

SIMU-LC : A LIGHT-CURVE SIMULATOR FOR COROT

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ABSTRACT

In order to prepare the analysis of COROT data, it has been decided to build a simple tool to simulate the expected light curves. This simulation tool takes into account both instrumental constraints and astrophysical inputs for the COROT targets. For example, granulation and magnetic activity signatures are simulated, as well as p modes, with the expected photon noise. However, the simulations rely sometimes on simple approach of these phenomenons, as the main goal of this tool is to prepare the analysis in the case of COROT data and not to perform the most realistic simulations of the different phenomenons.

Key words: Data analysis, simulation, p modes, granulation, magnetic activity, photon noise

2. PHOTON NOISE

The photon noise is certainly the easiest component of the noise to simulate... as far as it has the expected behaviour: a true white noise with a variance depending on the photon counts. The COROT specifications impose a level of photon noise of $B_0 = 0.6$ ppm in the amplitude spectrum of a star of magnitude $m_0 = 5.7$ for an observing duration of 5 days. The stellar flux for a given magnitude m being related to the stellar magnitude by:

$$F = F_0 10^{(m_0 - m)/2.5} \quad (1)$$

and knowing that the level of noise varies as the square root of the flux, the level of noise is related to the star magnitude by:

$$B = B_0 10^{(m_0 - m)/5}. \quad (2)$$

As indicated above, this simple relation can model the photon noise but not some other photon counting perturbations as slow (periods of hours or more) trends in photon countings which might contribute to the low frequency spectrum of the noise.

1. INTRODUCTION

Simulating the data that a space instrument like COROT will provide might look presumptuous. Indeed, it is certainly, when comparing to previous comparable instruments like IPHIR or GOLF. These two examples show that the nominal behaviour of the instrument is not always reached, but this does not prevent this instrument to provide very interesting data. However, despite some technical problems, IPHIR and GOLF yielded a wealth of scientific results. Thus, what is the interest of simulating COROT data? How close to reality these simulations will get? This might not be the most important fact as the preparation of these simulations will help us to prepare the analysis of real data and to be ready in case of unexpected technical behaviour of the instrument perturbing the data, or unexpected physical behaviour of the targets of the instrument. A consequence of that is that the simulation tool must include technical and physical aspects, making the task even more difficult. These aspects cover: photon noise, p modes excitation, granulation signal, stellar activity signal, orbital perturbations, stellar rotation...

The software presented here is freely available at:
www.lesia.obspm.fr/~corotswg/simulightcurve.html

3. SOLAR-LIKE OSCILLATIONS

The solar-like oscillations are stochastically excited and simulated here following the recipe of Anderson et al. (1990), recalled below. Each solar-like oscillation is a superposition of a large number of excited and damped proper modes. Each solar-like oscillation can then be decomposed as:

$$\sum_j A_j \exp[-2i\pi\nu_0 t] \exp[-\eta(t - t_j)] H(t - t_j) + \text{c.c} \quad (3)$$

where A_j is the amplitude at which the mode j with proper frequency ν_0 is excited by turbulent convection, t_j at which it is excited, η is the (linear) mode damping rate, H is the Heaviside function and "c.c." means complex conjugate. The Fourier transform of Eq.3 yields the spectrum:

$$f(\nu) \simeq \frac{U}{1 + 2i(\nu - \nu_0)/\Gamma} \quad (4)$$

whith $\Gamma = \eta/\pi$, $U \equiv \sum_j \tilde{A}_j$ and \tilde{A}_j is a complex number proportional to $A_j \exp[i t_j]$.

As the excitations are random, t_j is random and hence \tilde{A}_j has a random phase. As the excitation are very numerous, according to the central limit principle the real and

imaginary parts of the complex number U are distributed according to a normal statistics. Hence the spectrum of the oscillations, $f(\nu)$, can be simulated by generating the real and imaginary parts of the complex number U according to a normal statistics. An inverse Fourier transform is next applied in order to simulate the oscillations in the time domain. This is the principle of the Anderson et al. (1990)'s recipe.

The question is now that is the constraints on the mean and variance of U . From Eq. 4 we deduce the power spectrum $P(\nu)$ of the oscillation :

$$P(\nu) = \frac{\|U\|^2}{1 + (2(\nu - \nu_0)/\Gamma)^2} \quad (5)$$

The mean mode profile $\langle P \rangle$ is obtained by averaging Eq. 5 over a larger number of realizations:

$$\langle P(\nu) \rangle = \frac{H}{1 + (2(\nu - \nu_0)/\Gamma)^2} \quad (6)$$

where $H \equiv \langle \|U\|^2 \rangle$, which is by definition the variance of the complex number U since $\langle U \rangle = 0$. We see from Eq. 6 that the mean profile has a Lorentzian shape as it is – in first approximation – observed.

According to the Parseval-Plancherel relation, H is related to the mean square of the mode intrinsic amplitude in terms of luminosity fluctuations, A_L^2 , as (see eg. Baudin et al. 2005):

$$H = \frac{C_\ell^2 A_L^2}{\pi \Gamma} \quad (7)$$

where C_ℓ is a visibility factors, which depends on the degree ℓ . Once the mode intrinsic amplitude A_L , the mode line width Γ and C_ℓ are given, we have then a constraint on the variance of U .

Note that the derivation of Eqs. 4-7 assumes that the mode life-time ($\sim \eta^{-1}$) is much shorter than the duration of the observation (T_0), that is $T_0 \eta \gg 1$. For life-times much longer than T_0 , different expressions are derived but the principle of the simulation remains the same.

For sake of simplicity, we use the adiabatic assumption formulated by Kjeldsen & Bedding (1995) to deduce the maximum of A_L from the maximum of the root mean square mode velocity V_{\max} according to:

$$(A_L)_{\max} = (A_L)_{\odot, \max} \frac{V_{\max}}{V_{\odot, \max}} \sqrt{\frac{T_{\odot, \text{eff}}}{T_{\text{eff}}}} \quad (8)$$

where T_{eff} is the effective temperature and the symbol \odot refers to quantities related to the Sun. We take for the Sun the *rms values* $(A_L)_{\odot, \max} \simeq 4$ ppm (see table 2 in Kjeldsen & Bedding (1995) and more recently Barban et al. (2004)) and $V_{\odot, \max} \simeq 27$ cm/s according to Chaplin et al. (1998)'s seismic observations.

In turn the root mean square of the mode intrinsic velocity, V , is related to the rate \mathcal{P} at which energy is injected into the mode by turbulent convection and the mode damping rate $\eta = \pi \Gamma$ as:

$$V^2 = \frac{\mathcal{P}}{2 \eta \mathcal{M}} \quad (9)$$

where $\mathcal{M} \equiv I/\xi_r^2(h)$ is the mode mass, I the mode inertia, ξ_r the mode radial displacement and h the height above the photosphere where oscillations are measured (we consider $h = 0$). The way the quantities involved in Eq. 9 are model is explained in the next two Sects.

3.1. MODE EXCITATION AND DAMPING RATES

Theoretical mode damping rates are obtained from the tables calculated by Houdek et al. (1999). These calculations rely on the non-local and time-dependent formulation of the convection of Gough (1977, 1976) (for more details, see Houdek et al. 1999).

The computation of the excitation rates \mathcal{P} is performed according to the model of stochastic excitation of Samadi & Goupil (2001). The calculations assume – as in Samadi et al. (2003a) – a Lorentzian function for modelling the convective eddy time-correlations. Furthermore, the characteristic wavenumber k_0 involved in the theory is assumed to be constant according to the simplification proposed by Samadi et al. (2003b). Its value is related to the value $k_{0, \odot} \simeq 3.6 \text{ Mm}^{-1}$ inferred in Samadi et al. (2003b) from a 3D simulation of the Sun as: $k_0 = k_{0, \odot} \Lambda_{\odot}/\Lambda$ where Λ is the mixing-length evaluated at the layer the convective velocity is maximum.

3.2. MODE MASS AND MODE FREQUENCY CALCULATION

The solar model we consider is calculated with the CE-SAM code (Morel 1997) and appropriate input physics, described in details in Lebreton et al. (1999). In particular, convection is modelled according to the classical mixing-length theory (Böhm - Vitense 1958, hereafter MLT) with a mixing-length $\Lambda \equiv \alpha H_p$ where H_p is the pressure scale height and α_c is the mixing-length parameter. In contrast with Lebreton et al. (1999), the atmosphere is restored from the Eddington classical gray atmosphere and microscopic diffusion is included according to the simplified formalism of Michaud & Proffitt (1993). The calibration of the solar model, in luminosity and radius for an age of 4.65 Gyr, fixes the initial helium content $Y = 0.2751$, metallicity $Z = 0.0196$ and the MLT parameter $\alpha_c = 1.76$.

The oscillation eigenfunctions and hence the mode masses, \mathcal{M} , in Eq. 9 are calculated with the adiabatic ADIPLS pulsation code (Christensen-Dalsgaard et al. 1996).

4. STELLAR GRANULATION SIGNAL

Granulation can be considered as noise if the aim of the observation is the stellar oscillations, but it carries some information about the physics of the star, and very valuable information as the convection is a very badly described phenomenon in stellar modelling. Thus, we call it a *signal* and not a noise.

However, granulation can be described only with a statistical approach. Moreover, its contribution in the Fourier

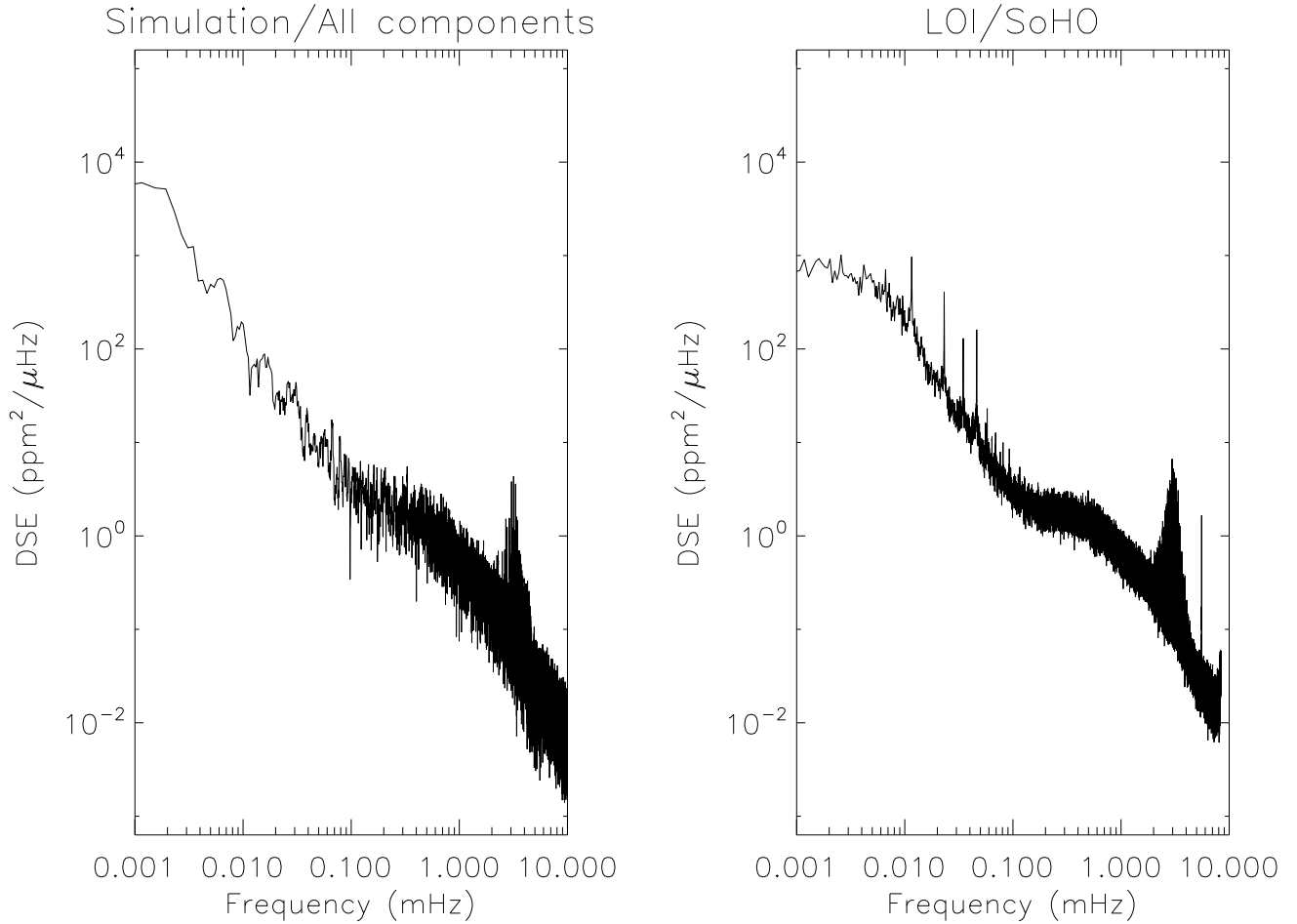


Figure 1. Left: Fourier spectrum of the output of the light curve simulator described here, in the case of the Sun; Right: Fourier spectra of observation of the Sun obtained with the photometer LOI onboard SoHO (the high peaks at low frequency are daily aliases due to SoHO synoptic observations performed every 24 hours). This comparison shows a reasonably good agreement, except maybe for the activity signature

domain is not independent of frequency: it will contribute more at low frequency. A common description is to consider that signal S as a random signal with some memory: then, its autocorrelation function is:

$$ACF_S(t) = A^2 e^{-|t|/\tau} \quad (10)$$

where A is the amplitude of this signal (to be related to its variance) and τ a characteristic time. Knowing that the Fourier transform of the autocorrelation function is the squared modulus of the function S , one has:

$$\|S(\nu)\| = \frac{2A^2\tau}{1 + (2\pi\tau\nu)^2}. \quad (11)$$

Thus, the Fourier spectrum of the granulation is modelled as a Lorentzian which, if it is summed over all frequencies, yields the variance of the signal and then its relation with A :

$$\sigma^2 = \int \|S(\nu)\| \Rightarrow \sigma = A/\sqrt{2}. \quad (12)$$

So, knowing the intensity standard deviation due to granulation and its characteristic time, it is possible to model

its Fourier spectrum (and considering its phase is random, it is possible to model the corresponding time series by an inverse Fourier transform). This is in fact the approach developed by Harvey (1985) to model the solar noise spectrum, including granulation, mesogranulation and supergranulation. It can be used to describe any noise with memory (some electronic noise for example).

In the case of granulation, the required parameters are estimated from theory of convection. The eddy size Λ_{eddy} is assumed to be equal to $\Lambda = \alpha H_p$ where α is the mixing-length parameter and H_p the pressure scale height. The eddy overturn time is related to the eddy convective velocity, v , by $\tau_{\text{eddy}} = \Lambda_{\text{eddy}}/v$ and is considered as the characteristic time of granulation τ . The number of number of eddies seen at the star surface, N_{eddy} , is $2\pi(R_*/\Lambda_{\text{eddy}})^2$ where R_* is the star radius. This relation ignores of course that the medium is highly anisotropic. According to the mixing-length theory (see eg. Cox 1968), the eddies contrast (border/center of the granule), $(\delta L/L)_{\text{eddy}}$, can be

related to the difference between the temperature gradient of the eddy, ∇' and that of the surrounding medium, ∇ , according to the relation:

$$(\delta L/L)_{\text{eddy}} = \frac{\nabla - \nabla'}{\nabla} \quad (13)$$

In turn, Eq. 13 can be reduced to:

$$(\delta L/L)_{\text{eddy}} = (4/9) \frac{\lambda}{(1 - \lambda)} \frac{1}{\gamma} \quad (14)$$

where γ is the convection efficiency and λ the ratio between the convective flux and the total flux of energy. This relation is finally calibrated in order to match the solar constraints, and the intensity standard deviation for the whole observed disc is

$$\sigma = (\delta L/L)_{\text{eddy}} / \sqrt{N_{\text{eddy}}} \quad (15)$$

All the quantities are obtained on 1D stellar models computed as explained in Sect. 3.

5. STELLAR ACTIVITY SIGNAL

The stellar magnetic activity will induce intensity variation in time, mainly due to the presence of starspots crossing the observed disc because of the rotation. However, some other sources of intensity variations are expected (flares for example).

A first approach is to consider the intensity variations as described by the same way than granulation. Knowing the standard deviation and the characteristic time of intensity variations allows to build the Fourier spectrum of these variations as a Lorentzian. This approach has been used for example by Aigrain et al. (2004). The difficulty in the case of magnetic activity is that there is no theoretical description of the phenomenon. Thus, the parameters describing it are empirically derived.

The characteristic time is taken as the period of rotation of the star, or, if the rotation is slow, the intrinsic lifetime of a spot. The latter is arbitrarily chosen as the one in the solar case, computed by Aigrain et al. (2004). The rotation period is computed from an empirical law involving the age and the $B - V$ color index of the star described in the same reference.

The standard deviation of intensity variations due to magnetic activity is also derived from an empirical law. However, this law involves the Rossby number R_0 (ratio of rotation period P_{rot} and the overturn convective time at the base of the convection zone τ_{bcz}). This number can be derived empirically (Aigrain et al. 2004), but in the present simulations, we derive τ_{bcz} from models. Then we use empirical laws to estimate σ .

Another approach is to simulate in the intensity time series the influence of individual spots, to estimate their number and contrast (as for example Lanza et al. (2004)). The expected result in the Fourier spectrum should be similar to the first approach, but this detailed simulation should allow for rotation measurements. This approach will be included in a further version of our simulation software.

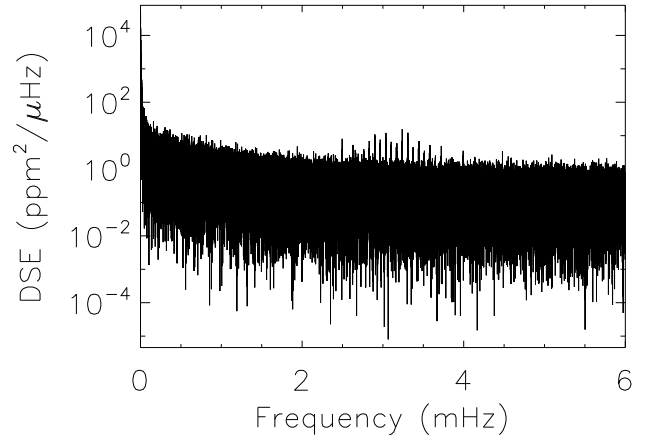


Figure 2. Fourier spectra of the output of the simulator described here, in the case of a Sun seen with a magnitude $m = 6$, showing that the p modes are easily detected, as well as granulation and activity

6. EXAMPLES

A first example of the output of this software is shown in Fig.1: a simulation of a Sun with a magnitude $m = 0$ (in order to have a very weak photon noise) is compared with a spectrum from the LOI photometer (Appourchaux et al. 1997) onboard SoHO. The agreement between simulation and observation is good, except maybe at very low frequency: the activity component of the signal is overestimated in the simulation (this is explained as the Sun does not fit very well the empirical law used for activity estimation).

The following example is a sun with a magnitude $m = 6$, showing that both p modes and activity should be visible with COROT for such a star (Fig.2). Another example is shown in Fig.3 for a star of 1.5 solar mass and an age of 2.4 Gyr: again, p modes and activity are detectable.

7. CONCLUSION

This simulator software will continue to evolve with time. As indicated above, intensity modulation due to starspots will be included, as well as other stellar or instrumental signals, as for example instrumental perturbations due to orbital variations. Moreover, this effort of simulation will not end with the delivery of first data but will be continued after that. The comparison with real data will allow to check for the validity of physical hypothesis used to simulate the different signals of astrophysics origin in the data. This should bring a great amount of information on our knowledge of these often not well known phenomena, which stellar simulation is often derived from the solar case. In parallel, the simulation of instrumental components of the signal will be improved to help the interpre-

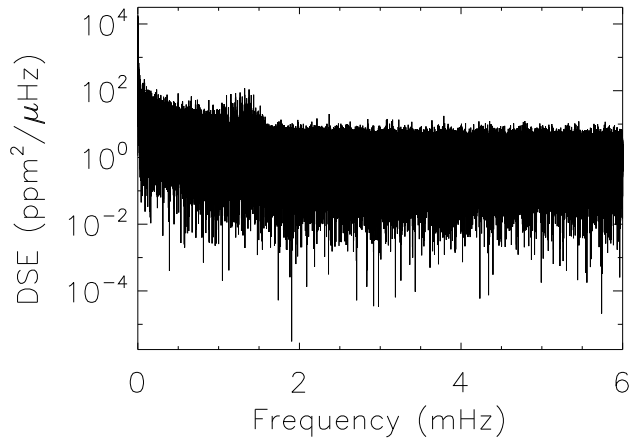


Figure 3. Same than Fig. 2 for a star with a mass $M = 1.5M_{\odot}$ and a magnitude $m = 8$, showing also p modes, granulation and activity signatures

tation of real data. All these reasons justify in our opinion the need for the simulation tool presented here.

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