Moderate and Fast Rotation: where do we stand ?

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ABSTRACT

Recent investigations of the pulsation properties of a rotating star are briefly presented and what remains to be done is outlined in view of the forthcoming space seismic mission COROT.

1. Introduction

Determining the rotation profile $\Omega(r, \theta)$ and the sound speed within stellar interiors by means of seismic data inversions are two major goals of asteroseismology. In order to achieve these objectives, one however needs to know beforehand what are the effects that rotation induces on the oscillation frequencies of a star, hereafter denoted as ν_{nlm} where the subscripts have their usual meaning: n, the radial order, ℓ the degree and m the azimuthal order of the mode. These effects arise from the centrifugal and Coriolis forces in a co-rotating frame (Unno *et al.* 1989). The importance of their roles depends on whether the rotation is slow, moderate or fast. It is therefore convenient to consider the parameters characterizing the strength of the rotation through the centrifugal and Coriolis forces respectively as

$$\epsilon = \frac{\Omega}{(GM/R^3)^{1/2}}; \qquad \mu = \frac{\Omega}{\omega}. \tag{1}$$

In case of a slow rotation, ϵ and μ are small and a perturbation approach can easily be used. Pioneer works are due to Ledoux (1945); Cowling & Newing (1949); Ledoux (1951). It is sufficient to stop the expansion at the first order in Ω, $O(\Omega)$ i.e. to include only the dominant effects of Coriolis force which is to lift the mode degeneracy in the azimuthal order. If the rotation angular velocity does not have latitudinal dependence and the rotation is slow enough, the multiplets of modes show a Zeeman-like equidistance structure i.e. multiplets of modes equally spaced in frequency. For a mode with given radial order and degree n, ℓ (omitted below), the split frequencies in a co-rotating frame are obtained as

$$\nu_m = \nu_0 + m < \Omega >; \quad m = -\ell, \ell, \tag{2}$$

where $< \Omega >$ represents some weighted average of the rotation rate which depends on the mode and the stellar model. A variational formulation allows to compute $< \Omega >$ from the zeroth order eigenfunctions (Lynden-Bell & Ostriker, 1967; Hansen *et al.* 1977). The splittings $\nu_m - \nu_0$ can in principle be easily inverted to yield the rotation profile. This has successfully been done with solar data (see for instance, Corbard *et al.* 1998 and references therein).

One speaks of moderate rotation when the rotation can be treated as a perturbation but one has to go to higher order terms in power of ϵ, μ in the perturbative expansion, i.e beyond the first order correction to the

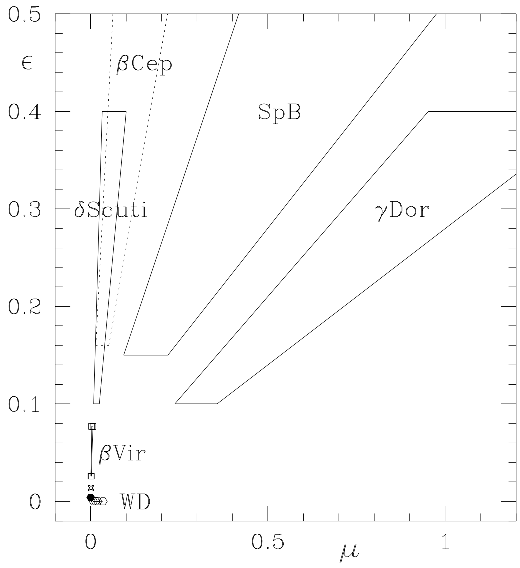

Fig. 1.— Diagram $\mu - \epsilon$: the cross corresponds to a 1.5 $M_{\odot}$ model representative of Procyon and the solid line ending with open squares delimits the case of a 1.3 $M_{\odot}$ model representative of βVir; the full circle indicates the solar case; open circles stand for a typical variable white dwarf. Other domains are labelled with the corresponding type of star: β Cephei (dashed line), δ Scuti, SPB, γ Dor stars (solid lines)

eigenfrequency and, accordingly, beyond the zeroth order for the associated eigenfunction. In a moderate or fast rotating star, the angular dependence of a given mode can no longer be simply represented by a single spherical harmonic $Y_{\ell,m}$.

For large ϵ and/or μ, i.e. for fast rotation, one must turn to a nonperturbative approach or a 2D computation for treating the waves and/or a 2D computation for the equilibrium configuration.

Hence two questions naturally arise: which star is a slow, moderate or fast rotator with respect to its oscillations ? (Sect.2). In practice how do we take effects of rotation into account for moderate and fast rotation ? (Sect.3-5).

2. Centrifugal versus Coriolis effects

The magnitude of the rotational perturbation of an oscillation frequency depends on the nature of the associated oscillation mode. Different types of variable stars oscillate with different types of modes, which are not equally sensitive to the centrifugal and Coriolis forces.

As an illustration, Fig.1 shows the (μ, ϵ) domains relevant for several types of oscillating stars. The values of the parameters (μ, ϵ) are obtained from Eq.(1), using values for the rotational velocity and frequencies of the modes, in the ranges expected for each type of stars. The Sun and solar-like stars, such as β Vir and Procyon for instance, are oscillating with high radial order p-modes but rotate slowly enough and both (μ, ϵ) are very small. A first order perturbation theory is enough in this case. The same is true for the variable white dwarfs, which oscillate with high radial order g-modes.

On the other hand, the large majority of modes for most δ Scuti and β Cephei stars, correspond neither to small ϵ nor to small μ. For these stars, both the centrifugal and Coriolis forces must be taken into account at best with a perturbative approach up to the second or even the cubic order, and in the worst case with a fully non perturbative 2D numerical approach.

For γ Dor and slowly pulsating B (SPB) stars which oscillate with low frequency high order g-modes, the parameter μ is quite large and a nonperturbative approach is obviously required (see Rieutord these proceedings).

3. Moderate rotation

For moderate rotation, we need to include quadratic contributions $O(\Omega^2)$ for interpreting current ground-based observations, and cubic contributions $O(\Omega^3)$ will be required for the interpretations of future space seismic observations.

3.1. Perturbative approach

Rotation distorts the stellar structure via the centrifugal force which decreases the effective gravity $\mathbf{g}_{\text{eff}}$ everywhere except on the rotation axis.

3.1.1. Perturbation of the spherical equilibrium configuration

Changes of the spherical structure by the averaged centrifugal force according to

$$g_{\text{eff}} = g - \frac{2}{3} r\Omega^2 \qquad (3)$$

can be computed directly at the cost of slight changes in an evolutionary code (Kippenhahn & Weigert, 1994). One consequence on the oscillation frequencies is that the position of the centroid mode does not coincide with that of the nonrotating models in a Fourier spectrum. This 'frequency shift' was computed by Simon (1969) and Saio (1981) for oscillation modes of polytropes. Magnitude of this effect has been estimated by means of a scaling law by Christensen-Dalsgaard & Thompson (1999). It is included along Kippenhahn & Weigert's prescription in numerical modelling of the pulsation frequencies of δ Scuti rotating stars and shows that period ratios of consecutive radial modes for instance are significantly affected (Goupil *et al.* 2000).

The equilibrium stellar configuration must also take into account the rotationally induced instabilities which can cause changes in the structures (Tassoul, 1978). The effect on the stellar structure and its evolution of a depth-dependent rotation and diffusion model for rotationally induced mixing and transport of angular momentum have been investigated by Endal and Sofia (1976, 1979); Zahn (1992); Talon et al. (1997); and the series of papers by Meynet & Maeder (1997), Maeder & Zahn (1998); Maeder (1999) (see Zahn, these proceedings). To which extent these effects affect the oscillation frequencies remains to be investigated.

3.1.2. Perturbation of the oscillation frequencies

Quadratic effects in Ω, due to the latitudinal effect of the centrifugal force and Coriolis contributions, cause a departure from equidistance: the frequencies within a given multiplet of modes are no longer equally spaced. This destroys the regular patterns in the frequency distribution in a Fourier spectrum (Saio 1981, Smeyers and Martens 1983; Unno *et al.* 1989; Dziembowski and Goode 1992, Lee 1993) easily recognisable in case of a slow rotation. Quadratic effects for some specific stars have been computed by Chlebowski (1978), Hansen *et al.* (1978), Saio (1981), Engelbrecht (1986) for uniform rotation and for a shellular rotation in a realistic solar model by Gough and Thompson (1990).

Near-degeneracy of modes (which couples modes with different degrees ℓ) due to the Coriolis force and a latitudinal rotation were considered in Dziembowski & Goode (1992).

Soufi *et al.* (1998) carried out the perturbation up to 3rd order for a shellular rotation. They also explore the consequences of near-degeneracy of modes due to the centrifugal force. As a result, the oscillation frequency of a given mode (omitted subscripts n, ℓ) in an inertial frame can be schematically written as:

$$\begin{aligned} \nu_m \;&= \nu_0 \Big(1 + m\,(C_1 - 1)\,\mu + (D_0 + m^2 D_1)\,\mu^2 \\ &\qquad + m\,(T_0 + m^2 T_1)\,\mu^3\Big) \end{aligned} \tag{4}$$

where ν_0 is the frequency including only the spherically symetric centrifugal distorsion and $\mu = \bar{\Omega}/\nu_0$ and $\bar{\Omega}$ is the mean rotation rate. The coefficients C, D, T depend on the mode.

3.2. Orders of magnitude

Numerical calculations using Eq.(4) quantify the effect of a *uniform* rotation on oscillation frequencies of typical models for the main target stars of the COROT seismological program.

3.2.1. Solar-like oscillations

For a 1.3 $M_\odot$ model oscillating with asymptotic p-modes, a moderate rotation means a rotational velocity of $v \sim 25 - 30$ km/s. The cubic order contribution to frequencies remains small (~ 0.01 μHz). On the other hand the distorsion effect amounts to $\sim 0.1 - 0.5$ μHz and increases with the order of the mode (due to the larger concentration of the mode in the surface layers where the effect of the centrifugal force is larger) (Dziembowski & Goupil, 1998).

For these modes, the small separation between frequencies of a $\ell = 2$ and a radial ($\ell = 0$) modes for instance behaves as:

$$\nu_{\ell=2,m=0} - \nu_{\ell=0,m=0} \sim const.\ \epsilon^2 \sigma \tag{5}$$

for large n where $\sigma = 2\pi\nu/(GM/R^3)^{1/2}$. The rotational contamination of the small separation increases linearly with σ hence with the radial order n. This linear increase should be easily identified. With a suitable frequency combinations of splitting components, it is possible to remove this contamination and to recover a small separation free of rotational perturbances.

The rotation rate is also recovered up to 0.01 μHz with the splittings defined for instance as:

$$\frac{\nu_{n,\ell,m} - \nu_{n,\ell,-m}}{2m} \tag{6}$$

3.2.2. Delta Scuti stars

For a 1.8 $M_\odot$ star, a typical rotational velocity of $v \sim 70 - 150$ km/s corresponds to a moderate rotation for modes in the vicinity of the fundamental radial mode.

Figure 2 compares the frequency distributions for a typical model of a δ Scuti star, when rotation is and is not taken into account. The rotation is quite moderate in this exemple, corresponding to a rotational velocity $v = 100$ km/s. The frequencies of the centroid modes $\nu_{l=0,m=0}$ and $\nu_{l=2,m=0}$ are significantly shifted from the values of the frequencies of the same modes in absence of rotation. This illustrates that period ratios cannot be trusted for mode identification without a careful look at the models. Cubic order contributions are large i.e ~ 0.1 to 1μHz and must be taken into account. Their effects can be seen on the $m \neq 0$ components of the $\ell = 2$ multiplets.

Here again, suitable combinations of frequencies can help to remove undesirable contaminations from high order contributions and to recover the rotation profile.

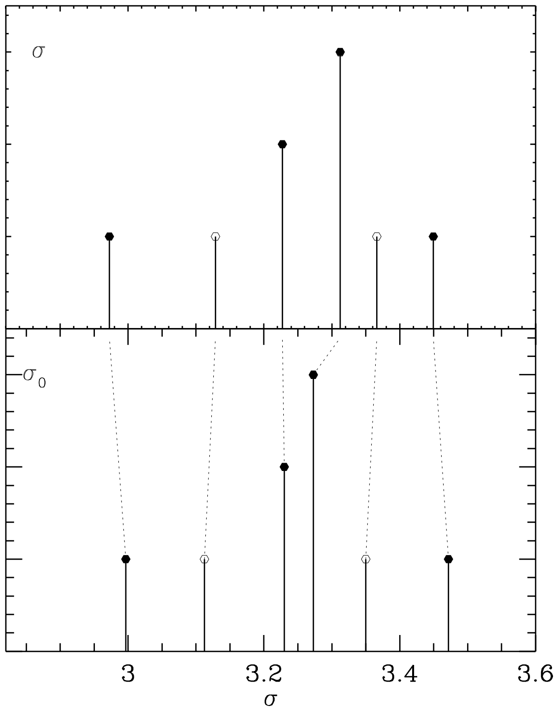

Fig. 2.— Effect of a moderate rotation on the frequency distribution for a typical δ Scuti model (1.8 $M_{\odot}$, $logT_{eff} = 3.8760$, $R = 2.125$). **Bottom**: with no rotation **Top**: with a rotation rate corresponding to a rotational velocity of 100 km/s. Frequencies are in normalized units $\sigma = 2\pi\nu/(GM/R^3)^{1/2}$ and amplitudes are arbitrary. The highest peak is the $\ell = 2, m = 0$ mode, the intermediate one is the $\ell = 0$ mode and the shortest modes are the $m \neq 0$ components of the $\ell = 2$ multiplets The dashed lines join modes with same n, ℓ, m numbers. (*adapted from Goupil* et al. *2001)*

4. Fast rotation:

Methods for computing an equilibrium configuration including the effect of the centrifugal force have been developped for a polytrope by Monaghan & Roxburgh (1965), for more realistic stellar models by Roxburgh *et al.* (1965), Faulkner *et al.* (1968); in the case of a conservative rotational potential by Kippenhahn & Thomas (1970) and used by Claret & Gimenez (1993) and Claret (1999). More recently, 2D numerical codes have been developped in order to compute equilibrium configurations (Deupree, 1990, 1995, 1998; Clement 1994). The next step will be to study the oscillations of such 'fast rotating models' (see Scuflaire, these proceedings). This could be carried out with the help of a variational formulation (Lynden-Bell & Ostriker 1967; Clement, 1989).

A non perturbative approach uses an expansion with an infinite series of spherical harmonics (with different ℓ) which in practice is truncated at some level. The series has been limited to a coupling of 2 to 4 spherical harmonics for low degree modes by Lee & Saio (1986), Lee (1993), Lee & Baraffe (1995) who studied a uniform rotation and neglected the centrifugal effect of rotation on the evolution of the structure. In a series of papers, Clement (1981, 1984, 1986) worked out such an approach with a coupling of about 10 spherical harmonics, neglecting the centrigugal perturbation.

High radial order g- and gravito-inertial modes have been studied by Berthomieu *et al.* (1978), Unno *et al.* (1989), Clement (1981, 1989). Dintrans & Rieutord M. (2000) have recently developped a nonperturbative ray method which takes into account the Coriolis force and neglects the centrifugal force in view of studying γ Dor stars (cf Rieutord these proceedings)

Finally, it must be mentionned some attempts to develope numerical 2D computations for generating stellar oscillations: Ipser & Lindblom (1990, 1991), Clement (1998).

5. Others effects of rotation

There is a number of other issues that must be dealt with, before a useful seismic study can properly be carried out.

5.1. Influence of rotation on the emergent flux :

Because the star is distorted, rotation affects the emergent flux and therefore the determination of the stellar parameters of the star. Methods for computing the emergent flux from a rapidly rotating star were developped by Collins (1963), Roxburgh & Strittmatter (1965), Maeder & Peytremann (1970). Numerical calculations based on such methods in the case of δ Scuti stars have been performed by Michel *et al.* (1999), Perez-Hernandez *et al.* (1999) (cf Suarez et al., these proceedings).

Claret (2000) investigates the gravity darkening effect, i.e. the rotation dependent relation between T_{eff} et surface gravity g.

Even with equal intrinsic amplitudes, integration of the emergent flux over the stellar disc yields different values for the amplitudes of modes with different degree ℓ (visibility effect) (Dziembowski, 1977; Pesnell, 1985). Methods based on this property have been proposed for determining the degree ℓ of observed modes (mode identification) independently of the angle of the line of sight (Dziembowski, 1977; Watson, 1988; Garrido, 2000). This requires high quality atmosphere models (Kupka *et al.*, Barban *et al.*, Garrido et al. works currently in progress)

These methods lie on the assumption that the mode can be written with a single spherical harmonic. This is no longer true for moderate rotators for which visibility effects of oscillation modes depend on the angle of the line of sight (Balona, Dziembowski, Goupil, unpub.)

5.2. Influence of rotation on modal selection:

Pulsational stability, taking into account a slow or moderate rotation, have been investigated in specific stellar cases by Hansen *et al.* (1978); Caroll & Hansen, (1982); Lee & Baraffe (1995), Lee (1998), Saio *et al.* (2000).

Theoretical works on nonlinear resonant coupling, and in particular the role of rotation in saturating the oscillation amplitude growth, were performed by Dziembowski & Krolikowska (1985), Buchler *et al.* (1995), Buchler *et al.* (1997), Goupil *et al.* (1998).

6. Going further

A rotation, which is faster in the inner layers, than in the outer layers as is probably the case for a δ Scuti star, is expected to yield an even more complicated frequency distribution that in the case of uniform rotation. Indeed, low frequency modes (g-modes or mixed) are less sensitive to the centrigugal distorsion that higher frequency modes which have amplitudes only in the outer layers. However, frequencies of low frequency modes will be as much or even more affected if the rotation rate is larger in the inner part of the star. This remains to be confirmed and quantified (Suarez *et al.* in progress).

Once the high order rotational effects are identified, the strategy is to find appropriate combinations of frequencies so as to remove first the high order contamination, then to invert these frequency combinations for the sound speed and the rotation profile.

Work on visibility effects and mode identification for a moderate or fast rotating pulsating star must be completed.

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