Fluid modeling for the slow dynamics of collisionless space plasmas

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OUTLINE

Space plasmas: Main features and some debated questions

How to model collisionless plasmas ?

Landau fluid models

- The various models and their capability of describing linear waves
- 3D simulations of meso-scale turbulence : role of ion Landau damping
- 1D simulation of non-resonant ion heating and constraining effect of the mirror instability

Conclusions

Space plasmas: Main features and debated questions

Space plasmas are magnetized and turbulent

 $\beta \approx 1, M_{s} \approx 1$

Fluctuations: power-law spectra extend to ion gyroscale and below

Dispersive and kinetic effects cannot be ignored.

Presence of coherent structures (filaments, shocklets, magnetosonic solitons, magnetic holes) with typical scales of a few ion Larmor radii.

Among the debated questions:

The concepts of waves make sense even in the strong turbulence regime.

- Spectral energy distribution and its anisotropy
- Dissipation mechanisms
- Heating of the plasma: temperature anisotropy and resulting micro-instabilities
- Particle acceleration



How to model collisionless plasmas ?

Solar wind is almost non collisional

Vlasov-Maxwell simulations: hardly possible on the present day computers in three space dimensions (6 variables + time, and **a broad range of time scales**).

Gyrokinetics (*Howes, ApJ* **651**, 2006, *Schekochihin et al., ApJ Supp.,* **182**, 310, 2009), concentrates on the quasi-transverse dynamics and averages out the fast waves. (Applicability to space plasmas still to be validated).

Gyrokinetic simulations (*G. Howes, PoP* **15**, 055904, 2008; *PRL* **107**, 035004, 2011) are now feasible and show the presence of cascades both in the physical and velocity spaces in the range $k_{\perp}p\geq 1$, but they remain challenging numerically and difficult to interpret.

One needs a fluid model that

- retains low-frequency kinetic effects: Landau damping and FLR corrections
- can be integrated relatively fast,
- allows for strong temperature anisotropies
- does not a priori order out the fast magnetosonic waves.

Question: Can kinetic effects be ignored at large scales?

Fluid description retaining low-frequency kinetic effects: Landau fluid models

- Introduced by *Hammett & Perkins (PRL 64, 3019, 1990)* as a closure retaining linear Landau damping.
- Applied to large-scale MHD by Snyder, Hammett & Dorland (PoP 4, 3974, 1997) to close the hierarchy of moment equations derived from the drift kinetic equation:
 Large-scale (LS) Landau fluids: applied to the Magneto-Rotational Instability Quataert et al., ApJ 577, 524 (2002), Sharma et al., ApJ 596, 1121 (2003); ApJ 637, 952 (2006).
- Extended to dispersive MHD with Hall effect and large-scale FLR corrections (Passot & Sulem, PoP 10, 3906, 2003; Goswami, Passot & Sulem, PoP 12, 102109, 2005; Passot, Sulem & Hunana, PoP, 19, 082113, 2012):
 Meso-scale (MS) Landau fluids.
- Inclusion of quasi-transverse scales extending beyond the ion gyroscale, under the gyrokinetic scaling
 (Passot & Sulem, PoP 14, 082502, 2007; Passot, Sulem, Hunana, PoP 19, 082113, 2012):
 FLR-Landau fluids.

In contrast with gyrokinetics, Landau fluids retain fast waves that are accurately described, except close to resonances.

Landau-fluid can describe regimes with strong temperature anisotropy.

Landau fluids

For the sake of simplicity: neglect electron inertia, use quasi-neutrality.

Ion dynamics: derived by computing velocity moments from Vlasov Maxwell equations.

$$\begin{cases} \text{Decompose the ion pressure as:} \\ \mathbf{p}_p = p_{\perp p} \mathbf{n} + p_{\parallel p} \boldsymbol{\tau} + \prod, \text{ with } \mathbf{n} = \mathbf{I} - \widehat{b} \otimes \widehat{b} \text{ and } \boldsymbol{\tau} = \widehat{b} \otimes \widehat{b}, \text{ where } \widehat{b} = \mathbf{B} / |\mathbf{B}|. \\ & \uparrow \\ & \mathsf{FLR \ corrections} \end{cases}$$

Electron pressure tensor is taken gyrotropic (considered scales >> electron Larmor radius)

Use exact equations for the gyrotropic pressures (that include heating/cooling due to the work of the non-gyrotropic pressure forces).

Simplifications are introduced at the level of the heat flux equations (see Ramos PoP 12, 052102 (2005) for a set of nonlinear equations that nevertheless still assumes an ordering)

Terms that involve the non-gyrotropic pressure and heat fluxes are kept only when they appear linearly.

The completion of this type of fluid model requires the determination of:

(1) closure relations to express the 4th-rank cumulants $\tilde{r}_{\parallel\parallel}$, $\tilde{r}_{\parallel\perp}$, $\tilde{r}_{\perp\perp}$ (closure at lower or higher order also possible)

Only issue when dealing with the Large-Scale Landau fluid model (Snyder, Hammett & Dorland, PoP 4, 3974, 1997).

(2) (non gyrotropic) FLR corrections to the various moments.

(1) Brief description of the hierarchy closure

The 4th-rank cumulants are obtained from the linearized kinetic theory, assuming small frequencies with respect to the ion gyrofrequency.

This requires either long wavelengths with respect to the ion gyroradius or quasi-perpendicular directions.

IN PRACTICE:

The kinetic expressions typically depend on electromagnetic field components and involve the plasma dispersion function (which is nonlocal both in space and time).

These various expressions are expressed in terms of other fluid moments in such a way as to minimize the occurrence of the plasma dispersion function.

The latter is otherwise replaced by suitable Padé approximants, thus leading to local-in-time expressions. At some places, a Hilbert transform with respect to the longitudinal space coordinate appears, that modelizes Landau damping.

This procedure ensures consistency with the low-frequency linear kinetic theory, up to the use of Padé approximants.

$$\widetilde{r}_{\parallel \perp} = \frac{p_{\perp}^{(0)2}}{\rho^{(0)}} [1 - R(\zeta) + 2\zeta^2 R(\zeta)] \Bigg\{ [2b\Gamma_0(b) - \Gamma_0(b)$$

 $-2b\Gamma_1(b)\Big]\frac{b_z}{B_0}+b[\Gamma_0(b)-\Gamma_1(b)]\frac{e\Psi}{T_\perp^{(0)}}\bigg\}$

For example, from kinetic theory

leads to, in the LF model:

$$\widetilde{r}_{\parallel \perp p} = -\frac{\sqrt{\pi}}{2} v_{\mathrm{th}\parallel p} \mathcal{H} \Big[q_{\perp p} + \frac{1}{\Omega_p} \times \frac{\overline{p}_{\perp p}}{\rho_0} (\overline{p}_{\perp p} - \overline{p}_{\parallel p}) (i \vec{k}_{\perp} \times \frac{\vec{b}_{\perp}}{B_0})_z \Big]$$

Two methods to determine the non-gyrotropic elements of the tensors

First method

Solve the (coupled) algebraic equations that result from the projection of the tensorial pressure equations, orthogonally to the gyrotropic "directions".

To obtain an explicit solution, this procedure requires an expansion in a small parameter, usually taken as the time and space scale separation with the ion gyroscales.

Leads to the Meso-Scale Landau fluid model.

This approach has the great advantage of being fully nonlinear. Its algebraic complexity, however precludes an easy numerical implementation. This is even more cumbersome at the level of the heat flux which enters this formula.

A second order solution was explicited in a linear setting in: Goswami, Passot & Sulem, PoP **12**, 102109 (2005).

It is shown that all non-gyrotropic contributions are necessary to reproduce the linear growth rate of the mirror instability, and in particular the restabilization at small scales. (*Passot, Sulem & Hunana, PoP* **19**, 082113 (2012))

Note that the mirror-instability threshold is captured by LS Landau fluids (including only Landau damping).



Mirror growth rate:

In the case of cold electrons, an asymptotic linear analysis near threshold from the MS-Landau fluid model leads to



In the fluid framework, its correct determination requires the heat flux contribution σ .

The growth rate then identifies with that of the kinetic theory.

Contributions originating from the gyroviscosity cancel out. The system is stabilized at small scales by the nongyrotropic correction R_{NG} to the fourth-rank cumulants.

In the case of warm (and possibly anisotropic) electrons, comparison with the low-frequency kinetic theory (Kuznetsov, Passot & Sulem, PoP, in press)



This Meso-Scale Landau fluid model, nevertheless leads to spurious instability (beyond its range of validity) for KAWs when temperature anisotropy is too large.



Although the instability occurs beyond the spectral validity range of the model, such unstable scales are usually present in simulations not limited to the largest MHD scales.

In such a regime, an accurate description of the small scales is required, at least at a linear level.



The other possiblity to determine FLR contributions is to use the linear kinetic theory in the low-frequency limit (in a way similar to what is done for the closure of the hierarchy) : $\omega/\Omega \sim \epsilon \ll 1$

In this case, the expansion is valid for:

• quasi-transverse fluctuations $(k_{\parallel}/k_{\perp} \sim \epsilon)$ with $k_{\perp}r_L \sim 1$

• hydrodynamic scales with $k_{\parallel}r_L \sim k_{\perp}r_L \sim \epsilon$.

 r_{I} : ion Larmor radius



No need of an arbitrary truncation: all linearized fluid equations are satisfied when plugging the fluid moments directly calculated from the LF kinetic theory, except the perpendicular velocity equation: it reduces to the perpendicular pressure balance condition, as in gyrokinetics.

• The model conserves the total energy:

$$E = \int \Big[\frac{\rho |u|^2}{2} + \frac{|b|^2}{2} + \beta_{\parallel i} \Big(p_{\perp i} + p_{\perp e} + \frac{1}{2} (p_{\parallel i} + p_{\parallel e}) \Big) \Big] d\mathbf{x}$$

Conservation of energy is independent of the heat fluxes and subsequent equations, but requires retaining the work done by the FLR stress forces.

- Implementation of the Landau damping via Hilbert transforms, and also of the FLR coefficients as Bessel functions of $k_{\perp}\rho$, is easy in a spectral code.
- Electron Landau damping is an essential ingredient in many cases (limiting the range of validity of isothermal electrons often used in hybrid simulations).
- All linearized fluid equations are satisfied when plugging the fluid moments directly calculated from the LF kinetic theory, except the perpendicular velocity equation: it reduces to the perpendicular pressure balance condition, as in gyrokinetics.
- Possibility of including weak collisions (Gross and Krook 1956, Bhatnagar 1962, Green 1973) in a form that preserves energy conservation.



Frequency and damping rate of Alfvén waves:



FIG. 2: Normalized frequency $\omega_r/(k_{\parallel}v_A)$ (left) and damping rate $-\omega_i/(k_{\parallel}v_A)$ (right) for KAWs with $\theta = \tan^{-1}(1000)$, $\tau = 1$, versus $k_{\perp}r_L$ for $\beta = 0.1$ (top), $\beta = 1$ (middle), $\beta = 10$ (bottom). $\theta = 89.9^{\circ}$



Does not capture

FIG. 3: Normalized frequency $\omega_r/(k_{\parallel}v_A)$ (left) and damping rate $-\omega_i/(k_{\parallel}v_A)$ (right) for KAWs with $\tau = 0.01$, $\beta = 1$ versus $k_{\perp}r_L$ for $\theta = \tan^{-1}(10)$ (top), $\theta = 60^o$ (bottom).



Meso-Scale Landau fluid is correct up $k_{\perp} r_{L} \approx 1$

Comparison FLR-Landau fluid with full kinetics

For large β and angles close to 90°,

the frequency can exceed the ion gyrofrequency without encountering resonance. In this case, the FLR-Landau fluid remains valid.



 θ =80° (red), θ =83° (green), θ =86° (blue), θ =89° (magenta)

Comparison FLR-Landau fluid (crosses) with full kinetics (continuous line)

The isothermal electron equation of state leads to drastically different results already when $k_{\perp}\rho$ <1



magnetic compressibility: $\chi(k_{\perp}r_L) = |B_z(k_{\perp}r_L)|^2 / |B(k_{\perp}r_L)|^2$ electric field polarization: $\mathcal{P} = \operatorname{Arg}(E_y/E_x)/\pi$. $\begin{cases} \mathcal{P} < 0 \\ \mathcal{P} > 0 \end{cases}$ left polarizized wave right polarized wave



Proton beta is 0.1, 0.5, 1, 2, 4, 10

Polytropic bi-fluid : incorrect even at large scales; Landau damping is not sufficient to reproduce kinetic theory. FLR-Landau fluid provides a precise agreement with kinetic theory. (Hunana et al. ApJ, submitted).

Three-dimensional simulations of decaying turbulence: role of ion Landau damping

3D MS-Landau fluid simulations in a turbulent regime

(simplified model) (Hunana, Laveder, Passot, Sulem & Borgogno, ApJ 743, 128, 2011).

Freely decaying turbulence (temperatures remain close to their initial values)

- Isothermal electrons
- Initially:

no temperature anisotropy; equal ion and electron temperatures incompressible velocity.

Pseudo-spectral code Resolution: 128³ (with small scale filtering)

Size of the computational domain: 32π inertial lengths in each direction Initially, energy on the first 4 velocity and magnetic Fourier modes kd_i= m/16 (m=1,...,4) with flat spectra and random phase.

Comparison of MS-Landau fluids and Hall-MHD simulations

Compressibility reduction by Landau damping



FIG. 6: Compressibility for Hall-MHD (red line) and FLR-Landau fluid (blue line) evaluated as $(\sum_{k} |\mathbf{k} \cdot \mathbf{u}_{k}|^{2}/|\mathbf{k}|^{2})/\sum_{k} |\mathbf{u}_{k}|^{2}$ for $\beta_{0} = \beta_{\parallel} = 0.8$. Because of the start with the identical initial condition where the velocity field is divergence free. The figure shows that the compressibility is clearly inhibited in the Landau fluid simulation.

Important in solar wind context: Although solar wind is a fully compressible medium, the turbulent fluctuations behave as is there were weakly compressible.



Strong reduction of the parallel transfer

Frequency analysis:

Hall-MHD

10⁻¹ frequency (ω)



10⁻¹ frequency (ω)

10

Slow waves are strongly damped

Alfvén waves for the propagation angle of 45°

10⁰



 $k_y = 0, k_x d_i = k_z d_i = m/16$, where m = 1 (red), m = 2 (green), m = 4 (blue), m = 6 (black)

Development of temperature anisotropy



Proton magnetic moment versus the heliocentric distance (Marsch, Living Review Solar Phys., 2006)

Non-resonant heating:

Simulation of perpendicular ion heating under the action of given randomly phased KAW with wavelength comparable to the ion Larmor radius on particles for $\beta \lesssim 1$

(Chandran et al. ApJ **720**, 503, 2010 see also Bourouaine, 2008, 2011)



Need for a fully nonlinear approach.

Hybrid simulations of quasi-perpendicular turbulence show preferential perpendicular heating of ions, not directly related to the gyroradius but to a temporal scale (Markovskii & Vazquez, ApJ 739, 22 (2012)).

Ion distribution functions elongate préferentially along or across magnetic field near regions of strong magnetic activity (current sheets) in 2D hybrid simulations (Servidio et al. PRL 108, 045001 (2012)).

> **Goal**: Study (nonresonant) heating due to KAWs within LF simulations and identify its physical origin

Parameters of the 1D FLR-Landau fluid simulations:

- Angle of propagation: 80° with respect to the ambient magnetic field
- White noise in time random driving around k_{inj}, applied on the perpendicular velocity component (u_y) each time the sum of kinetic and magnetic energy falls below a given threshold: it is intended to simulate the injection of energy from the end of the solar wind Alfvén wave cascade.

Resulting root mean square of the transverse magnetic field fluctuations is of the order of 0.12 times the magnitude of the ambient field (realistic for the solar wind)

- Isotropic initial temperatures; various parallel proton β .
- Size of the domain L measured in units of ion inertial length
- Number of grid points: typically N=256 (after partial desaliazing).
- No artificial dissipation is added.

Laveder, Marradi, Passot & Sulem, Geophys. Res. Lett. 38, L17108 (1974)



Fixing $k_{ini}/k_o = 0.087$ (relatively small scale) and varying β (thus changing the domain size)

Time variation of the (space averaged) ion temperatures

Green: β = 1.2, L=16 π

Perpendicular heating and (early time) parallel cooling of ions,

with a larger efficiency as β is reduced

(in agreement with simulations of the action of prescribed KAW, Chandran et al. ApJ 2010)

With injection at larger scales, there is a critical value of β , below which parallel ion heating and above which perpendicular heating dominates.

Efficient parallel electron heating at small β



Parallel electron temperature (same conditions)



Perpendicular heating

Both parallel and perpendicular heat fluxes contribute to the variation of $\frac{|b|^2 p_{\parallel}}{a^2}$

Comparison with solar wind data

A large majority of the observational measurements in the case of a predominant ion perpendicular heating are limited from above by the curve

 $T_{\perp i}/T_{\parallel i} - 1 - a/(\beta_{\parallel i} - \beta_0)^b = 0$

$$a = 0.77, b = 0.76$$
 and $\beta_0 = -0.016$

that fits the contour associated with the growth rate $\gamma = 10^{-3}\Omega_{i}$ of the mirror instability in linear kinetic computations assuming bi-Maxwellian ions and isothermal electrons.



FLR-Landau fluid simulations (with weak collisions in the form of a BGK operator):



Equal parallel and perpendicular electron temperatures.



With collisions, points characterizing the stateof the system follow the threshold curve

Summary

MS and FLR Landau fluids suitable for plasma dynamics at the ion gyroscale.

They retain

hydrodynamic nonlinearities + linear approximation of low-frequency kinetic effects (Landau damping & FLR corrections)

Consistent with the quasi-transverse character of the turbulent cascade. Suitable for simulations of the solar wind turbulence, especially with small wave amplitude (for which PIC simulations are not suited) and in absence of resonance.

Landau damping: - depletes compressible effects and inhibits longitudinal transfer - leads to a correct description of the mirror instability threshold.

FLR corrections: - arrest of mirror instability at small scales

- affect KAWs and magnetosonic dispersion relations and polarization even at MHD scales.

A sufficiently accurate description of the FLR is needed to prevent small-scale spurious instabilities.

FLR-Landau fluid accurately reproduces dispersion relation of (quasi-transverse) kinetic Alfvén waves and mirror instability at all the scales.

The FLR-Landau fluid model allows for a study of heating due to Alfvén wave turbulence. (Simulations are possible without addition of artificial dissipation, at least in 1D).

Main results :

- Perpendicular ion heating is possible in nonresonant situation with KAW driving, when turbulence typical scales are close enough to the ion gyroradius. It generates a self-regulated state where mirror instability constrains the growth of the temperature anisotropy.
- At constant injection scale, heating efficiency increases as β decreases.
- With injection at larger scales, heating can preferentially affect ion parallel temperature (as well as the electron parallel temperature) at small values of β .
- A diagram of the plasma state in (β_{//}, T_⊥/T_{//}) plane, similar to the one observed in the slow solar wind, is obtained, especially in the presence of a weak level of collisions that helps to constrain the plasma close the mirror instability threshold.

Forthcoming developments: Three-dimensional FLR-Landau fluid simulations.