Turbulent convection: comparison of Reynolds stress models with numerical simulations

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ABSTRACT

Numerical simulations of turbulent convection have become a common tool in stellar astrophysics, but their application is limited to two cases: the study of basic properties of compressible convection, and stellar atmospheres. Fully nonlocal convection models based on the Reynolds stress approach have come into use only more recently, but their applicability extends to a much wider range of astrophysical problems. Here, we review the origin of the most important limitations of both methodologies. Despite their very different computational costs and the concomitantly different range of applications, they have occasionally been regarded as competitors. In reality, they complement each other, because for most problems in stellar astrophysics numerical simulations are still unaffordable while for problems accessible to both methodologies simulations can be used to gain insight into the reliability of statistical assumptions made in the Reynolds stress approach. We describe some of the tests which we have performed to investigate the qualitative and quantitative capabilities of this new generation of fully nonlocal convection models. One of these models has already been applied to convection in the envelope of A-type main sequence stars.

1. Introduction

Convective instability is an ubiquitous phenomenon in stars. High opacity and/or high luminosity can cause a temperature gradient to become steep enough to create convective instability. Either of them occurs in most stars in the HR diagram. The primary effect of a convective instability is a nonradiative heat transfer which essentially decreases the temperature gradient. As a consequence, convective heat transfer affects stellar structure and stellar evolution. For stars with a convective photosphere, e.g. the Sun, it is possible to observe the inhomogeneous spatial structure of a turbulent convective flow directly through the emitted radiation (line profiles, etc.). Naturally, convection zones are well mixed. Unless a convective flow is inhibited by some boundary, it extends beyond regions of unstable stratification (i.e. overshooting). One possible consequence thereof are increased stellar life times in case of convective stellar cores. As convection maintains a fluid flow, it is coupled to other processes involving velocity fields: driving, damping, and modulation of stellar pulsation, the transport of angular momentum, differential rotation, and the solar dynamo are well-known examples. This explains the physical importance of stellar convection and the ongoing interest in its better qualitative and quantitative understanding.

However, the goal of a better understanding of stellar convection has proven extremely difficult since the very beginning of astrophysics. Convection is a highly nonlinear process which manifests itself in a highly turbulent flow. The inherent complexity has inhibited major progress with heuristic methods, like the classical mixing length theory (MLT). On the other hand, any approach to stellar turbulent convection derived from the governing Navier-Stokes equations (NSE) faces the challenge of adequately treating the nonlinear interactions by approximations to the problem, as a complete analytical treatment has remained impossible.

Here, we compare the two most powerful methods currently available for describing stellar convection: numerical simulations and the fully nonlocal convection models based on the Reynolds stress approach (Canuto 1992, Canuto 1993, Canuto and Dubovikov 1998, and Canuto 1997). We begin with a discussion of the problem of widely varying spatial and temporal scales which appear in turbulent convection and continue with the different averaging concepts that have been developed to tackle this problem. It is possible to investigate the approximations made in the Reynolds stress approach using numerical simulations of turbulent convection for which all relevant spatial and temporal scales can be resolved. Recently, such comparison work has been done for a physical scenario relevant to stars (Kupka 1999a, Kupka 1999b, Kupka 2001, Kupka and Muthsam 2001). We give some examples to explain the underlying ideas and procedures of such a comparison. As the results were quite satisfactory, the code developed for this purpose has now been applied to convection zones in envelopes of A-type main sequence stars (Kupka and Montgomery 2001).

2. The scale problem. Statistical approach.

Stellar convection is characterized by high Reynolds numbers $\mathrm{Re} = UL/\nu$ (U is a mean velocity, L the largest length scale, and ν the kinematic viscosity). As a dimensionless measure of inertial to viscous forces, Re is also a measure for the strength of the nonlinear (advection) terms in the NSE compared to the linear (viscous) terms. Very large Re numbers are found in many astrophysical and geophysical problems (Table 1). The huge ratio of the largest length L of these problems to the length scale l_{d}, where viscous forces become dominant, entails an enormous amount of grid points, if these length scales are resolved in a numerical simulation. We note that the values quoted for the planetary boundary layers correspond to pure convection which is rarely found in the earth's atmosphere (e.g. day time in a desert when there is no wind). Other instability mechanisms create still larger structures (e.g. tropical storms). However, while in the earth's atmosphere most of the turbulent kinetic energy resides close to the largest scales, the same is not the case for the ocean. There, the largest scales, called "gyres", characterize important global properties of the system such as pole-equator heat exchange. But the bulk of turbulent kinetic energy resides in the "mesoscales" (10 to 100 km) and convective cells are even much smaller than that (few km and less). Thus, the bulk of kinetic energy cannot be resolved in low resolution numerical ocean simulations which have to rely heavily on mesoscale modeling. This is analogous to the solar case, where it is not possible to resolve solar differential rotation and solar surface granulation simultaneously with computational resources of the present and the near future. One feature which distinguishes stellar convection from its geophysical counterparts is the importance of radiative transfer,[1] characterized by a very low Prandtl number $\mathrm{Pr} = \nu/\chi$ (Pr compares viscosity to thermometric conductivity). Consequently, stellar convection is highly turbulent even for inefficient convection, i.e. in a situation where it would already be laminar in a corresponding geophysical flow (neglecting shear instabilities). For further discussion on the spreading of length scales as well as for data and references on geophysical flows we refer to Bryan (1991) and to Canuto (1994).

bulk parameters	solar convection zone	earth's atmosphere (PBL)	earth's oceans
L	$\sim 180\,000$ km	~ 1 km	up to few $1\,000$ km
l_{d}	$\sim 1 \ldots 10$ cm	~ 1 mm	~ 1 mm
Re	$10^{10} \ldots 10^{14}$	$\sim 10^{8}$	$\sim 10^{12}$
Pr	$10^{-6} \ldots 10^{-10}$	~ 0.7	~ 6

Table 1: A comparison of bulk parameters of convection zones of the Sun, the earth's atmosphere (convective planetary boundary layer), and the earth's oceans (turbulent flow sustained by several different instability mechanisms). The largest size L found in the unstable region, the smallest length scale of the flow as estimated from the dissipation scale l_{d}, and Reynolds as well as Prandtl numbers are given.

The ratio of time scales reveals a similar spreading. For the Sun, the relevant scales range from a few seconds (typical for radiative transport through the photosphere) to several minutes (solar oscillations) to the Kelvin-Helmholtz time scale for the outer half of the solar envelope, i.e. about 10^6 years. This is comparable to the oceans of the earth for which time scales range from 0.1 seconds to half a century or even 10^5 years.[2] Using relations between L, l_{d}, and Re (see Canuto 2000) it follows that 10^{20} to 10^{27} grid points are required to resolve all the relevant spatial scales in the Sun. With time steps of a few seconds which are typical of simulations of solar granulation one would require about 10^{12} steps to achieve thermal relaxation, in agreement with the difference in time scales. The dominance of the nonlinear interactions, which is mainly responsible for this large spreading of scales, also prohibits simple remedies such as taking a coarser grid and larger time steps, i.e. to ignore the small length scales and the short time scales. Energy would rapidly pile up at the scale of the grid spacing and blow up the calculations. Recalling that within a stellar structure model convection is just part of a larger problem, it is obvious that direct numerical simulation (DNS) of stellar (and geophysical) convection is totally outside our present computational capabilities.

However, neither all the length scales nor all the time scales are of astrophysical interest. Even for the Sun the horizontal spatial resolution of observations is only a few 100 km, and the vast majority of stellar observations are averages over the entire disk. In this sense, the available observations are already statistical averages. And for the majority of applications the successful prediction of basic statistical properties such as mean values and skewness would already be a major achievement. Applying a statistical method to the

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[1] The other important features are first the huge density and pressure contrasts due to gravitational stratification, which give rise to non-Boussinesq (compressibility) effects. And second, the transition from an optically thick to an optically thin fluid in convection zones that reach into stellar atmospheres which leads to the observed strong inhomogeneity.

[2] The last two values refer to the coupling of the Pacific with the Atlantic and with the Arctic Ocean.

NSE based on some form of averaging is thus the logical approach for investigating stellar convection.

Two types of averaging are currently in use: one of them involves volume averages of the NSE. The idea behind this approach is to compute only the most important length scales, i.e. those which carry the bulk of the turbulent kinetic and potential energy. Numerical simulations using this idea are known as large eddy simulations (LES). In their most common form (e.g. Xie and Toomre 1993, Chan and Sofia 1996, Stein and Nordlund 1998, cf. Canuto 1997 and Kupka 1999b for further references) they resolve the up- and downdrafts which transport most of the heat energy and which are found in all convective flows. The small scale fields are modeled by some (physically or numerically derived) enhanced viscosity. The second approach is to consider an ensemble average of the NSE. The goal of this method is to compute low order moments of the (fluctuating) temperature and velocity fields from equations directly derived from the NSE using additional assumptions on statistical properties of the flow. The ensemble average is the basis for the fully nonlocal Reynolds stress-type convection models we discuss below.

If we consider each numerical solution obtained for a time t_k as a particular realization of a solution of the NSE, we can compare the results of both methodologies. An ensemble can be constructed from the simulations by averaging over many t_k. For an LES, the assumptions about the unresolved scales essentially introduce an additional statistical uncertainty. Otherwise, the interpretation of the results remains unaltered, because any numerical simulation has a finite resolution, and the numerical solution on a grid point can be considered as an average over its surrounding volume.

Contrary to LES, the ensemble average approach can be used to compute the physical variables of interest as a function of only the vertical coordinate (and time). In addition to vertical resolution, the computational costs of convection models based on this type of averaging depend on whether they entail algebraic or differential equations. On a modern workstation, the local models may be solved in a few seconds for the entire convective envelope of a star. Non-local models typically require a few minutes to a few hours (cf. Kupka and Montgomery 2001). On the other hand, the volume average based 3D LES require between a few weeks and a few months assuming a vertical resolution common in stellar atmosphere and structure calculations provided that the Kelvin-Helmholtz time scale does not exceed a couple of days. Some savings are possible by reducing the resolution, but at the disadvantage of no longer numerically resolving the temperature gradient accurately enough. This limits hydrodynamical simulations to stellar atmospheres and to very shallow envelope convection zones,[3] as measured in terms of total stellar mass. LES in 2D saves some time, but the physics of the problem is different (Canuto 2000, Canuto 2001) and so are the quantitative results such as the extent of the overshooting region (cf. Kupka and Muthsam 2001, Kupka and Montgomery 2001). Thus, for the majority of the problems in stellar astrophysics, convection models based on an ensemble average are the only feasible approach.

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[3]Occasionally, simulations have been performed which use artificially increased radiative conductivity or are based on an incomplete thermal relaxation. But the former do not include the real microphysics of stars which puts them into the realm of idealized studies of compressible convection while nonrelaxed simulations may be arbitrarily off from the actual, thermally relaxed flow state.

3. The Reynolds stress approach

The idea behind the Reynolds stress approach is to describe the flow by low order moments of the basic physical variables of the NSE and their thermodynamical counterparts (such as temperature). The moments are obtained from an ensemble average and are given as functions of space and time and involve the gradients of the mean (average) variables. To derive the governing equations, fields $A(t, x, y, z)$ are split into their average and their fluctuating parts: $A = \overline{A} + A'$. An equation for $\overline{A}$ is then obtained by applying the average to the equation for A, while an equation for A' is obtained by subtracting the equation for $\overline{A}$ from the original equation. However, because of the nonlinear terms in the NSE, the equations e.g. for the averaged velocity field $v_i(t, x, y, z)$ depend on products of the general form $\overline{v_i' v_j'}$. An equation for these second order moments may be derived using the equations for v_i' (Canuto 1992), but the new equations depend on moments of even higher order. Continuing this procedure, one ultimately ends up with an infinite hierarchy of moments. To obtain a finite set of moments given by a corresponding number of equations requires to express the higher order moments in terms of lower order ones, which is the well-known closure problem of turbulence theory.

The most advanced Reynolds stress models close the hierarchy at fourth order (Canuto 1992, Canuto 1993, and Canuto 1997)[4] by relating them to second order moments. Hence, they are also known as second order closure models. Each of the moments of lower order is either provided by a dynamical equation or expressed in terms of other moments by algebraic relations. The moments have a well-defined physical meaning. Averages of the basic variables such as the temperature $\overline{T}$ describe the mean structure of the star. The second order moments describe how convection modifies this mean structure by transporting part of the energy. A minimum nonlocal convection model consists of the turbulent kinetic energy K, the mean square of temperature fluctuations $\overline{\theta^2}$ (thermal potential energy), $\overline{w\theta}$ (where $F_c = c_p \rho \overline{w\theta}$ is the convective flux), the turbulent kinetic energy of the vertical velocity components $\overline{w^2}$ (where $p_t = \rho \overline{w^2}$ is the turbulent pressure), and the dissipation of turbulent kinetic energy ϵ. The third order moments characterize the anisotropy and the inhomogeneity of the transport process. This includes nonlocal transport of second order moments, their skewness (which can be related to the filling factor, the relative horizontal area of upward-moving fluid), and various pressure correlations. Thus, each quantity describes an important property of the flow, which at least for the case of the earth, in a laboratory, or in evaluations of numerical simulations, can also be measured directly.

In Canuto and Dubovikov (1998), the (one-point) Reynolds stress model was rederived from a general turbulence model. Using renormalization group techniques and a few physical assumptions,[5] the latter provides a closed system of equations for the spectra of the second order moments in case of homogeneous turbulent flows. Reynolds stresses can be computed from integration over all wavenumbers. The homogeneous part of the Reynolds stress-type convection model of Canuto (1992) can be obtained by performing an approximate analytical integration. The resulting equations have to be supplemented by a

[4] See also Grossman (1996) and Xion (1997) for models which, however, employ a mixing length for the dissipation of turbulent kinetic energy.

[5] Basically, they are locality of energy transfer in wavenumber space and small anisotropy of turbulent forces in comparison with the anisotropy of the external forces.

model for the third oder moments for nonlocal transport and the dissipation of turbulent kinetic energy to obtain a complete, fully nonlocal convection model. The procedure employed by Canuto and Dubovikov (1998) has the advantage to provide values for some of the closure constants of the (Canuto 1992) Reynolds stress model and extends the latter to include the effects of radiative losses on some of the moments. But instead of invoking heuristic arguments for each specific term which has to be closed in the standard approach, they use a general turbulence model that has successfully passed some 100 tests for very different turbulent flows. This provides a more reliable basis on which a convection model can be built that also includes inhomogeneity, compressibility, and other effects.

4. A modern convection model and its comparison with numerical simulations

We have studied the Reynolds stress convection model of Canuto and Dubovikov (1998), extended by earlier results of Canuto (1992) and Canuto (1993), in detail. To describe the mean structure, this model employs differential equations for the mean pressure and for the mean flux and temperature (Canuto 1993 and Canuto and Dubovikov 1998, see equations (7)–(8) of Kupka 1999b). Together with the equation for the mass one obtains a fourth order system of coupled differential equations. The effect of convection on the mean structure is described by five more coupled and nonlinear differential equations, each of second order in space. They give the second order moments K, $\overline{\theta^2}$, $\overline{w\theta}$, $\overline{w^2}$, and ϵ (see the summarizing equations (1)–(5) in Kupka 1999b). For inefficient convection and at $\mathrm{Pr} \ll 1$, the effect of heat losses on damping terms in the equations for $\overline{\theta^2}$ and $\overline{w\theta}$ can be included as in Canuto and Dubovikov (1998). Compressibility as well as anisotropy effects on pressure correlations can also be included (Canuto 1993). The coupled, fourteenth order system of equations describes the change of $\overline{T}$ and the second order moments as a function of time, e.g. $\partial K/\partial t$. Without a mean flow and without any external forcing other than a constant input flux at the bottom, the equations always attain a stationary solution after evolving for about a Kelvin-Helmholtz time scale from any reasonable initial solution.

A variety of models for the third order moments (TOMs), which yield the fluxes of the second order moments (such as $\overline{w\theta^2}$), has also been implemented and tested. For details on these studies we refer to Kupka (2001) and Kupka and Muthsam (2001). Here, we just mention that some of the algebraic models for third order moments which were derived from full, dynamic equations yield results which compare satisfactorily to computations based on the latter. This provides us with a cheap alternative as long as we are only interested in stationary solutions. The considerable reduction in computational costs is particularly important for implicit solvers which are indispensable when dealing with problems that feature large relaxation times.

To test the moment equations of the Reynolds stress model we have used bench-mark data from direct numerical simulations of pure convection by H. J. Muthsam. They assume a plane parallel geometry with a constant gravity pointing towards the bottom of the simulation box. A perfect gas with a constant c_p/c_v is assumed. The efficiency of convection is determined by prescribing the radiative conductivity, which is held fixed for all points of each horizontal plane for all times t. Radiative transfer is assumed to be diffusive. The resulting system is thus most closely related to (shallow) envelope convection below stellar photospheres.

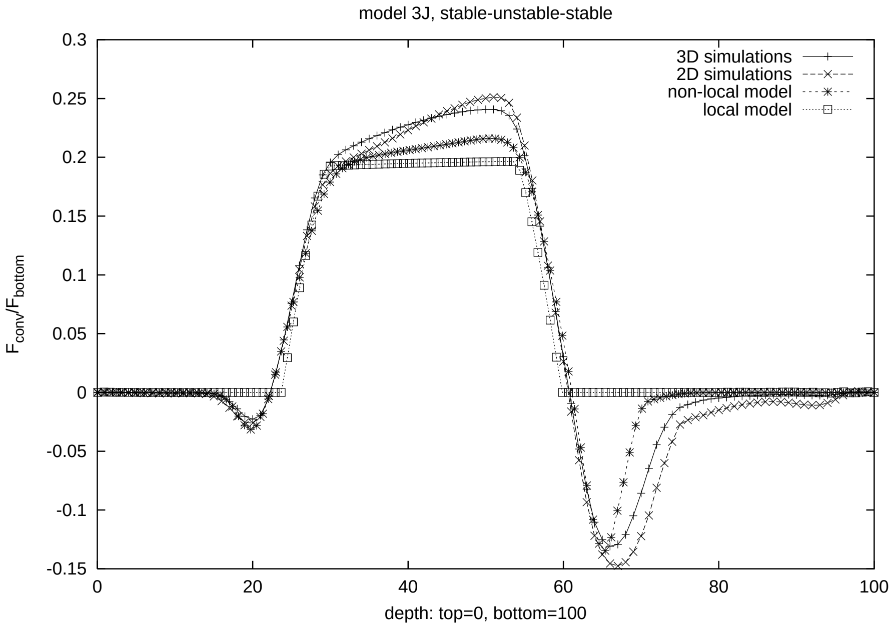

Fig. 1.— Convective flux in units of total flux as a function of depth for a convection zone embedded between stable layers. A 3D and a 2D simulation with otherwise identical parameters are compared with the convection model of Canuto and Dubovikov (1998, as described in Kupka 1999b) and with the local limit of this model. Use of the 2D simulation data is by courtesy of H. J. Muthsam.

Top and bottom of the simulation box are considered impenetrable and stress free with $\overline{T}$ fixed at the top and $\partial\overline{T}/\partial z$ fixed at the bottom, i.e. there is a constant input flux and thus input luminosity at the bottom. Horizontal boundary conditions are periodic with effective aspect ratios (vertical to horizontal extent) of 4 to 6 for the convectively unstable regions. As the instability is driven by a low local radiative conductivity rather than by a boundary layer, and as we only compare to models which have a stably stratified layer attached to at least one side of the convection zone, the flow becomes turbulent already at a very low Rayleigh number (cf. Zahn et al. 1982). For Pr of order unity or less, this implies a low critical Reynolds number. Thus, it is possible to simulate fully turbulent convection already with a resolution of $72 \times 50 \times 50$ to $125 \times 100 \times 100$ grid points. For a Pr of 1 or 0.25 at Rayleigh numbers $\mathrm{Ra} \sim 10^5$ to 10^6, which is some 3 to 4 orders of magnitude above the critical value, it is hence possible to resolve all relevant length scales.

Using such simulations as the equivalent of laboratory data, we can find out whether a convection model reproduces the mean fields such as $\overline{T}$, the second and third order moments, as well as topological features such as skewness. We can assess the overall performance of an entire model as well as the reliability of individual closure assumptions. The model problems we used for our comparison work are from a library of numerical simulations created by H. J. Muthsam (see e.g. Muthsam et al. 1995 and Muthsam et al. 1999). So far, the following cases have been investigated:

- isolated convection zones (embedded between two stable layers) and interacting zones (separated by a stable layer in between and from below);
- inefficient convection (where the radiative flux F_{rad} carries more than 75% of the input flux at all layers) and efficient convection (where the convective flux F_{conv} attains more than 75% at least in some layers);
- shallow convection zones, i.e. $T_{\mathrm{top}}/T_{\mathrm{bottom}} = 0.1 \ldots 0.25$ and $\rho_{\mathrm{top}}/\rho_{\mathrm{bottom}} = 0.05 \ldots 0.1$.

The entire domain thus spans between 4.2 and 4.8 pressure scale heights H_p where the convection zones themselves have a thickness of 0.8 to 3.0 H_p. Presently, deep convection zones where $T_{\mathrm{top}}/T_{\mathrm{bottom}} < 0.05$ and $\rho_{\mathrm{top}}/\rho_{\mathrm{bottom}} < 0.01$ such as the one studied by Chan and Sofia (1996) can only be handled through LES. First results have already been published (Kupka 1999a, Kupka 1999b). A complete description will be given in Kupka (2001) and Kupka and Muthsam (2001). Here, we give two results to illustrate the comparison procedure and the performance of the convection model.

Figure 1 displays the convective flux for a convection zone embedded between two stable layers. These calculations were already discussed in Kupka (1999b). The results for the 3D simulation presented here are based on further relaxation of this model (1.1 rather than 0.45 thermal time scales) which reduced the maximum convective flux from 26% to 24% of the total flux. Also, a longer averaging was used (0.14 thermal time scales instead of 0.01 — one thermal time scale is equal to about 10 000 sound-crossing times). The size of these changes is typical also for other quantities, thus the overall results have remained more or less unaltered. In the same figure, we give the results for an equivalent 2D simulation (briefly discussed in Muthsam et al. 1995) which can be clearly distinguished from the 3D case by a much larger overshooting (OV). Contrary to the 3D case, the 2D overshooting would extend even much further if there were no solid plate which stops the downward flow (see Kupka and Muthsam 2001, for an extensive comparison between 2D and 3D simulations). The solution of the nonlocal model compares fairly well with the 3D simulation. In particular, it predicts an OV zone that remains constrained well inside the simulation box. We note that the higher order closures employed in this convection model are exactly the same as in Kupka (1999b), except for the quantity $\tau_{p\theta}$, which was increased by a factor of three. This change was motivated by the result that in comparison with any of the simulations available to us, the value derived by Canuto and Dubivikov (1998) overestimates the damping of the convective flux. Most likely, this overestimation is due to the anisotropy assumptions made for the analytical derivation of $\tau_{p\theta}$. As a net result, the convective flux increases by about 1%. On the other hand, the solution of the local limit of the model is much further off from the simulations almost everywhere. We also note that a traditional mixing length model would yield results similar to this local convection model.

Figure 2 demonstrates how we test closure relations. It displays the dissipation of $\overline{\theta^2}$, ϵ_θ, as obtained from numerical simulations, and compares it with the most popular closure for this quantity. This closure was also rederived in Canuto and Dubovikov (1998). We note that because the simulations correspond to $\mathrm{Pr} = 1$, we have only used the "high efficiency" limit of their expression, because for very low efficiences (reached at the far end of the OV region) the problem becomes dominated by molecular viscosity. For stars, this transition

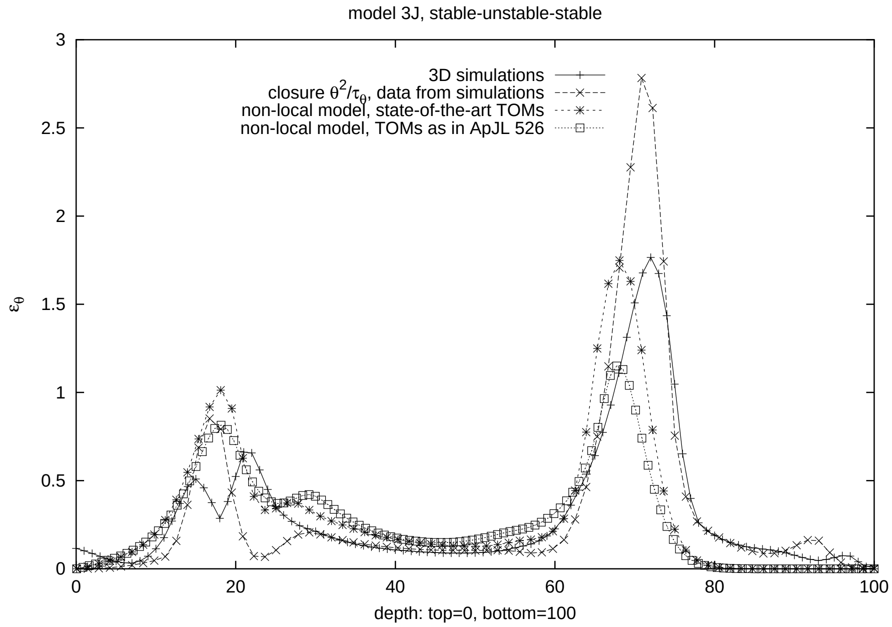

Fig. 2.— Dissipation of temperature fluctuations ϵ_θ as a function of depth for a convection zone embedded between stable layers. A 3D simulation is compared with the closure $\epsilon_\theta = \overline{\theta^2}/\tau_\theta$, rederived by Canuto and Dubovikov (1998), when data are taken from simulations, and with solutions of the full convection model (assuming two different closures for third order moments).

occurs at much lower velocities and efficiencies, which are usually considered to have no importance in stellar modeling. The closure results displayed in Fig. 2 were obtained in three different ways: first, from putting simulation data for the second order moments into the closure. This is a self-consistency test. If the closure were perfect (and simulations not affected by any numerical viscosity), the corresponding curve would match exactly the one obtained from computing ϵ_θ directly from the simulations. Indeed, inside the convection zone the closure does an excellent job. The slightly larger deviations in the OV regions are at least partially due to the fact that molecular viscosity becomes important in the outer OV region which has not been taken into account in deriving the closure relation. This, however, is of no relevance when applying the closure in modeling stellar convection. The other two curves exhibit the performance of the Reynolds stress convection model when using this closure. Over most of the convection zone the performance is still very good, although deviations are larger now, particularly in the OV region. This can be explained from the smaller OV obtained from the model in comparison with the simulations (Fig. 1). The two curves representing the Reynolds stress convection model are based on different third order moments for the description of nonlocal transport. However, the difference when using the third order moments from Canuto et al. (2000) instead of those used in Kupka (1999b, based on Canuto 1993) is within the overall uncertainties. For many other scenarios (such as interacting convection zones) we have found this closure to perform equally well.

5. Conclusions

The nonlinear interactions present in any highly turbulent flow are responsible for the very wide range of spatial and temporal scales that contribute to the dynamical evolution of the flow. At the same time, the strong coupling of the different scales prohibits any perturbation approach. Consequently, both numerical and analytical treatments of stellar turbulent convection are extremely difficult and entail approximations to the problem.

The two most powerful methodologies available for treating stellar convection are based on different methods of statistical averaging over the various scales. But whereas the large eddy simulations can only be applied to a limited range of problems characterized by short thermal relaxation time scales, the fully nonlocal convection models based on the Reynolds approach could in principle be applied to most convection related problems in stellar astrophysics using computational equipment available today.

However, the Reynolds stress models entail further statistical (closure) assumptions. In addition to their success in describing convection in the earth's atmosphere (Canuto et al. 1994) the models must be tested within a stellar-like scenario. We have performed such tests using idealized microphysics and considering a parameter range accessible to direct numerical simulations (Kupka 1999a, Kupka 1999b, Kupka 2001, Kupka and Muthsam 2001). The results are promising enough to justify the application of such models to stars. Although in principle the parameter range of the tests could be extended using large eddy simulations, the additional statistical assumptions about the unresolved scales made in such simulations reduces the significance of such tests. And moreover, the parameter ranges of most astrophysical problems, for example those encountered in convective stellar cores, cannot be attained by numerical simulations anyway.

Hence, the obvious step was to modify the program developed for solving the equations for the Reynolds stress convection model so that it can be linked to a stellar code (Kupka and Montgomery 2001). Although we cannot directly measure the moments of the hydrodynamic equations for convection zones inside a star, indirect observational tests will provide the clue to assess whether the new generation of nonlocal convection models keeps up to the expectations from the comparisons with idealized numerical simulations.

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