Effect of fast rotation on stellar emergent flux and photometric indexes

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ABSTRACT

Some hot stars, Be or δ Scuti stars are known as fast rotators, and this has been stressed to have, for seismological studies, a non-negligible effect on the determination of their photometric parameters (Michel et al. 1998). This has stimulated updated studies in the estimation of photometric parameters for δ Scuti stars (Pérez Hernández et al. 1999). In this contribution we propose to introduce briefly this work as a tool for seismological studies, not only of δ Scuti stars but generalizing to fast rotators, in the framework of COROT. We comment the main lines of the physical assumptions and discuss their application in the COROT perspective.

1. Introduction

In the last decade, it became clear that fast rotation, which is common in stellar pulsators of moderate and high mass on the main sequence, had to be considered as a key parameter in seismological studies. Its influence on the eigenfrequency values has been regarded with more and more concern and numerical development toward higher orders were proposed to extend the domain of reliable numerical computation of eigenfrequencies toward high rotation rates (see e.g. Soufi et al., 1998). Another aspect of fast rotation is its influence on the measured photometric indexes and, consequently, on the determination for observed stars of fundamental parameters like luminosity L and effective temperature T_{eff}. This problem, with implications far beyond the seismology domain, has been addressed by numerous authors (see e.g. Tassoul, J.L., 1978). In the specific context of stellar seismology, a large number of studies are based on a comparison of the observed oscillations with the ones computed for a set of models representative of the observed star. For such comparison exercises, it is generally important to have a precise determination of the ranges of fundamental parameters characterizing the relevant set of models. In the case of fast rotators, it was shown (Michel et al. 1998) that overlooking this effect could in many cases make irrelevant the comparison. In a recent work (Pérez Hernández et al. 1999), an updated and refined estimation of this effect has been realized and proposed under a form intended to allow application in the framework of relevant seismology studies of fast rotators.

This presentation is intended to describe the main lines of the method proposed by Pérez Hernández et. al (1999) in the perspective of the COROT mission.

2. Physical description of the phenomenon

2.1. Geometrical description of the stellar surface

The fast rotating star is no longer spherical but oblate and the centrifugal force yields a new potential to be added to the gravitational one. In Pérez Hernández et al. 1999, it is assumed that, for the uppermost layers, the gravitational potential can be approximated as resulting from a punctual mass concentrating all the stellar mass at its center. Then, in the case of rigid rotation to which this work is restricted, the global effective potential Φ of the uppermost layers can be assumed to be,

$$\Phi = -\frac{GM}{R(\theta)} - \frac{1}{2}\Omega^2 R^2(\theta)\sin^2\theta \tag{1}$$

where M is the global stellar mass and $R(\theta)$ the radius depending on θ the colatitude to the rotation axis.

The surface is then a Roche surface and can be described by,

$$x(\theta) = \frac{R(\theta)}{R_\mathrm{p}} = \frac{3}{\omega\sin\theta}\cos\left(\frac{1}{3}[\pi + \arccos(\omega\sin\theta)]\right) \tag{2}$$

where R_p the polar radius of the star, $\omega = \frac{\Omega}{\Omega_\mathrm{c}}$, where Ω is the rotation velocity and Ω_c the critical rotation rotation for which the centrifugal force balances the gravity at the equator. It can be shown that,

$${\Omega_c}^2 = \frac{8GM}{27{R_\mathrm{p}}^3} \tag{3}$$

2.2. Derivation of the local monochromatic specific intensity

At each point on the surface described hereabove, the atmosphere is approximated by a plane-parallel atmosphere in hydrostatic equilibrium. The emergent flux can then be estimated with classical atmospheric models, provided the effective gravity $\vec{g}(\theta) = \mathrm{grad}\Phi$, the effective temperature $T_\mathrm{eff}(\theta)$ and the chemical composition.

From Equation 1 one can obtain the local effective gravity,

$$g(\theta) = \frac{GM}{{R_\mathrm{p}}^2}g_n(\theta) \text{ with } g_n(\theta) = \frac{8}{27}\left((\frac{27}{8x^2} - \omega^2 x\sin^2\theta)^2 + (\omega^4\mathrm{x}^2\cos^2\theta\sin^2\theta)\right)^{1/2} \tag{4}$$

The effective temperature is considered to follow a gravity-darkening law of the form,

$${T_\mathrm{eff}}^4 = C(\omega)g^\beta \tag{5}$$

where $C(\omega)$ and β are constants for a given rotating star. The β coefficients have been obtained from Claret (1998), where they have been computed as a function of mass, age and chemical composition and present a good agreement with the observational values obtained from eclipsing binaries by **??** for the extreme cases of a fully radiative atmosphere and a fully convective atmosphere respectively. In addition to this, in a recent work of Claret (2000) these computations are extended to low mass stars, opening more ways for exploiting this tool in the COROT perspective.

The $C(\omega)$ coefficient can be expressed in terms of the intrinsic luminosity L, by using the Stefan's law and integrating over the stellar surface A,

$$C(\omega) = \frac{L}{\sigma}\left(\int_A g^{\beta} dA\right)^{-1} \tag{6}$$

where σ is the Stefan's constant.

Then, using Equation 5 and integrating on the Roche surface A, one obtains for $T_{\text{eff}}(\theta)$,

$$T_{\text{eff}}{}^4(\theta) = \frac{L}{4\pi\sigma {R_{\text{p}}}^2} {g_n}^{\beta} \left(\int_0^1 \frac{{g_n}^{\beta+1} x^4}{1-\frac{8}{27}\omega^2 x^3 \sin^2\theta} d(\cos\theta)\right)^{-1} \tag{7}$$

With these values of $T_{\text{eff}}(\theta)$ and $g(\theta)$ the local specific intensity $I_\nu(\mu)$ is computed, using ATLAS 9 models (Kurucz 1996).

$\log T_{\rm e}$ ω	$\log g_{\rm e}$ i	B	U	V	$B1$	$B2$	$V1$	G
3.8	3.60							
0.3	0	-0.0081	-0.0067	-0.0149	-0.0059	-0.0103	-0.0147	-0.0160
0.3	10	-0.0065	-0.0051	-0.0131	-0.0043	-0.0086	-0.0129	-0.0141
0.3	20	-0.0044	-0.0030	-0.0119	-0.0021	-0.0068	-0.0116	-0.0131
...								

Table 1: Magnitude differences between rotating and non-rotating copartners for a grid of models with solar metallicity.$T_{\rm e}$ is given in K and $g_{\rm e}$ in $\rm cm\,s^{-2}$ (Only available in electronic format at the CDS).

2.3. *Estimation of the effect on the flux measured in different bands*

For a given rotating star with an inclination i, the specific intensity can then be projected on the line of sight (μ the projection factor), integrated spatially over the visible stellar surface of area A and finally integrated in frequency taking into account the response function $T_m(\nu)$ of the relevant filter,

$$l_m(\omega, i) = \int_0^{\infty}\int_0^{\pi/2}\int_0^{\pi} T_m(\nu) I_\nu(\theta,\phi) x^2(\theta) \sin\theta \mathrm{f}(\theta,\phi) \mathrm{d}\theta \mathrm{d}\phi \mathrm{d}\nu \tag{8}$$

where the function $f(\theta, \phi)$, at a given point of the surface depends only on ω and i (see Pérez Hernández et al. 1999 for detailed expression). The result l_m corresponds to the flux measured in a given band with a multiplicative constant B_m depending on the filter considered.

2.4. The tables of differential photometric magnitudes

In order to provide results which can be used independently on the selected calibrations, Pérez Hernández et al. (1999) presented the effect of rotation differentially, considering as reference a ”non-rotating” copartner defined as a zero-rotation star with the same composition, mass and luminosity that the considered rotating star and a radius R equal to $R_{\rm p}$. It is stressed that this copartner does not necessarily correspond to any self-consistent interior model. These non-rotating copartners are only introduced to express the photometric results in terms of differences.

The difference in the magnitude in the filter m between the rotating model and its non-rotating counterpart is given by,

$$\Delta M_m(\omega, i) = -2.5 \log \left[\frac{l_m(\omega, i)}{l_m(0, i)}\right] \tag{9}$$

These results are available under the form shown in Table 1 (Pérez Hernández et al. 1999). The ranges of $(T_{\rm e}, g_{\rm e})$ are considered as,

$$\begin{array}{ll} 3.80 \leq \log T_{\rm e} \leq 4.04, & \text{steps of } 0.02 \\ 3.30 \leq \log g_{\rm e} \leq 4.35, & \text{steps of } 0.05 \end{array}$$

where $T_{\rm e}$ and $g_{\rm e}$ are defined in Pérez Hernández et al. (1999) as,

$$g_{\rm e} \equiv \frac{G\,M}{R_{\rm p}^2} \tag{10}$$

2.5. Effects due to changes in the interior structure

In the case of non spherical interior rotating models, when taking into account the centrifugal force, the structure quantities $(p, \rho, \phi...)$ have a colatitude dependence proportional to $P_2(cos\theta)$. We take the radius of the spherically symmetric rotating model $R_{\rm ss} = R(\theta_0)$ where θ_0 is given by $P_2(cos\theta_0) = 0$. Therefore, by using Equation 1 we obtain the polar radius as:

$$R_{\rm p} = \frac{R_{\rm ss}}{1 + \dfrac{\Omega^2\, R_{\rm ss}^3}{3GM}} \tag{11}$$

In order to analyze set of models of different rotation or if rotating models are not available, one can estimate L and $R_{\rm p}$ from their values for non-rotating models (see Pérez Hernández et al. 1999). Considering simple laws see e.g. Tassoul(1978) we have:

$$L(\omega) = L(0)(1 - C_{\rm L}\omega^2) \tag{12}$$

$$R_{\rm p}(\omega) = R_{\rm p}(0)(1 - C_{\rm R}\omega^2) \tag{13}$$

Where the quantities (0) refer to the non-rotating models. Here, $\omega = \frac{\Omega}{\Omega_c}$ as seen in section 2.1, but in this case the definition of Ω_c is slightly different from (3) :

$$\Omega_c{}^2 = \frac{8GM}{27R_{\rm pc}{}^3} \tag{14}$$

Where $R_{\rm pc}$ is the polar radius a star with $\omega = 1$ would have, that is $R_{\rm pc} = R_{\rm p}(0)(1 - C_{\rm R})$.

An estimation of the coefficients $C_{\rm L}$ and $C_{\rm R}$ of 0.055 and 0.013 respectively for a $2M\odot$ model at the ZAMS is proposed by Sackmann I.J., 1970. Recent computations by us give similar results for models between $1.6M_\odot$ and $2.2M_\odot$ on the main sequence. For evolved stars, in particular from the Red Edge, these values are not longer valid.

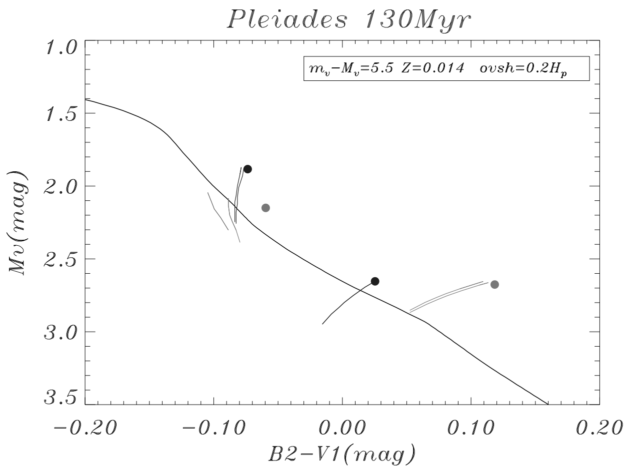

Fig. 1.— Colour-Magnitude diagram for δ Scuti stars of the Pleiades cluster with the correction of rotation for models at 130 Myr. Filled circles represent observed values. Suárez et al. 2001b.

3. Applications in the COROT perpective

As commented at the beginning of this contribution, there are several candidates, in the framework of COROT, which are considered as fast rotators: individual objects and stars belonging to clusters.

As an example of what can be done in this last case, we present here the corrections explained hereabove, for a selection of δ Scuti stars of the Pleiades cluster (Suárez et al. 2001b). Figure 1 represents a colour-magnitude diagram with the observed stars (filled circles). Continuous line corresponds to an isochrone of 130 Myr. Segments are the potential non rotating counterparts location, for a given $vsini$ (table 1), computed as function of the angle of inclination i, from i=90° (top) to $i(\tilde{\omega}_{max} = 0.95)$ (bottom).

Models at the intersection of segments with the isochrone will be our non-rotating counterparts. Considering the error on $vsini$ typically around $\pm 10\%$ and neglecting other errors (i.e. on photometric parameters, ages...,etc), we obtain two segments per observed star giving intervals in $M_{\rm v}$ and $B_2 - V_1$. In addition to the correction in M_v and $B2 - V1$ photometric parameters commented above, the technics used here provides an estimation of ω and i for each star separately.

On the other hand, applying the correction for rotation in the photometric parameters of individual objects, we can obtain a range of possible corrected values in function of the i and ω, since there is a close relation between i and ω, possible ranges of one of these parameters can be obtained fixing the other, i.e. estimating ω from rotational splitting.

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DISCUSSION

After questions by I. Rouxbourgh concerning the limits of the validity of this method, a discussion has been opened (work in progress).