Nonlocal convection using the Reynolds stress approach: δ Scuti-type stars.

Friedrich Kupka
Institute for Astronomy, University of Vienna, A-1180 Vienna, Austria
Institute of Mathematics, University of Vienna, A-1090 Vienna, Austria

Michael H. Montgomery
Institute of Astronomy, Madingley Road, CB3 0HA Cambridge, United Kingdom
Institute for Astronomy, University of Vienna, A-1180 Vienna, Austria

ABSTRACT

In this paper, we present some preliminary results of the first application of a fully nonlocal convection model to realistic stellar envelopes. The model has previously been studied by a detailed comparison with numerical simulations. Our first results were derived for the He II convection zone of a 2.1 solar mass star along the evolutionary track from the zero-age main sequence to the turn-off point, thus crossing the region populated by the δ Scuti variables in the classical instability strip which are among the primary target stars proposed for the COROT programme. Although we are only able to provide a consistency test with earlier analytical results, in the near future asteroseismology may be one of the most important tools for directly testing this new generation of convection models.

1. Introduction

Turbulent convection is both one of the most complex and one of the most important physical processes encountered in stellar astrophysics. The complexity is due to two facts. First, the strong nonlinearity of the physical problem limits linearizations to a mere tool for stability analysis, while perturbation methods are inapplicable due to the lack of any obvious smallness parameter. This makes analytical treatment of turbulent convection extremely difficult. Second, again due to the nonlinearity of the problem, turbulent convection entails vastly different spatial and temporal scales which makes a complete numerical treatment of turbulent convection not easier than an analytical one and in fact always entails substantial approximations to the problem (see also Kupka 2001a).

But in spite of its inherent difficulty, turbulent convection in stars cannot be ignored: it is responsible for heat transport in at least some part of most stars, it provides an important mixing mechanism, and it couples to rotation, pulsation, and magnetic fields, which it either generates, or inhibits, or at least modulates.

Over the recent years, several local convection models, in particular the well-known mixing length theory, have shown their limitation. These models only entail algebraic equations and are expressed in terms of local physical quantities alone.[1] However, the attempts to account for fundamentally nonlocal processes such as convective overshooting within such models have been judged to have more or less failed (Renzini 1987, Canuto 1993). And despite the fact that some of these models are more successful than others in simultaneously matching several observational quantities such as solar radius and solar Balmer line profiles without readjusting model parameters (Bernkopf 1998), there are experimental data which cannot be explained by any of the local convection models (cf. the surface gravity determination from photometry of the Hyades as well as the temperature dependence of the metallicity index of Strömgren photometry, Figs. 5 and 6 in Smalley & Kupka 1987).

nonlocal modeling of some of the basic variables describing turbulent convection has been pursued for quite some time, e. g., in Xiong (1986). However, only more recently have a series of models of increasing complexity been proposed by Canuto (1992), Canuto (1993), Canuto & Dubovikov (1998), and Canuto (1997) which provide a fully nonlocal description for all the required second order moments without invoking a mixing length.

A code for solving these equations has been developed by one of us (Kupka 1999a, Kupka 1999b). The code and equations have stood extensive and successful comparisons with numerical simulations of fully compressible convection (Kupka 1999b, Kupka 2001a, Kupka 2001b, Kupka & Muthsam 2001). We have recently linked this code with a stellar envelope computation program to solve the "hydrodynamic moment equations in Reynolds stress form" for envelopes of A-type main sequence stars.

Here, we describe some first results for the case of a 2.1 solar mass star and compare the overshooting below the bottom of the lower helium convection zone with results obtained from earlier, analytical expressions which however lacked predictive power (Zahn 1991). A more detailed description of the code will be given in Kupka (2001b) and an extensive discussion of results for A stars in Kupka & Montgomery (2001). We hope that in the near future, asteroseismology will provide data that put more stringent conditions on this new generation of convection models.

2. The model

The equations we solved are those given by Canuto & Dubovikov (1998) for the relevant second order moments of velocity and temperature fields. They are the turbulent kinetic energy K, the mean square of temperature fluctuations $\overline{\theta^2}$ (thermal potential energy), $\overline{w\theta}$ (where $F_c = c_p\rho\overline{w\theta}$ is the convective flux), the turbulent kinetic energy of the vertical velocity components $\overline{w^2}$ (where $p_t = \rho\overline{w^2}$ is the turbulent pressure), and the dissipation of turbulent kinetic energy ϵ. The equations of Canuto & Dubovikov (1998) were supplemented by terms which account for compressibility effects (Canuto 1993) and have already been summarized in Kupka (1999b) (equations (1)–(5)) in this form. They constitute a system of 5 coupled nonlinear differential equations, each of second order in space. They give the time evolution,

[1] They also may involve a measure for distance to the nearest locally stable layer or the size of the convection zone to provide a minimum account for nonlocality.

i. e., ($\partial K/\partial t = f(T(r), ...)$, etc.). Without a mean flow and without any external forcing other than a constant input luminosity at the bottom, the equations always attain a stationary solution after evolving for about a Kelvin-Helmholtz time scale from any reasonable initial solution, which may either be provided by a local convection model or by a slightly perturbed radiative solution. The equations are supplemented by those for the mean pressure and for the mean flux and temperature as given in Canuto (1993) and Canuto & Dubovikov (1998) (see equations (7)–(8) of Kupka 1999b). Together with the mass budget, the mean structure equations form a coupled system of differential equations of fourth order which is standard in stellar structure calculations, but they are coupled now to a tenth order system describing turbulent convection rather than to just a few algebraic equations.

In these first experiments, we have restricted ourselves to the down-gradient approximation (see Canuto & Dubovikov 1998 for a definition) for the fluxes of K, $\overline{\theta^2}$, $\overline{w\theta}$, and $\overline{w^2}$, similar to what was done in the first comparison of solutions of these equations with numerical simulations (equations (9) and (10) in Kupka 1999a). The only exception is the flux of dissipation, $\overline{\epsilon w}$, which we set to be $0.6\overline{q^2 w}/\tau$ where $\tau = 2K/\epsilon$ and $\overline{q^2 w}$ is half the flux of K, as this closure was found to yield excellent results in comparison with a large set of numerical simulations (cf. also Canuto 1992 as well as Kupka & Muthsam 2001, details will be published elsewhere). It even decreases computational costs and increases stability in comparison with the down-gradient expression (Canuto & Dubovikov 1998, equation (6) of Kupka 1999b).

The details of the numerical solution procedures will be described in Kupka (2001b), and a discussion specific to the problems of solving these equations for A-type main sequence stars will be given in Kupka & Montgomery (2001). Here, we just mention that the solution method involves centered finite differences on a staggered mesh which is mapped to points equispaced in the logarithm of the surface mass, and a spherical geometry is assumed. Alternatively, plane parallel geometry, grids equidistant in space, or any combinations thereof can also be used. In addition to explicit time stepping (Kupka 1999b), a semi-implicit method (Kupka 2001b) was developed. This allows the equations for the lower helium convection zone to be solved in times ranging from a minute to a half an hour on a modern workstation. The inclusion of the above-lying hydrogen convection zone is also feasible, though at higher computational costs of a few hours (Kupka & Montgomery 2001); this is due to the denser zoning which is required in order to adequately resolve the steep opacity changes in the hydrogen ionization zone.

3. Our steps towards a complete coupling of the codes thus far

We have supplemented the moment equation solver of Kupka (1999b) with an interface for the OPAL opacity and equation of state routines, as taken from the Warsaw-New Jersey stellar evolution code of Paczyński, Sienkiewicz, and Kozłowski (described in Pamyatnykh 1999), which itself is a modernized version of the original stellar structure code of Paczyński (1969) and Paczyński (1970). The initial state is taken from a stellar envelope model computed with the stellar structure code using a local convection model. We keep the pressure, temperature, and radius (or mass) fixed at the upper boundary and impose a constant input luminosity at the lower boundary. Boundary conditions for the second order moments are

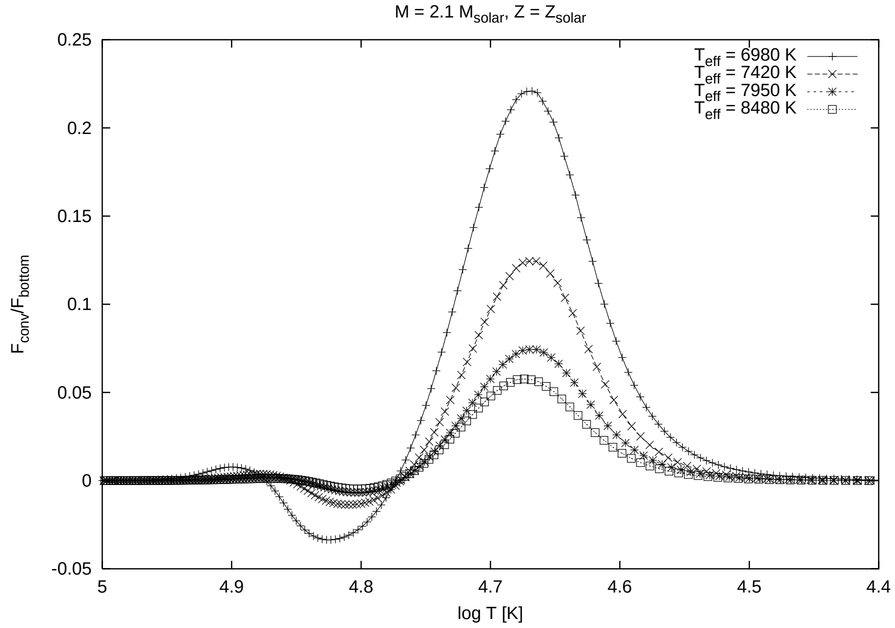

Fig. 1.— Convective flux in units of total flux as a function of mean temperature (and thus depth) for the lower helium convection zone. In this calculation, effects of radiative losses on the dissipation of thermal energy and on the correlation of (fluctuating) temperature and pressure gradient have been neglected.

taken from Kupka (1999b). The moment equation solver employs this input envelope to derive an initial state. A sufficiently large region around the convectively unstable part of this input envelope is then chosen and relaxed towards a stationary state. The new envelope region can then be attached to the upper part of the envelope represented by the upper boundary conditions, and downward integration to the stellar core can be continued from the bottom of this compound part of the stellar envelope by the stellar structure code to obtain a complete stellar envelope. To link the moment equation solver to any other stellar envelope code based on downward integration is straightforward, once an appropriate interface to the moment equation solver has been designed.

4. Some results relevant for δ Scuti stars

We have computed a set of envelopes relevant for a 2.1 solar mass star along its evolutionary trajectory from zero-age main sequence to the turn-off point. Figure 1 compares the convective flux at the different evolutionary stages as found for the lower convection zone, which is due to the ionization of He II.

Clearly, for the more evolved star the lower effective temperature makes the convection more intense despite the lower surface gravity for the more evolved models. From Fig. 2 we see that the corresponding convective flux in Fig. 1 is still positive in a domain where $\nabla - \nabla_{\rm ad}$ is already negative, in agreement with what is found in numerical simulations of convection using a perfect gas and a prescribed radiative conductivity and in solutions of the moment equations for the same physical scenario (Kupka 1999a). However, this region remains very small and is succeeded by a much larger region of negative convective flux near the base of the convection zone, which continues until the convective flux and velocity fields eventually vanish.

We can measure the size of this overshooting region in different ways. Zahn (1991) defines L_p as the distance from where F_c attains its first zero[2] to the local minimum where $F_c < 0$ (see his Fig. 1 which corresponds to a point below $\log T \sim 4.8$ in our own Fig. 1). In Tab. 1 we compare this distance L_p in units of pressure scale heights with the total overshooting (OV). We defined the latter as the entire region for which $\nabla - \nabla_{\mathrm{ad}} < 0$ and for which the turbulent kinetic energy and the vertical velocity are not negligibly small ($(\overline{w^2})^{1/2} \geq 1$ cm/s, which is 10^{-5}–10^{-6} of the value found for the middle of the convection zones computed for Fig. 1).

T_{eff}[K]	$\log g$	L_p found	L_p (JPZ)	total OV found	x
6980	3.53	0.45	0.40	1.5	1.6
7420	3.63	0.35	0.30	1.3	0.9
7950	3.77	0.27	0.25	1.25	0.6
8480	4.04	0.25	0.20	1.2	0.5

Table 1: Overshooting (OV) below the He II convection zone for a 2.1 $M_\odot$ star in units of H_p. We note that for strict validity of the criterion (3.9) of Zahn (1991) we should find $x = x(\sigma, \overline{w^2}, \overline{w^3}, H_p, K_{rad}) \gg 1$.

For comparison, we also give the distance L_p computed using equation (3.9) of Zahn (1991). This formula requires the filling factor σ and the mean velocity at the point where F_c changes sign, which the (Zahn 1991) model cannot provide. On the other hand, these quantities can be computed from our solution of the moment equations; the filling factor can be computed following the prescription described in Canuto & Dubovikov (1998) which relates the skewness of the velocity field $S_w = \overline{w^3}/(\overline{w^2})^{3/2}$ to the relative area filled by upward moving fluid, while the mean turbulent velocity is obtained directly as a solution of the moment equations.

In this sense, the comparison is a self-consistency test that confirms the validity of the solution at least if the deviations from adiabaticity are not too large. Such deviations are small, if the criterion (5.12) of Zahn (1991) is fulfilled, i. e., $x \gg 1$ where x means the product on the left-hand side of (5.12) which is related to $\nabla/(\nabla_{\mathrm{ad}} - \nabla)$ by (5.10) of Zahn (1991). As one can see from the last column of Tab. 1, the criterion becomes marginally satisfied at least for larger convection zones. Nevertheless, the values of L_p we found compare quite well with the criterion of Zahn (1991).

The decay of the velocity field in the far OV region where F_c approaches zero is roughly exponential as in Freytag et al. (1996), but at a shorter e-folding distance, i. e., at much higher rate, the physical reason

[2]following the radius from an unstable to a stably stratified (overshooting) region

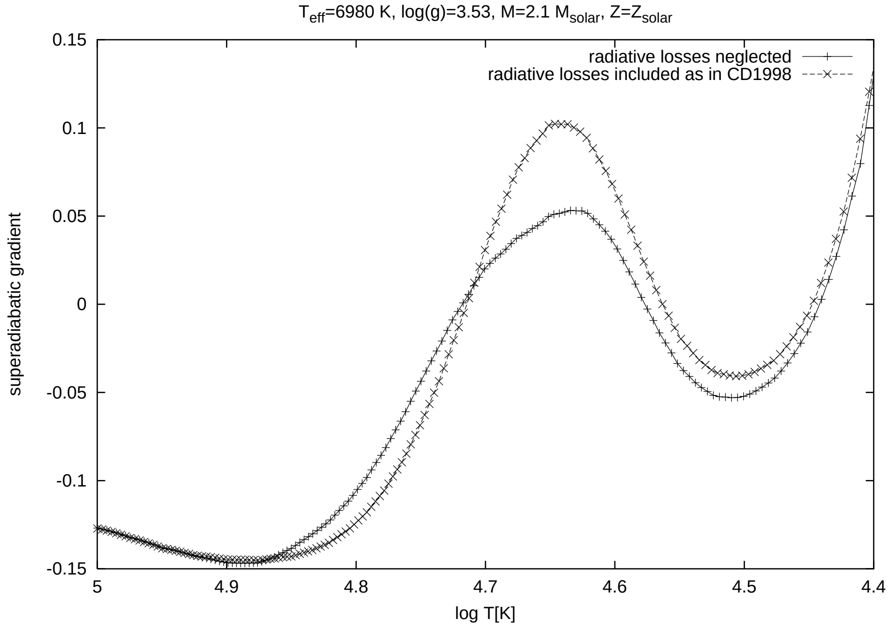

Fig. 2.— Superadiabatic gradient $\nabla - \nabla_{\rm ad}$ as a function of mean temperature (and thus depth) for the lower helium convection zone. The effects of radiative losses on the dissipation of thermal energy and on the correlation of (fluctuating) temperature and pressure gradient are clearly visible, as their inclusion provides a higher superadiabatic peak in the convection zone and a more stable temperature gradient in the lower overshooting region (at $\log T \sim 4.8$).

having been discussed by Canuto (2001). In an extended region for which $F_c < 0$, it is also exponential, but at a different rate, while it is nonexponential in-between and where $F_c = 0$, near the beginning of unstable stratification.

The overshooting extent and even more so the convective flux are significantly reduced when taking into account the effect of radiative losses on the dissipation of $\overline{\theta^2}$ and on the correlation between the fluctuating components of the temperature, θ, and the pressure gradient, $\partial p'/\partial r$. If we consider, for instance, the most evolved model with $T_{\rm eff} = 6980$ K, we find that F_c is almost reduced to 1% of the values displayed in Fig. 1. As a result, $L_p \sim 0.1$ rather than 0.45, if we use equations (34b-c) of Canuto & Dubovikov (1998) to model the radiative loss effects. The latter probably provide a lower limit; more details and a different formulation will be discussed in Kupka & Montgomery (2001).

The difference between the inclusion and neglect of radiative losses is also evident from Fig. 2, which compares $\nabla - \nabla_{\rm ad}$ for the model with $T_{\rm eff} = 6980$ K. Similar differences are found for the less evolved models.

5. Discussion and conclusions

How can one assess the reliability of the results obtained with the new convection model? For the case of idealized microphysics, it is possible to use direct numerical simulations (Kupka 1999a, Kupka 1999b, Kupka 2001b, Kupka & Muthsam 2001). For the envelope convection zones of A-type main sequence stars, only large eddy simulations of convection zones are affordable. However, the relaxation time for this problem is quite long and the resolution requirements for the hydrogen convection zones are even more demanding than for the solar case. Thus, currently only simulations in 2D exist (Sofia & Chan 1984, Freytag et al. 1996). Since 2D simulations yield OV's which are too large and kinetic energies and velocities which are too high (Kupka & Muthsam 2001), due to the fundamentally different physics of turbulence in two dimensions (see also Canuto 2000), such comparisons have to be considered with caution. Nevertheless, in reasonable agreement with the fact that the simulations naturally include effects of radiative losses (at least within the framework of 2D physics) and overestimate fluxes and OV distances of the real 3D situation, the models of Sofia & Chan (1984) and Freytag et al. (1996) for the convective part of A star envelopes yield fluxes in between those of solutions of the moment equations with and without the radiative loss effects included as described above. A closer comparison would also require the entire envelopes (including the hydrogen convection zone) to be computed and the most complete closure models for the third order moments to be used, which we will discuss in a future paper (Kupka & Montgomery 2001).

Unfortunately, any such comparison will have only limited implications, as the physics cannot be completely and reliably captured by the available simulations. Because the helium convection zones are already located below the photosphere, direct observations of the velocity field (its skewness, etc.) are not possible. There are two other potential approaches whereby results such as those shown in Figs. 1 and 2 could be tested. An indirect test could be expected from diffusion calculations, because the size of the overshooting region also determines the amount of material in the outer envelope that is mixed. It can be expected to increase significantly during stellar evolution both in terms of depth and as a function of mass contained in that region. Hence, the depletion rate of He on the surface is changed. The He abundance, however, influences the efficiency of pulsational driving in the classical instability strip. Thus, diffusion calculations are linked to a second, more direct approach which provides stronger constraints on the model: asteroseismology. Of course, the ideal case would be to measure the temperature gradient in some way through seismology. Due to the nature of this technique and the concomitant extreme accuracy requirements this may not happen during the next years. However, asteroseismology from space could bring us closer towards this goal than ever before. A chemically normal star with 2.1 solar masses should readily exhibit δ Scuti-type pulsations during its main sequence evolution. The investigation of such pulsations is among the primary science goals proposed for the COROT project. A less challenging test would be to calculate the precise location of the instability strip using the new convection model for the stellar envelope. This would at least provide an indicator for the reliability of the temperature gradient predicted by the nonlocal convection model.

The present work shows that Reynolds stress convection models involving several coupled nonlinear differential equations can be linked to a stellar code and solved in short time on standard workstations. As this approach is physically substantially more complete than previous, local convection models, a real alternative to the traditional path of stellar modeling could develop over the next few years.

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