Constraints in term of stellar internal structure: report

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ABSTRACT

This paper gives a presentation of some recent works on the theme "Constraints in term of stellar internal structure" with emphasis on the results obtained by the members[1] of the COROT sismogroup created after the Kick-off meeting (see http://dasgal.obspm.fr/˜michel/COROT-SISMO.html for more information) involved in this theme. During an informal meeting which took place in November 1999 in Meudon, some actions to be done have been decided concerning: - the progress in stellar modeling by the introduction of radiative forces and the study of the limits of boundary of convective regions and -the relation between the limits of convective cores and the small frequency differences. A project of coupled inversion of structure and rotation involving two teams of the group has started up which is presented in the last section.

1. Introduction

The successful development of helioseismology encourages its extension to asteroseismology. Indeed, the large number of pulsating stars covering substantial areas of HR diagram gives potential tests of the theory of stellar structure and evolution. However there are a number of difficulties specific to asteroseismology. Observational ones are the small amplitudes of the oscillations in the spectrum and the noise (instrumental, stellar, atmospheric for observations from earth,..). Due to spatial filtering, only a limited number of modes is expected to be detectable, (degrees $l \leq 3$ for photometric observations). Moreover the identification of the detected frequencies will be difficult due to the theoretical properties of the modes.The uncertainties on mass, luminosity, chemical composition of the observed star might lead to a large number of possible models and different possible identifications of the observed frequencies.

2. About stellar models

Good stellar models are needed for the identification of the observed frequencies and their interpretation in terms of internal structure of the star.

The computation of a stellar model at given physics needs the following inputs: the mass $M_\star$, the initial chemical composition Z and X (or equivalent), and the age $t_\star$ (or equivalent $L_\star$,...). The uncertainties on the

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physics concern principally: the convection, which is described phenomenologically by the mixing length parameter Λ (MLT, Canuto-Mazitelli theory,...), the overshoot extent from the edge of the convective core described by ζ accounts for the penetration of convective core elements in the radiative envelope. Other uncertainty concerns the diffusion of chemical elements (microscopic, radiative, turbulent..). For stellar models with a mass larger than about 1.4 $M_\odot$, microscopic diffusion as described in the solar models leads to a very rapid sink of helium towards the central regions and the appearance of a hydrogen envelope which is not realistic (Turcotte S. et al. 1998). Arbitrary mixing is introduced in the codes to prevent that problem (Richer et al. 2000). Introduction of radiative forces does not help much (Morel & Alecian 2001). We know from helioseismology that additional mixing occurs in the transition regions between envelope convective zone and radiative interior. Other physical processes like the mass loss are not generally taken into account. Finally, other uncertainties are introduced by opacities tables, the EOS and the nuclear rates.

Constraints independent of models can be deduced from observation which are: the luminosity and effective temperature $L_\star \pm \Delta L_\star$, $T_{eff} \pm \Delta T_{eff}$, the chemical composition Z/X $\pm$ Δ (Z/X). Except for binary systems the estimation of the mass $M_\star$ is very crude. For binary systems, based on the hypothesis of a common origin for both components (same initial chemical composition and age), a common evolution of the star leading to a position in the HR diagram and present chemical composition within the observational constraints gives a determination of the age and initial composition. However, it has been shown that, for αCen, the increase of 5% in the mass, introduced by the new determination of masses by Pourbaix et al. (1999), leads to a factor 2 in the age and significant differences in the properties of the frequencies (Morel et al. 2000, Provost et al. 2000).

3. What will be observed

The spatial filtering selects the modes with degree $l = 0, 1, 2$ possibly 3. The number of detected modes will depend both on the noise and the intrinsic amplitude of the modes. The amplitude of stochastically excited pulsations of solar-like stars with masses from 0.9 to 2 $M_\odot$ have been estimated by Houdek et al. (1999), Samadi & Houdek (2000). For the same kind of stars, Trampedach et al. (1998) have estimated the stellar noise coming from the granulation by hydrodynamic simulations.

Using these results and an estimation of the COROT noise and taking account of the spatial filtering, a simulated spectrum has been constructed for an evolved solar-like star of 1.45 $M_\odot$ with age $t = 1.6$ Gy and central hydrogen content $X_c = 0.3$ (Berthomieu et al, 2000). It is plotted in fig.1. In order to predict the set of frequencies which can be detected, the level above which a peak will have 10% probability to be noise in a bandwidth of 35μHz has been determined. This level is indicated in fig.1. It is then assumed that the modes which have a given probability (here 90% set (a) or 50% set (b)) to have an amplitude larger than this level will be detected. For the solar-like star of 1.45 $M_\odot$ the frequencies and turning points are plotted in fig 2 for the two sets. The turning points are located at radius r_t below $0.25R_\star$ in the more optimistic case (set (b)).

The same kind of work has been made for δ Scuti by (Barban et al. 2000)

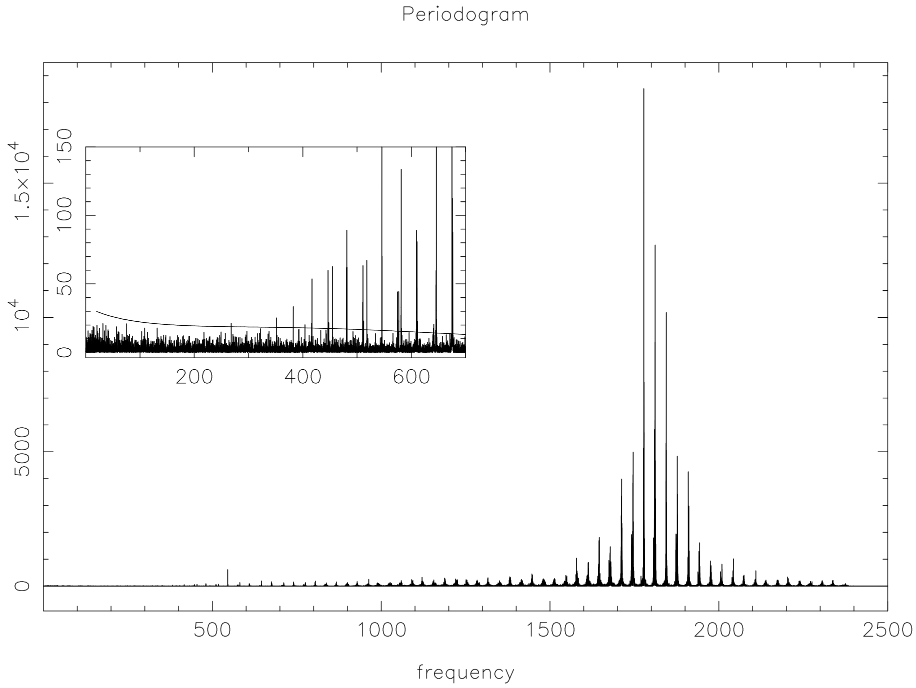

Fig. 1.— Simulated spectrum of an evolved 1.45 $M_\odot$ star .

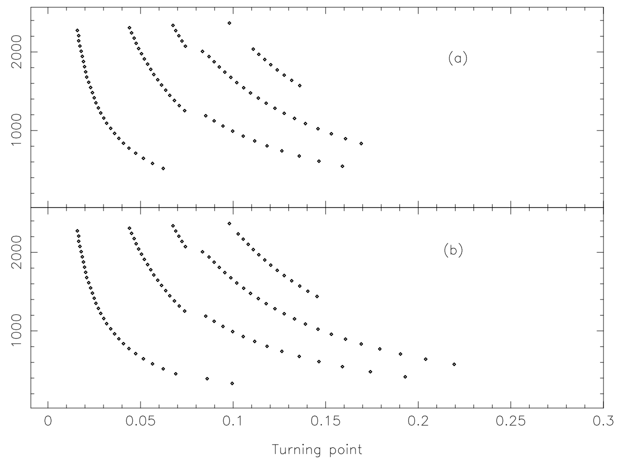

Fig. 2.— Frequencies (μHz) of predicted detectable modes versus their turning points defined by $r_t = \frac{(l+1/2)c}{2\pi\nu}$ (a) set of modes with $P_A > 90\%$, (b) set of modes with $P_A > 50\%$. The four curves correspond to $l = 0, 1, 2, 3$ from the left to the right side of each graph. Note the jump in turning points location near $r = 0.08R_\star$ corresponding to the boundary of the convective core.

4. How to extract information from identified frequencies

4.1. Global quantities

The acoustic low degree modes in the high frequency domain have an asymptotic behavior which is characterized by the quasi equidistance of the frequencies of modes of same degree and consecutive radial order, the large mean separation $\overline{\Delta\nu_l}$ which depends very few on the degree, and the quasi equidistance of modes with degree differing by 2 and consecutive radial order, the mean small separation $\overline{\delta\nu}_{l,l+2}$.

The mean large difference is related to the sound speed c in the star by:

$$\overline{\Delta\nu_l} \sim <\nu_{n,l}-\nu_{n-1,l}>\sim \left[2\int_0^R \frac{dr}{c}\right]^{-1} \tag{1}$$

It is proportional to the dynamical frequency of the star $\Omega_g = \sqrt{\frac{GM_\star}{R_\star^3}}$. Its value represents a strong constraint on the radius of the star at given mass independently of the chemical composition. As seen from its analytic expression, the mean large difference depends on the properties of the surface where the sound speed is small.

The mean small separations defined by

$$\overline{\delta\nu}_{l,l+2} \sim <\nu_{n,l}-\nu_{n-1,l+2}> \tag{2}$$

depend on the properties of the core of the star and are indicators of the age at given chemical composition. The mean difference $\overline{\delta\nu}_{0,2}$ is proportional to convective core mass m$_c$ and decreases when m$_c$ increases (Guenther & Demarque 2000). It is important to have a good analytic expression of this quantity. The asymptotic expression derived by Tassoul (1990) which is used for the Sun is not valid for stars with convective cores. A new formulation of $\delta\nu_{l,l+2}$ taking into account effect of gravity and a development in $l(l+1)$ has been derived by Roxburgh & Vorontsov (2000a).

Since modes with degree $l = 3$ might be hardly detectable, the mean small separation $\overline{\delta\nu}_{1,3}$ may be difficult to obtain with sufficient accuracy. Another intermediate quantity $\overline{\delta\nu}_{0,1}$ can be used involving only degrees $l = 0, 1$ (e.g. Gough 1998) defined by:

$$\overline{\delta\nu}_{0,1} \sim <\nu_{n,0}-2\nu_{n,1}+\nu_{n+1,0}> \tag{3}$$

4.2. Discontinuities.

Another way to extract information from the frequencies is the possibility to detect the presence of discontinuities in the structure of the star. Such discontinuities of the sound speed or its derivatives occur at the boundaries of convective regions, bases of the envelope convective zones or boundary of the convective core. A discontinuity located at radius r_d induces an oscillatory component in the deviation to regular spacing of frequencies of the form $\cos(2\pi\nu\tau_d)$ where $\tau_d = \int_{r_d}^{R} \frac{dr}{c}$ is the acoustic radius of the discontinuity. The observational determination of the period of the signal τ_d^{-1} gives the location of the discontinuity. The amplitude of the oscillatory signal is linked to the structure of the transition layer.

The periodic signature of the edge of the convective stellar envelope has been studied by Monteiro et al. (2000). For a 1$M_\odot$, the acoustic radius of this boundary varies from $\tau_d^{-1} \sim 550\mu$Hz at the ZAMS to $\tau_d^{-1} \sim 450\mu$Hz at solar age. It varies from $\tau_d^{-1} \sim 150\mu\text{Hz} \to 200\mu$Hz for $\sim$ 1.5$M_\odot$ evolved stars. They have shown that it is possible to detect this signature provided that more than 10 modes for each value of the degree l ($l = 0 \to 2$) are available with a precision of 0.1μHz or less on the frequencies.

The boundary of the convective core also introduces the same kind of oscillatory behavior but with an acoustic radius of the convective core much larger (about 10 times) than that of the convective envelop boundary. Indeed, for example, τ_d^{-1} is of the order of 2000μHz for a 1.5$M_\odot$ evolved star. This leads to a period in frequency of the sinusoidal signal too long to be detectable and to determine the core radius by this method.

4.3. Direct inversion of the frequencies

When sets of well identified low-degree mode frequencies are available, it is possible to carry out eigenfrequency inversions to investigate some properties of the internal structure of the star. However the small number of available degrees ($l < 4$), the fact that modes with $n >> l$ are asymptotically degenerate, and the errors on the frequencies render the inversions difficult. Moreover, the mass and the radius of the star are not well known.

Different ways of inverting the set of observed frequencies to infer the structure of the star can be used. Standard helioseismic differential methods which try to find the difference of structure between the observed star and a model have been used in the stellar case by Gough & Kosovichev (1993a,b) and Roxburgh et al. (1998) for a $0.8M_\odot$ star.

The linearization of the variational expression of stellar oscillations frequencies around a given reference model relates the relative frequency differences between the star and the model to the relative difference of sound speed $\frac{\delta c}{c}$ and density $\frac{\delta\rho}{\rho}$:

$$\frac{\delta\nu_i}{\nu_i} = \int_0^1 K_{c,\rho}^i \frac{\delta c}{c} \mathrm{d}x + \int_0^1 K_{\rho,c}^i \frac{\delta\rho}{\rho} \mathrm{d}x + \frac{F(\nu)}{Q_i} + \frac{\delta\Omega_g}{\Omega_g} + \varepsilon_i \tag{4}$$

where $x = \frac{r}{R_\star}$, is the nondimensionned radius, $K_{c,\rho}^i$ and $K_{\rho,c}^i$ are the respective kernels, $F(\nu)$ the term taking into account the effect on frequencies of the surface differences between star and model Q_i the energy of the mode divided by the energy that a radial mode ($l = 0$) at the same frequency would have. The term $\frac{\delta\Omega_g}{\Omega_g}$ takes into account the uncertainty on the mass and radius of the star. The errors ε_i in the data are assumed to be independent and Gaussian distributed with variances σ_i^2.

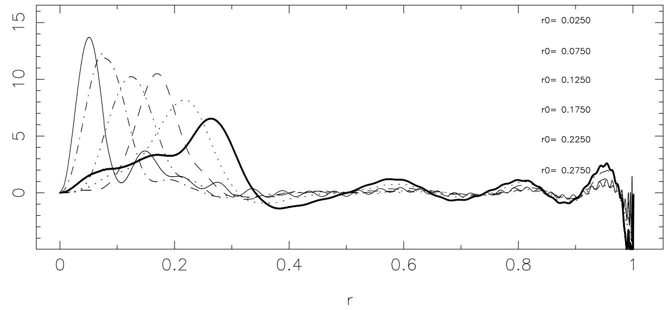

Fig. 3.— Sound speed averaging kernels for the set of modes with $P_A > 50\%$

The Optimally Localized Averaging (OLA) methods aims to find for each point r_0 of the star a set of coefficients $\alpha_i(r_0)$ $i = 1, N$ so that the linear combination of the observations represents the mean of the sound speed $< \frac{\delta c}{c}(r_0) >$ (or other quantity) around r_0.

$$< \frac{\delta c}{c}(r_0) >= \sum_{i=1,N} \alpha_i \frac{\delta \nu_i}{\nu_i} \tag{5}$$

with a kernel involved in the averaging of $\frac{\delta c}{c}(x_0)$ defined by:

$$\overline{\mathbf{K}}(r, x_0) = \sum_{i=1}^{N} \alpha_i(x_0) K^i_{c,\rho}(r) \tag{6}$$

and a variance of the result given by

$$\sigma(r_0^2) = \sum_{i=1,N} \alpha_i^2 \sigma(i)^2 \tag{7}$$

The averaging kernels can be constructed once the set of detectable modes and the predicted errors are known. As an example, kernels for sound speed inversion corresponding to the set of modes (set (b)) determined in section 2 are plotted in figure 3. In agreement with previous works, it is seen that the kernels are well peaked only in the core of the star, here below $x = 0.2$. (see for more details Berthomieu et al, 2000). Asymptotic method for stars has been developed by Roxburgh & Vorontsov (2000b) using additional assumptions on the structure of the star seem to give better results. (see Roxburgh these proceedings). Thus, more work is needed to adapt inversion methods to stellar case and to assess what might be learned about the internal structure of the star in this way. That is the aim of the inversion project presented in the following section.

5. Inversion project

5.1. Definition

A kind of hare and hound exercise of combined inversion of internal structure and rotation has been initiated in the group which up to now involves the two teams: the Meudon team (C. Barban, M.J. Goupil,

Y. Lebreton, E. Michel, I. Roxburgh) and the Nice team (G. Berthomieu, T. Corbard, G. Gonczi, P. Morel, J. Provost, T. Toutain). Both team provides to the other a set of identified frequencies and try in return to find by inversion of the frequencies it receives the properties of internal structure of the model.

At the first very simplified step, each group provides a set of identified frequencies in the domain: $l = 0, 1\ ;\ \nu > 300\mu Hz$ and $l = 2, 3\ ;\ \nu > 400\mu Hz$ for a stellar model within the following constraints in luminosity and effective temperature, metallicity, hydrogen abundance, mixing length Λ and overshoot ζ parameters

$$0.83 < log(L_*/L_\odot) < 0.89 \quad ; \quad 6400 < T_{eff} < 6600 \tag{8}$$

$$Z = 0.02 \quad ; \quad Y = 0.28 \quad ; \quad 1.55 < \Lambda < 1.85 \quad ; \quad 0. < \zeta < 0.3 \tag{9}$$

The models are computed with Caughlan et Fowler nuclear reaction rates, Eddington atmospheric temperature law, EOS: EFF, opacities OPAL, no diffusion.

This first part has been partly performed by the two teams. A set of frequencies has been provided by each group to the other one. Some preliminary results are given in the next section and in Provost 2000 (these proceedings).

5.2. Preliminary results

Some preliminary results have been obtained with the set of frequencies provided by I. Roxburgh, hereafter referred to "observed" frequencies, by the Nice team. At first step, some properties of the unknown model can be derived by using the constraints of the mean large and small separations. The constraints (8) and (9) on the models lead to a large domain of masses and ages. Among these models, a first selection can be made by considering those which have a mean large separation equal to that derived from the "observed" set. In figure 4 are plotted the quantities $\overline{\delta\nu_{02}}/\overline{\Delta\nu_0}$ and $\overline{\Delta\nu_0}$ for different models which have mean large separations close to the "observed" ones and different masses, overshoot and mixing length parameters.

The variation of the mean small differences relatively to the mass of the convective core for the different selected models are plotted in figure 5. In this figure, the "observed" values of the mean small differences are indicated by a straight line. As expected, the convective core mass depends essentially on the overshoot parameter, the positions of the models with different ζ are well separated. According to figure 5 left panel (respectively right panel), models with large ζ (respectively $\zeta = 0$) do not fit with "observations". The comparison of the two graphs favors a model with $\zeta \sim 0.1$ and shows how the small separations constrain the mass of the convective core. A detailed discussion of these results and the inference on the properties of the unknown model is given in Provost 2000 (these proceedings).

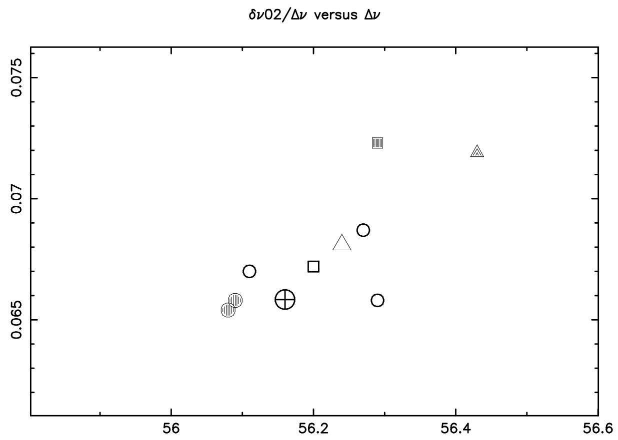

Fig. 4.— Plot of the mean small difference $\overline{\delta\nu}_{02}$ (normalized to mean large difference $\overline{\Delta\nu_0}$) versus mean large difference for selected models (from Provost these proceedings). Open circles correspond to models with 1.53 $M_{\odot}$, overshoot parameter $\zeta = 0.1$ and different values of the mixing length parameter λ ; full circles correspond to models with 1.53 $M_{\odot}$ and $\zeta = 0.15$ and $\zeta = 0.2$; open and full triangles correspond to models with 1.54 $M_{\odot}$ and respectively $\zeta = 0.1$ and $\zeta = 0.$; open and full squares to models with 1.525 $M_{\odot}$, $\zeta = 0.1$ and 1.55 M⊙, $\zeta = 0.$

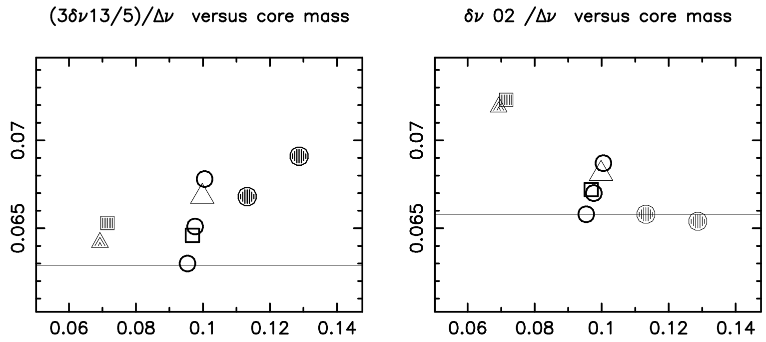

Fig. 5.— Plot of the mean small differences $\overline{\delta\nu}_{02}$ and $3\overline{\delta\nu}_{13}/5$ (normalized to mean large difference $\overline{\Delta\nu}_0$) as a function of the core mass with the same notations as in fig 4. The straight line corresponds to the values of mean small differences of the "observed" set.

6. Conclusion

In order to prepare the interpretation of the frequencies that COROT will provide in term of stellar internal structure, which is the task of the group, it is necessary to carry on both improvements on stellar modeling and on the knowledge of the stellar frequency properties. The preliminary results of the first step of the inversion project have already shown how some properties of the star can be derived from a large set of well identified frequencies. However this is an oversimplified case where strong assumptions have been made on the models (physics, chemical composition) and no errors on the frequencies have been taken into account. It has to be followed by a second step where a more realistic set of simulated data is considered taking into account instrumental constraints. stellar noise and errors on the frequency determinations.

Acknowledgments

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