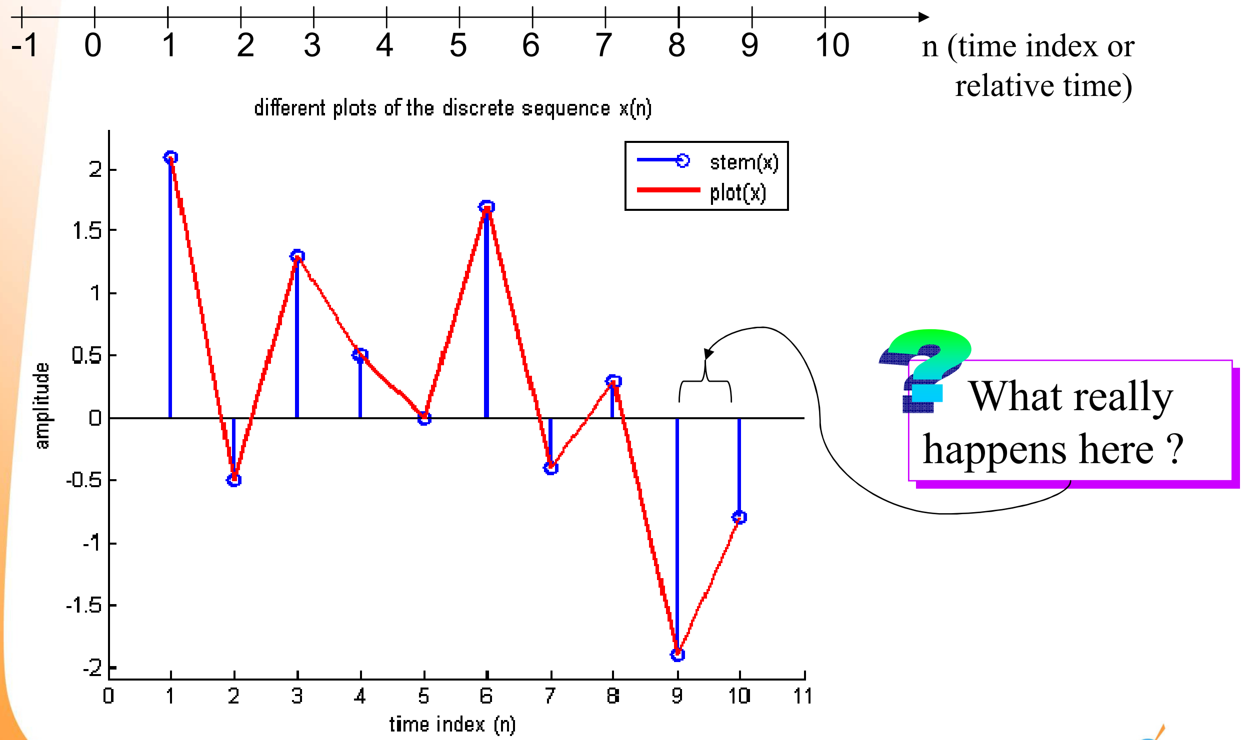
Temporal Description

x=[2.1 -0.5 1.3 0.5 0 1.7 -0.4 0.3 -1.9 -0.8];



Theoretical spectral description

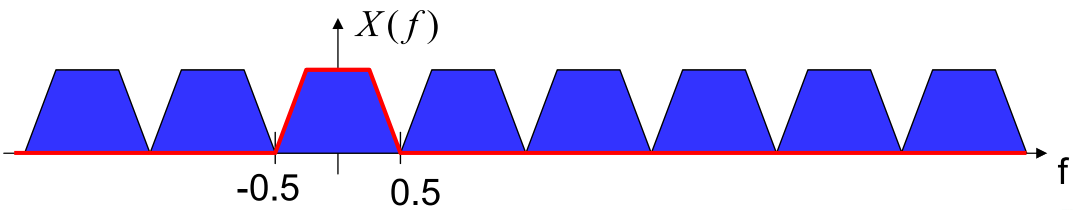
◆ **The answer :** the Fourier Transform $\Rightarrow$ $C_f = FT(x) = X(f)$

$$\boxed{FT(x) = X(f) = \sum_{n=-\infty}^{+\infty} x(n)e^{-2j\pi fn}, \quad f \in \mathbb{R}}$$

◆ **Remarks :**

$X(f+1)=X(f)$

- •periodicity of the spectrum
- •f can be limited to [0, 0.5] for real signal (spectrum symmetry)
- •f can be limited to [- 0.5 , 0.5] or [0 , 1] for complex signal



Basic spectral properties

◆ **Pure delay :**

$$y(n) = x(n-\tau) \qquad Y(f) = X(f).e^{-j2\pi f\tau}$$

◆ **Modulation or frequency shift:**

$$y(n) = x(n)e^{-2j\pi f_o n} = x(n).(\cos(2\pi f_o n) - j\sin(2\pi f_o n)) \qquad Y(f) = X(f+f_o)$$

Digital Down Conversion

◆ **Product and Convolution:**

$$y(n) = x(n)h(n) \qquad Y(f) = (X @ H)(f) \triangleq \int_0^1 X(\upsilon)H(f-\upsilon)d\upsilon$$

$$y(n) = (x @ h)(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k) \qquad Y(f) = X(f)H(f)$$

◆ **Parceval relation :**

$$\text{Energy} = \sum_{n=-\infty}^{+\infty} |x(n)|^2 = \int_0^1 |X(f)|^2 \, df$$

Practical spectral description (1)

◆ **Theoretical Formulation :**

$$TF(x) = X(f) = \sum_{n=-\infty}^{+\infty} x(n)e^{-2j\pi ft}, \quad f \in \mathbb{R}$$

No digital implementation possible

◆ **Approximations : The Discrete Fourier Transform**

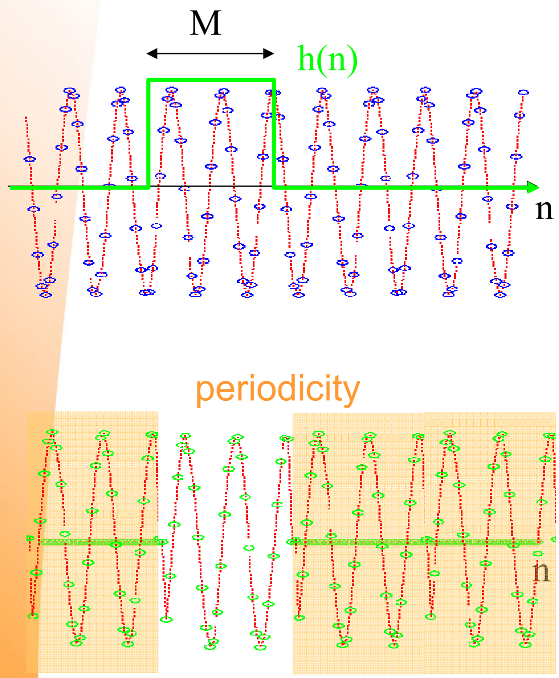
1. $n \in [0, M\text{-}1]$
2. Only f / $f=k/M$

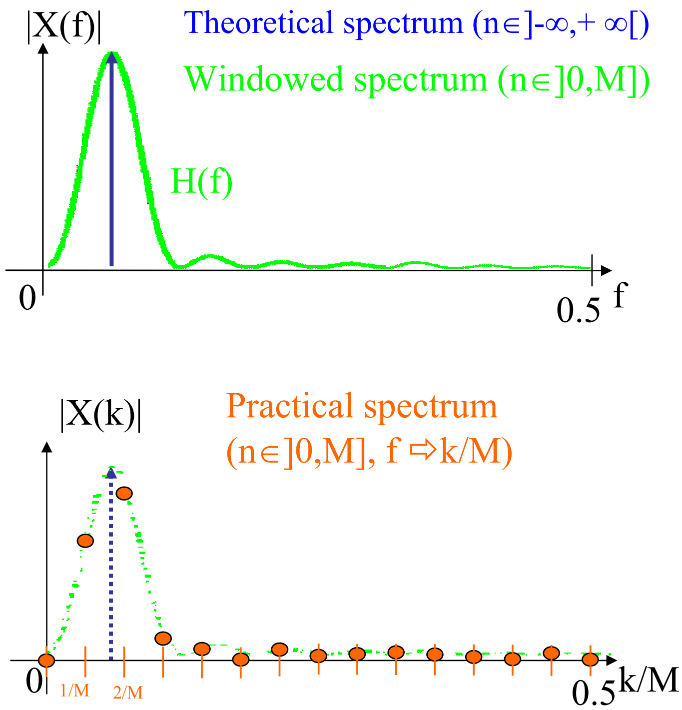
$$DFT(x) = X(k) \triangleq \sum_{n=0}^{M-1} x(n)e^{-2j\pi\frac{k}{M}n}, \quad k = 0,\cdots,M-1$$

X=fft(x);

$FFT(x) = DFT(x)$ with M a power of 2

Practical spectral description (2)





➡ Possibility to reduce distortion by using different kinds of windows, h(n)

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PART B.2: Rappels sur les filtres numériques

Linear Digital filtering

◆ **Formulations : temporal view** $x(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow y(n)$

b = [b_0 b_1 … b_N]; a = [1 a_1 … a_D]; y = filter(b,a,x);

$$\boxed{y(n) \triangleq \underbrace{\sum_{i=0}^{N} b_i x(n-i)}_{\text{non recursive part}} + \underbrace{\left(-\sum_{i=1}^{D} a_i y(n-i)\right)}_{\text{recursive part}}}$$

◆ **Basic properties :**

Linearity

if $x1(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow y1(n)$

if $x2(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow y2(n)$

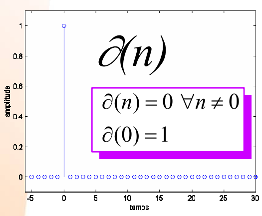
then $\alpha x1(n)+\beta x2(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow \alpha\, y1(n)+\beta y2(n)$

Temporal reproducibility

if $x(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow y(n)$ then $x(n\text{-}T) \longrightarrow \boxed{\text{Filter}} \longrightarrow y(n\text{-}T)$

Impulse response

◆ **Formulation :**

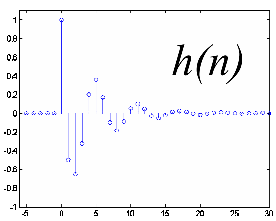


$\partial(n)$

$\partial(n)=0 \;\forall n \neq 0$
$\partial(0)=1$

$x(n)=\partial(n) \longrightarrow \boxed{\text{Filter}} \longrightarrow y(n)=h(n)$

The impulse response



$h(n)$

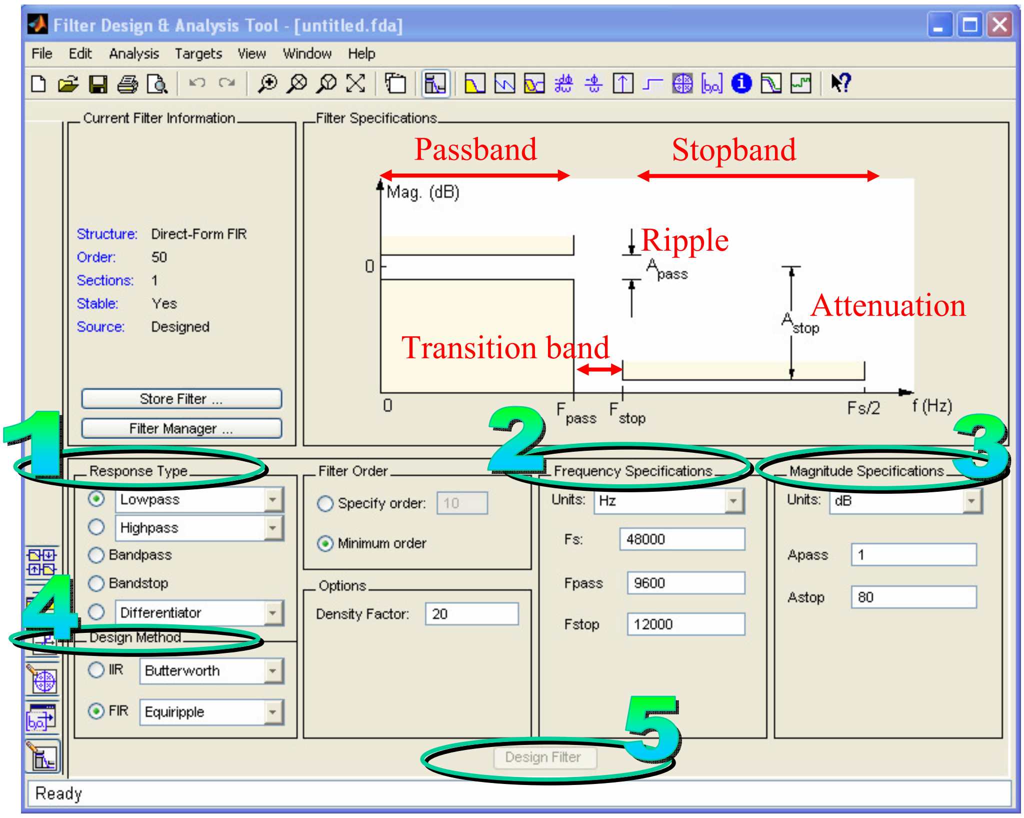
x = [1 zeros(1,29)]; h = filter(b,a,x);

◆ **Consequences :**

1 $y(n)=\sum_{i=0}^{N} b_i x(n-i)$ ⇒ Finite impulse response (FIR) $\begin{cases} h(k)=b_k & \text{for k=0,}\cdots\text{,N} \\ h(k)=0 & \text{else} \end{cases}$ always stable

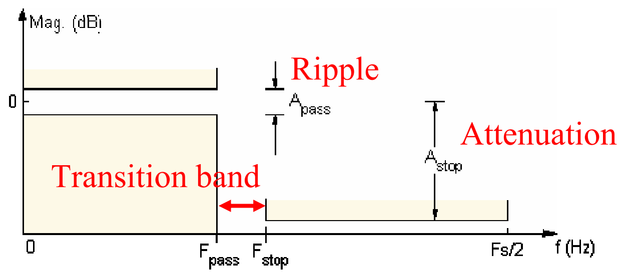
2 $x(n)=\sum_{k=-\infty}^{+\infty} x(k)\delta(n-k) \longrightarrow$ Filter with impulse response h(n) $\longrightarrow$ $\boxed{y(n)=\sum_{k=0}^{+\infty} h(k)x(n-k) \triangleq (h@x)(n)}$

Filter Specifications(1)



Filter Specifications (2)

- **General rules :**



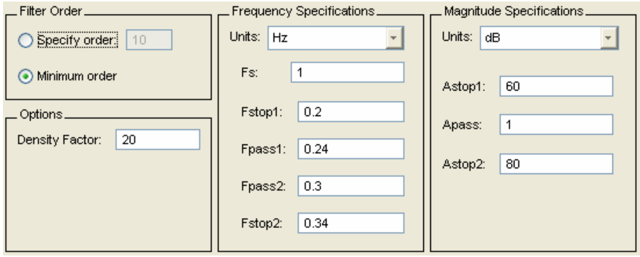
•If Attenuation or Ripple or Transition Band then Order or complexity

$$y(n)=\sum_{i=0}^{N} b_i x(n-i) - \sum_{i=1}^{D} a_i y(n-i)$$

• For a given specification, IIR is always less complex than FIR
But be careful to the stability (and the phase linearity) !

Basic Design Methods

Bandpass :

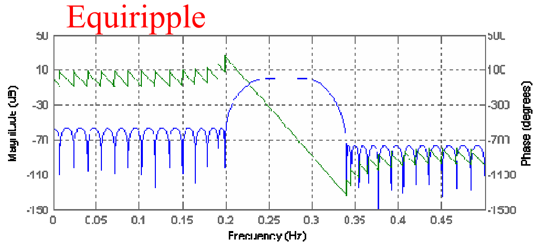


$|H(f)|^2_{dB}$

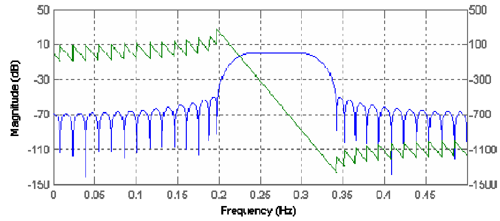
$phase_H(f)$

- **Basic FIR methods : ordre 63 ⇨ 64 coefficients, linear phase**

Equiripple

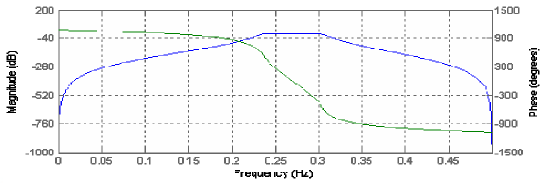


Least-square



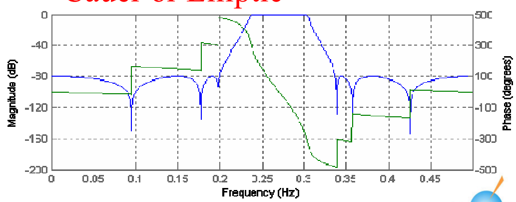
- **Basic IIR methods : order 24 ⇨ 50 coefficients**

Butterworth



order 12 ⇨ 26 coefficients

Cauer or Elliptic



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PART B.3: Filtrage Polyphase

The Z-Transform

◆ **Formulation :** $$\boxed{F(z) = ZT(f)(z) \triangleq \sum_{n=-\infty}^{+\infty} f(n) z^{-n} \;, z \in \mathbb{C}}$$

◆ **useful (and simple) properties:**

Given $X(z)=ZT(x)$ and $Y(z)=ZT(y)$

- linearity : $ZT(\alpha x(n) + \beta y(n)) = \alpha X(z) + \beta Y(z)$
- **convolution : $ZT[(x@y)(n)] = X(z)\, Y(z)$** ⟸ These make the ZT very interesting !
- $ZT(x(n+1)) = z\, X(z)$; $ZT(x(n+k)) = z^k X(z)$
- **$ZT(x(n-1)) = z^{-1} X(z)$; $ZT(x(n-k)) = z^{-k} X(z)$**
- **$ZT(x)$ for $z=\exp(j2\pi f) = TF(x)$**

◆ **Application to the filtering:**

$$(h@x)(n) = y(n) = \sum_{k=0}^{M-1} h(k)x(n-k) \quad \Rightarrow \quad ZT(h(n)) = \boxed{H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M-1} h(k) z^{-k}}$$

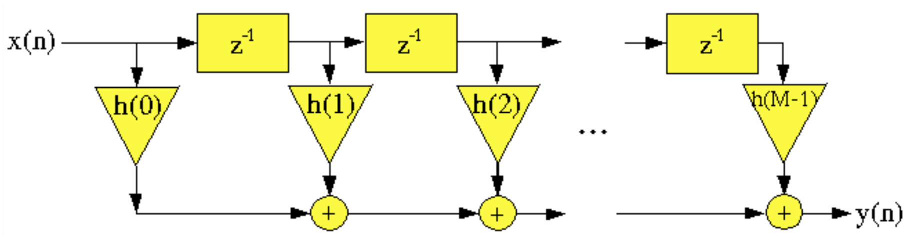
ZT

ZT

Filter structures

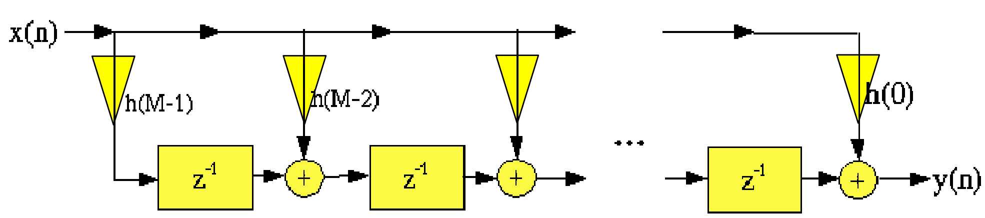
$$Y(z) = \sum_{k=0}^{M-1} h(k).(z^{-k} X(z))$$

Direct



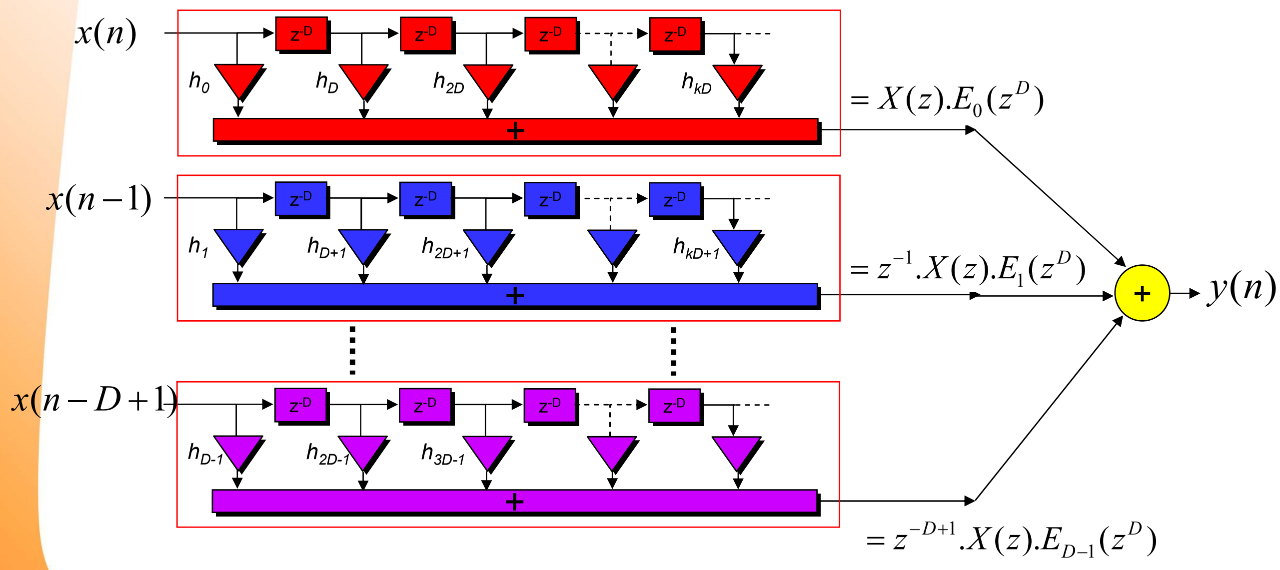
$$Y(z) = \sum_{k=0}^{M-1} (h(k).X(z)) z^{-k}$$

Transpose



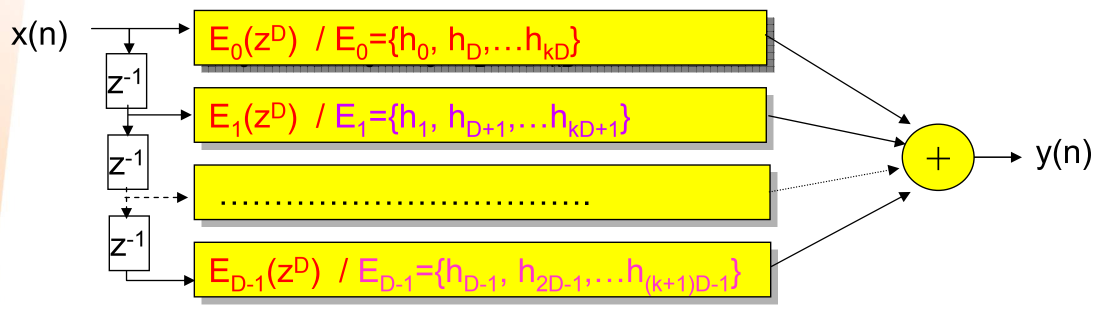
Polyphase structure (1)

$$y(n)=\sum_{k=0}^{N.D} h_k x(n-k)=\begin{cases} h_0x[n]+h_Dx[n-D]+h_{2D}x[n-2D]+\cdots+h_{kD}x[n-kD]\ldots \\ +h_1x[n-1]+h_{D+1}x[n-D-1]+h_{2D+1}x[n-2D-1]+\cdots+h_{kD+1}x[n-kD-1]\ldots \\ \vdots \\ +h_{D-1}x[n-D+1)]+h_{2D-1}x[n-2D+1)]+h_{3D-1}x[n-3D+1)]+\cdots+h_{(k+1)D-1}x[n-(k+1)D+1)]\ldots \end{cases}$$

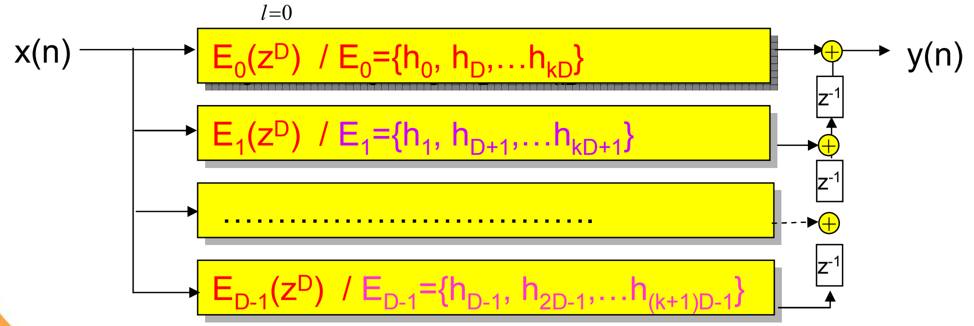


Polyphase structure (2)

$$Y(z)=H(z).X(z)=\sum_{l=0}^{D-1}(z^{-l}X(z))E_l(z^D)$$

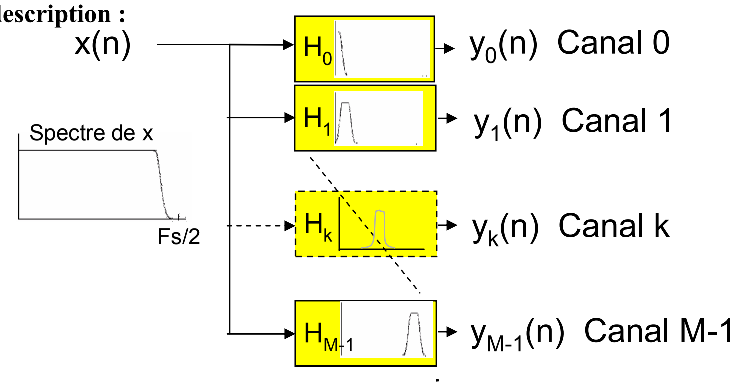


$$Y(z)=H(z).X(z)=\sum_{l=0}^{D-1}(X(z).E_l(z^D)).z^{-l}$$



Filter Bank

- **General description :**



Spectre de M canaux
de largeur théorique Fs/M

- **The idea :**

$$H_k(f) = H_0(f - \frac{k}{M})$$

$$\Rightarrow \quad H_k(z) = H_0(z.e^{-j2\pi\frac{k}{M}})$$

Polyphase filter bank (1)

1) $H_0(z) = \sum_{l=0}^{M-1} z^{-l}.E_l(z^M)$ Polyphase structure

2) $H_k(z) = H_0(z.e^{-j2\pi\frac{k}{M}})$ Frequency translation

$$\Rightarrow \quad H_k(z) = \sum_{l=0}^{M-1}\left[ze^{-j2\pi\frac{k}{M}}\right]^{-l}.E_l(\left(ze^{-j2\pi\frac{k}{M}}\right)^M)$$

$$H_k(z) = \sum_{l=0}^{M-1} z^{-l}.E_l(z^M)e^{j2\pi\frac{k.l}{M}}$$

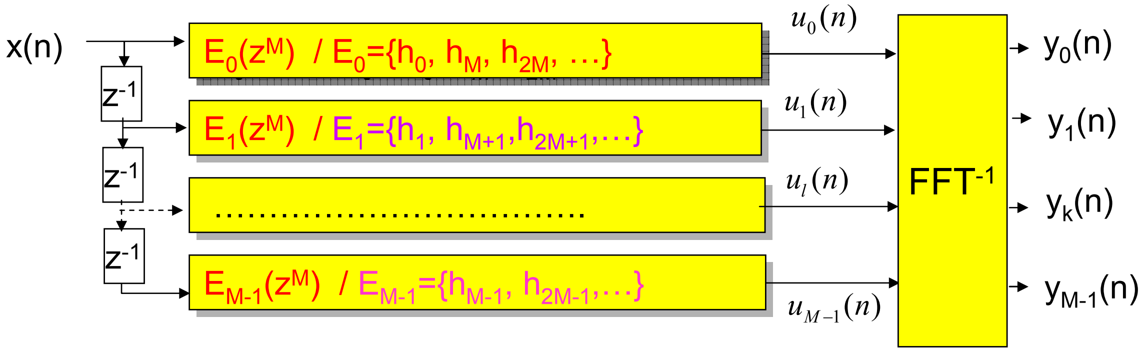
$$\Rightarrow \quad Y_k(z) = X(z).H_k(z) = \sum_{l=0}^{M-1} \underbrace{z^{-l}.X(z).E_l(z^M)}_{U_l(z)} e^{j2\pi\frac{k.l}{M}}$$

$$Y_k(z) = \sum_{l=0}^{M-1} U_l(z)e^{j2\pi\frac{k.l}{M}} = FT^{-1}(U_{l=0,\dots,M-1}(z)) \text{ at frequency } \frac{k}{M}$$

Polyphase filter bank

1) $U_l(z) = z^{-l}.X(z).E_l(z^M)$ for $l = 0,\dots,M-1$

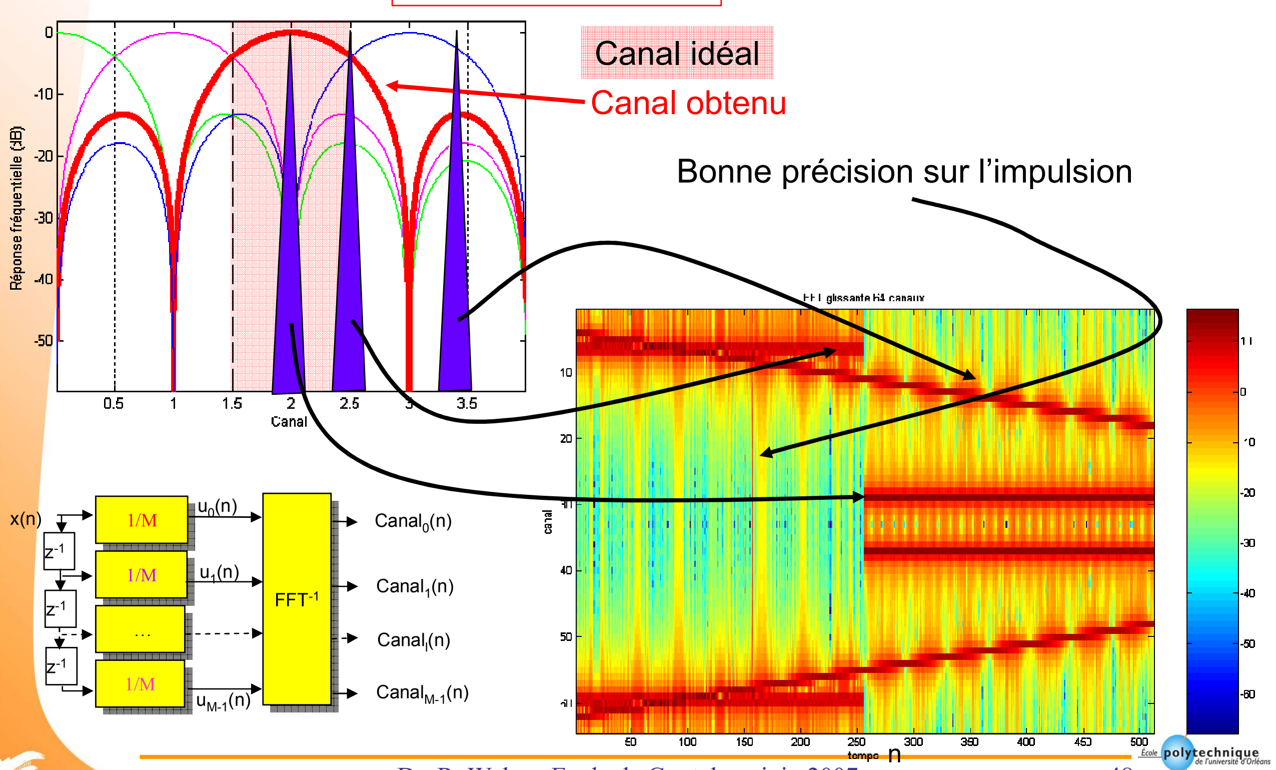
2) $Y_k(z) = FT^{-1}(U_{l=0,\dots,M-1}(z))$ at frequency $\frac{k}{M}$



Exemple 1

M=64, **H=fenêtre rectangulaire de taille M** ➡ $E_l=\{1/M\}$, $l=0,\dots,M-1$

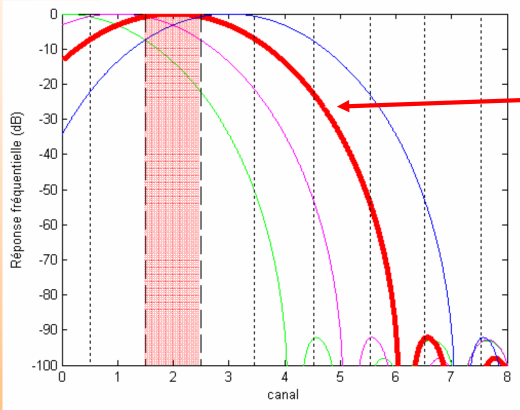
Banc de filtre = FFT



Exemple 2

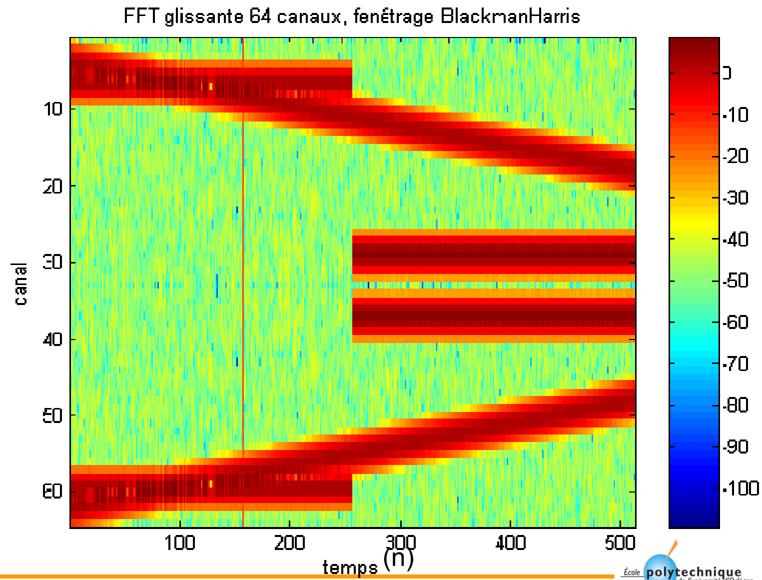
M=64, **H=fenêtre Blackmanharris de taille M** ➡ H_l={hi}, l=0,..,M-1

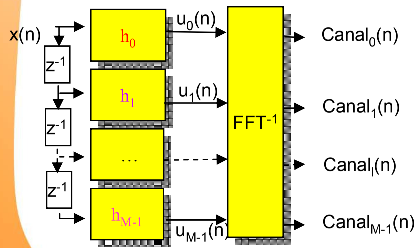
Banc de filtre = FFT fenêtrée par blocs sur M points



Canal idéal

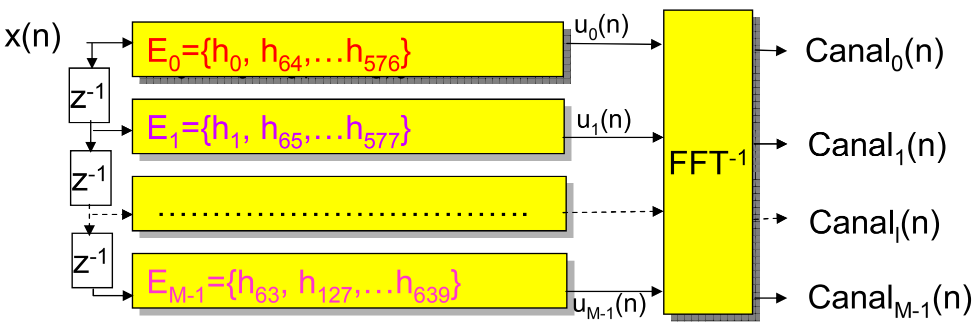
Canal obtenu

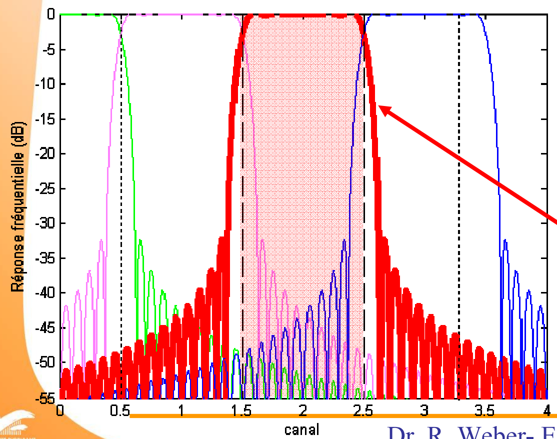




Exemple 3

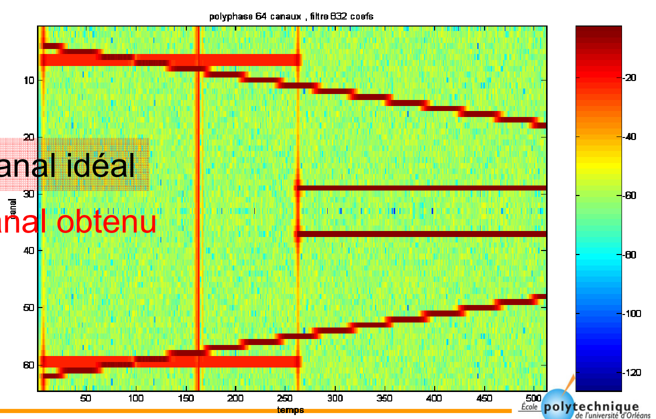
M=64, **H=filtre de 640 coefficients**





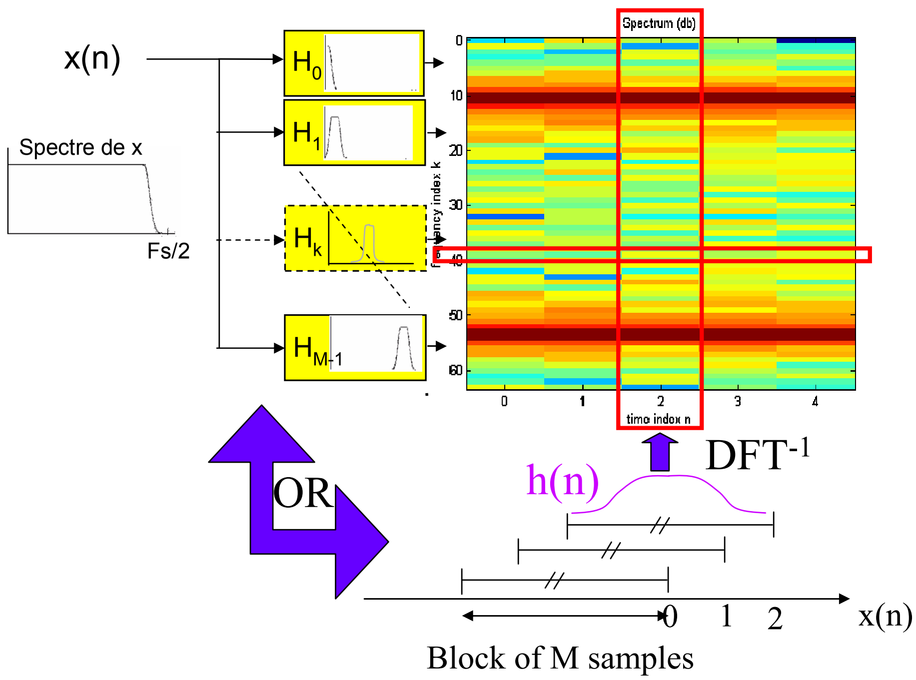
Canal idéal

Canal obtenu



Conclusion

◆ **2 formulations of the same spectral analysis tool :**



What happens if I keep only 1 time index every M ones ? ➡ Multirate System

PART B.4: Filtrage multicadence

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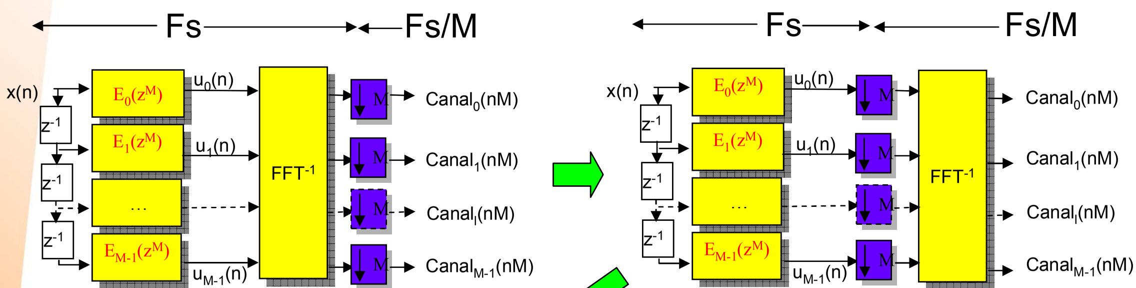
Mise en œuvre

Décimation par M : $s(n) \longrightarrow \boxed{\downarrow M} \longrightarrow s(nM)$

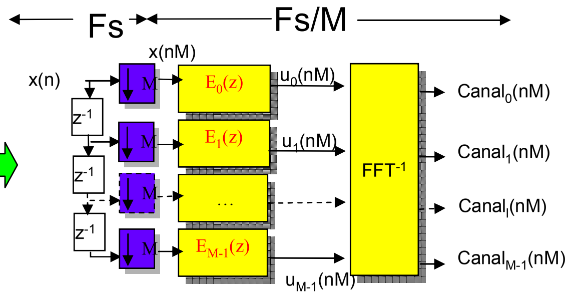


Fs : sampling frequency

$$u_0(n)=\sum_{p=0}^{P-1} h_{pM}\,x(n-pM)$$

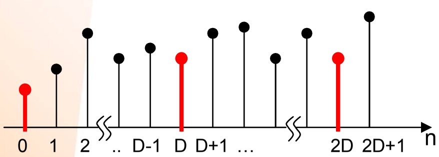
$$u_0(nM)=\sum_{p=0}^{P-1} h_{pM}\,x(nM-pM)$$

$$=\sum_{p=0}^{P-1} h_{pM}\,x((n-p)M)$$

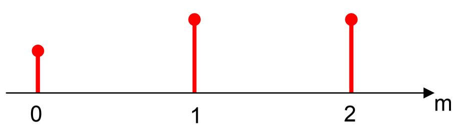


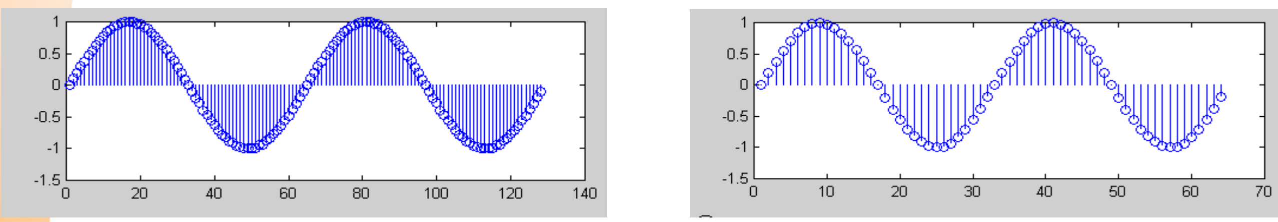
Conséquences d'une décimation

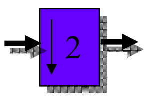
Aspect temporel



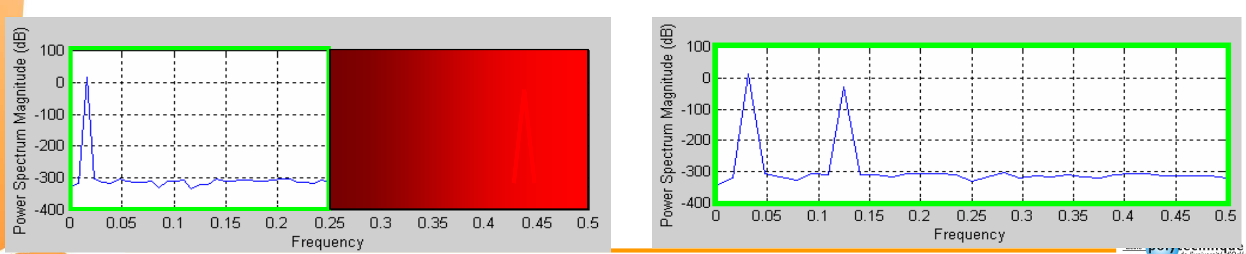
$x(n) \longrightarrow \boxed{\downarrow D} \longrightarrow x_D(m)$







Aspect fréquentiel



Vue théorique du décimateur

$$x(n) \rightarrow \boxed{\uparrow D} \rightarrow x_D(n) \xrightarrow{TZ} X_D(z)=\sum x_D(n)z^{-n}=\sum x(nD)z^{-n}=\sum x(n).\coprod_D(n)z^{-n}$$

$x(n) \xrightarrow{TF} \bar{x}(f)$; $x_D(n) \xrightarrow{TF} \bar{x}_D(f)$; $z=\ell^{j2\pi f}$

or $\coprod_D(n)=\frac{1}{D}\sum_{k=0}^{D-1}\ell^{j2\pi\frac{k}{D}n}$

$$X_D(z)=\frac{1}{D}\sum_{k=0}^{D-1}\sum_n x(n).\ell^{j2\pi\frac{k}{D}n}z^{-n}=\frac{1}{D}\sum_{k=0}^{D-1}\sum_n x(n).\left[z\ell^{-j2\pi\frac{k}{D}}\right]^{-n}=\frac{1}{D}\sum_{k=0}^{D-1}X\left(z\ell^{-j2\pi\frac{k}{D}}\right)$$

Avec X=Tz(x)

D'où

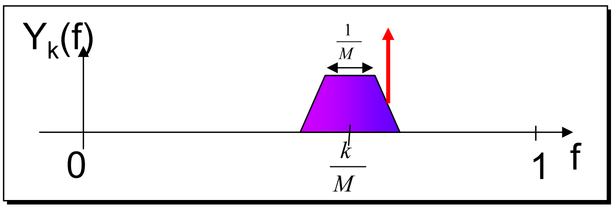
$$\boxed{\bar{x}_D(f)=\frac{1}{D}\sum_{k=0}^{D-1}X\left(\ell^{j2\pi(f-\frac{k}{D})}\right)=\frac{1}{D}\sum_{k=0}^{D-1}\bar{x}(f-\frac{k}{D})}$$

Application au banc de filtres

$$x(n) \rightarrow \boxed{H_k} \rightarrow y_k(n) \rightarrow \boxed{\downarrow D} \rightarrow y_k(Dn)$$

k=0,…,M-1

TF



D=M : Décimation critique

TF(yk(Dn))

D=M/2 : Overlap 50%