On the generation of oblique high-frequency waves by ion beams in the solar wind

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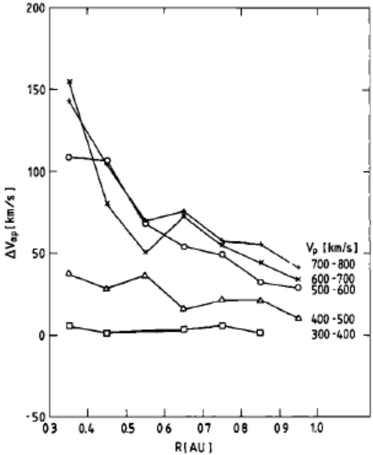
- Drift between protons and alpha particles in the collisionless fast solar wind
- Deceleration during transit through heliosphere
- Upper threshold: $\sim v_{\mathrm{A}}$



(Marsch et al., 1982)

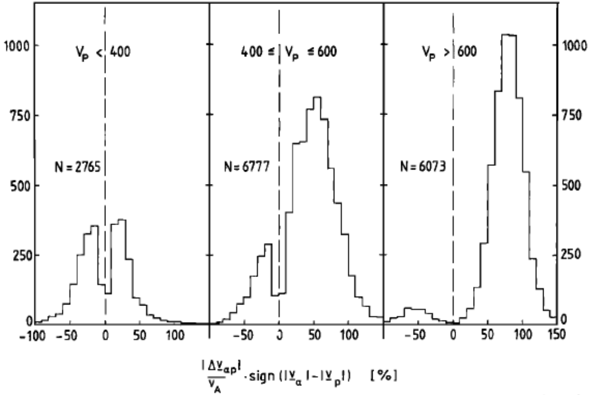
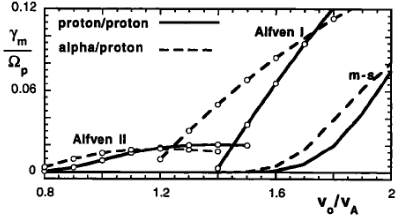


Fig. 11. Histograms of the ion differential speed in percentage of the local Alfvén speed for low, intermediate, and high-velocity streams.

(Marsch et al., 1982)

Alpha beam in the solar wind
Hybrid simulations

- The deceleration has not yet been fully understood
- Hybrid simulations can model the non-equilibrium physics of streaming ion species
- Several "streaming instabilities" have been found
- Instability thresholds: $\sim 1.7 v_{\mathrm{A}}$ for parallel propagation, $\sim v_{\mathrm{A}}$ for oblique propagation



(Gary et al., 2000)

$$\frac{\partial f}{\partial t} = \lim_{V\to\infty} \sum_{n=-\infty}^{+\infty} \frac{q^2}{8\pi^2 m^2} \int \frac{1}{V v_\perp} G v_\perp \delta(\omega_{\mathrm{r}} - k_\parallel v_\parallel - n\Omega) \times \\ \times \left|\psi_{n,k}\right|^2 G f \mathrm{d}^3 k$$

with the operator

$$G \equiv \left(1 - \frac{k_\parallel v_\parallel}{\omega_{\mathrm{r}}}\right) \frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega_{\mathrm{r}}} \frac{\partial}{\partial v_\parallel}$$

and the amplitude function

$$\psi_{n,k} = \frac{1}{\sqrt{2}} \left[E_{k,r} e^{i\phi} J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) + E_{k,l} e^{-i\phi} J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\right] \\ + \frac{v_\parallel}{v_\perp} E_{kz} J_n \left(\frac{k_\perp v_\perp}{\Omega}\right)$$

The delta function

$$\delta(\omega_{\mathrm{r}} - k_{\parallel} v_{\parallel} - n\Omega)$$

determines the resonant particles. Only particles with the resonance speed

$$v_{\parallel} = v_{\mathrm{R},n} \equiv \frac{\omega_{\mathrm{r}} - n\Omega}{k_{\parallel}}$$

of order n can resonate.

The characteristics of the operator

$$G \equiv \left(1 - \frac{k_\| v_\|}{\omega_\mathrm{r}}\right) \frac{\partial}{\partial v_\perp} + \frac{k_\| v_\perp}{\omega_\mathrm{r}} \frac{\partial}{\partial v_\|}$$

with constant phase speed $v_\mathrm{ph} = \omega_\mathrm{r}/k_\|$ are circles in velocity space with

$$\left(v_\| - v_\mathrm{ph}\right)^2 + v_\perp^2 = \mathrm{const.}$$

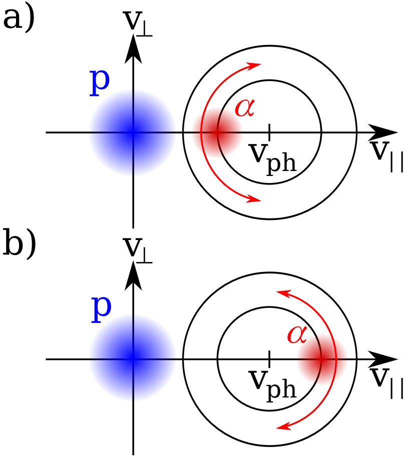
Therefore, the quasilinear evolution leads to a diffusion along circles around the phase speed of the interacting wave.

The amplitude function

$$\psi_{n,k} = \frac{1}{\sqrt{2}}\left[E_{k,r}e^{i\phi}J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) + E_{k,l}e^{-i\phi}J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\right] + \frac{v_\parallel}{v_\perp}E_{kz}J_n\left(\frac{k_\perp v_\perp}{\Omega}\right)$$

has for parallel propagation (since $J_i(0) = \delta_{i,0}$) only cyclotron-resonant contributions from $E_{k,r}$ for $n = -1$, from $E_{k,l}$ for $n = +1$, and Landau-resonant contributions from E_{kz} for $n = 0$. Higher orders are weak.
$E_{k,r} \equiv \frac{E_{kx} - iE_{ky}}{\sqrt{2}}$, $E_{k,l} \equiv \frac{E_{kx} + iE_{ky}}{\sqrt{2}}$

If the particles lose energy, the wave gains energy. That is an instability.



The wave-energy state

The wave energy (averaged over many periods) is given by

$$W = \frac{1}{8\pi}\left[\boldsymbol{B}_{\boldsymbol{k}}^{*}\cdot\boldsymbol{B}_{\boldsymbol{k}} + \boldsymbol{E}_{\boldsymbol{k}}^{*}\cdot\frac{\partial}{\partial\omega}(\omega\varepsilon_{\mathrm{h}})\boldsymbol{E}_{\boldsymbol{k}}\right]\Bigg|_{\omega=\omega_{\mathrm{r}}}.$$

This term can become negative depending on the contribution of the medium (due to $\frac{\partial}{\partial\omega}\varepsilon_{\mathrm{h}}$).

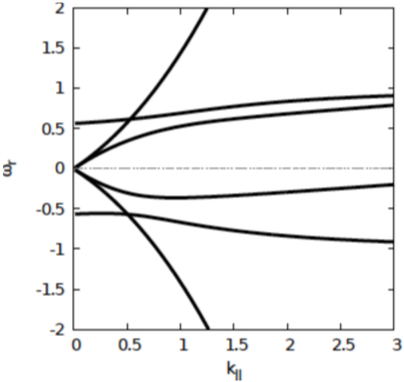
- Can only happen in media that are not in thermodynamic equilibrium
- Amplitude growth leads to decrease of energy of the medium
- Dissipation leads to growth instead of damping
- Total energy is (of course) conserved

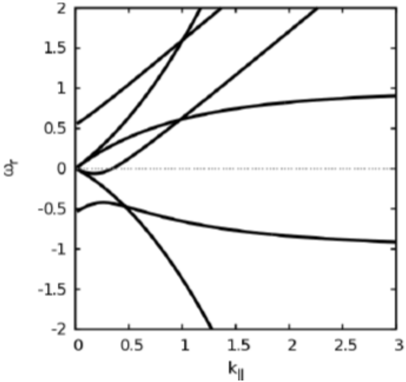
Instability thresholds

The following rules apply:

1. $0 < \omega_r / k_{\parallel} < U_i$
2. The resonances with $n = \pm 1$ and $n = 0$ of the alpha particles are the only relevant resonances
3. The resonance $n = +1$ needs a mode with $E_{k,l} \neq 0$, the resonance $n = -1$ needs a mode $E_{k,r} \neq 0$, and $n = 0$ needs $E_{k,z} \neq 0$
4. Modes with left-handed contributions are not unstable in the range of the proton resonance at $\omega_r \approx \Omega_p$
5. A wave, which is unstable according to points 1 through 4, is damped as long as $W < 0$ and vice versa

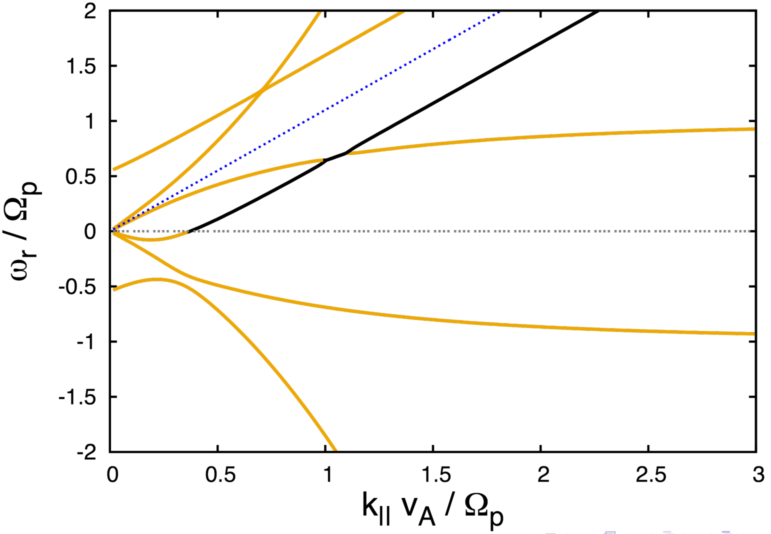
Examples of the dispersion





left: $U_{\rm i} = 0.1$, right: $U_{\rm i} = 1.1$

The wave-energy state



Cold plasma dispersion and polarization
Parallel case

Cold plasma dispersion and polarization

Parallel case with higher beta

Results

- The parallel magnetosonic instability threshold is at $U_{\mathrm{i}} \gtrsim 1.7 v_{\mathrm{A}}$ in our case
- It requires quite high (parallel alpha) betas
- The Alfvén mode is stable in parallel propagation
- In oblique propagation, its polarization is modified, and Alfvén I is unstable for $U_{\mathrm{i}} \gtrsim 1.2 v_{\mathrm{A}}$. For higher betas even earlier (be careful with dispersion!)
- It is also unstable at low wavenumbers due to Landau resonance (Alfvén II)
- Typical wavenumbers are in the range $k_{\|} v_{\mathrm{A}} / \Omega_{\mathrm{p}} = 0.2 \ldots 0.9$, and the frequencies are in the range $\omega / \Omega_{\mathrm{p}} = 0.2 \ldots 1.3$

Literature

S. P. Gary, L. Yin, D. Winske, and D. B. Reisenfeld. Electromagnetic alpha/proton instabilities in the solar wind. *Geophys. Res. Lett.*, 27:1355, May 2000. doi: 10.1029/2000GL000019.

E. Marsch, H. Rosenbauer, R. Schwenn, K.-H. Muehlhaeuser, and F. M. Neubauer. Solar wind helium ions - Observations of the HELIOS solar probes between 0.3 and 1 AU. *J. Geophys. Res.*, 87:35–51, January 1982. doi: 10.1029/JA087iA01p00035.

We use the dispersion of a cold plasma:

$$\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{E_k})+\varepsilon\boldsymbol{E_k} = \begin{pmatrix} S-n_z & -iD & J+n_xn_z \\ iD & S-n^2 & M \\ J+n_xn_z & -M & P-n_x^2 \end{pmatrix}\begin{pmatrix} E_{kx} \\ E_{ky} \\ E_{kz} \end{pmatrix} = 0$$

with

$$\varepsilon = \begin{pmatrix} S & -iD & J \\ iD & S & M \\ J & -M & P \end{pmatrix},$$

Summary
The unstable modes and their properties

Name	Resonance	Threshold	$\beta_{\parallel}$	Growth Rate
Parallel magnetosonic	$n = -1$	$\sim 1.7 v_{\mathrm{A}}$	high	high
Oblique magnetosonic	$n = 0$	$\sim 1.5 v_{\mathrm{A}}$	low	low
Alfvén I	$n = -1$	$\sim 1.2 v_{\mathrm{A}}$	low	high
Alfvén II	$n = 0$	$\sim 0.8 v_{\mathrm{A}}$	low	low